Solving Absolute Value Equations

STEP 1:

$|x-17| = 19$

Look at your equation.
Is the absolute value isolated?

YES → (circled)

NO →

Isolate the absolute value.

Rewrite the equation so that the absolute value is completely isolated (all by itself).

STEP 2:
Is the absolute value equal to a negative number?

NO → (circled)

YES →

STEP 3:

Split your isolated absolute value equation into two regular equations by taking the inside of the absolute value and setting it equal to the positive and negative version of the other side of the equation.

$x-17=19$ $\quad$ $x-17=-19$

An absolute value is always great than or equal to zero, therefore it can never equal a negative number!

There is NO solution to this equation! We write:

$\emptyset$

This is called the "Empty Set." It is a solution set that contains no solutions!

STEP 4:

Solve both equations.

$x-17=19$
$+17 \quad +17$
$x=36$

$x-17=-19$
$+17 \quad +17$
$x=-2$

STEP 5:

Check to see if any of your solutions are extraneous. Write your solution(s) in a solution set of the form x = { , }.

$|36-17|=19$
$|19|=19$
$19=19$

$|-2-17|=19$
$|-19|=19$
$19=19$

$x=\{36,-2\}$

Adapted from Stroh, A. (2019, September 21). Algebra 1 – unit 1 interactive notebook pages | the foundations of algebra. Math by the mountain. https://mathbythemountain.com/category/algebra-1/

Solving Absolute Value Equations

STEP 1:

$|x+2| = 5$

Look at your equation.
Is the absolute value isolated?

YES → NO

Isolate the absolute value.

Rewrite the equation so that the absolute value is completely isolated (all by itself).

STEP 2:
Is the absolute value equal to a negative number?

NO → YES

An absolute value is always great than or equal to zero, therefore it can never equal a negative number!

There is NO solution to this equation! We write:

$\emptyset$

This is called the "Empty Set." It is a solution set that contains no solutions!

STEP 3:
Split your isolated absolute value equation into two regular equations by taking the inside of the absolute value and setting it equal to the positive and negative version of the other side of the equation.

$x+2 = 5$ $\qquad$ $x+2 = -5$

STEP 4:
Solve both equations.

$x+2=5$
$-2 \quad -2$
$x = 3$

$x+2=-5$
$-2 \quad -2$
$x = -7$

STEP 5:
Check to see if any of your solutions are extraneous. Write your solution(s) in a solution set of the form x = { , }.

$|3+2| = 5$
$|5| = 5$
$5 = 5$

$|-7+2| = 5$
$|-5| = 5$
$5 = 5$

$x = \{3, -7\}$

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Solving Absolute Value Equations

STEP 1:

$|-3x+1|+12=8$

Look at your equation.
Is the absolute value isolated?

YES

NO

Isolate the absolute value.

Rewrite the equation so that the absolute value is completely isolated (all by itself).

$|-3x+1|+12=8$
$\quad -12 \quad -12$
$|-3x+1|=-4$

STEP 2:
Is the absolute value equal to a negative number?

NO

YES

STEP 3:
Split your isolated absolute value equation into two regular equations by taking the inside of the absolute value and setting it equal to the positive and negative version of the other side of the equation.

An absolute value is always great than or equal to zero, therefore it can never equal a negative number!

There is NO solution to this equation! We write:

$\emptyset$

This is called the "Empty Set." It is a solution set that contains no solutions!

STEP 4:
Solve both equations.

STEP 5:
Check to see if any of your solutions are extraneous. Write your solution(s) in a solution set of the form x = { , }.

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Solving Absolute Value Equations

STEP 1:

$2|3x-4|-8=6$

Look at your equation.
Is the absolute value isolated?

YES ← | → NO

STEP 2:
Is the absolute value equal to a negative number?

NO | YES

Isolate the absolute value.
Rewrite the equation so that the absolute value is completely isolated (all by itself).

$2|3x-4|-8=6$
$+8 \quad +8$
$2|3x-4| = 14$
$\div 2 \quad \div 2$
$|3x-4| = 7$

STEP 3:
Split your isolated absolute value equation into two regular equations by taking the inside of the absolute value and setting it equal to the positive and negative version of the other side of the equation.

$3x-4=7$ $\qquad$ $3x-4=-7$

An absolute value is always great than or equal to zero, therefore it can never equal a negative number!

There is NO solution to this equation! We write:

$\emptyset$

This is called the "Empty Set." It is a solution set that contains no solutions!

STEP 4:
Solve both equations.

$3x-4=7$
$+4 \quad +4$
$\frac{3x}{3} = \frac{11}{3}$
$x = \frac{11}{3}$

$3x-4=-7$
$+4 \quad +4$
$\frac{3x}{3} = \frac{-3}{3}$
$x = -1$

STEP 5:
Check to see if any of your solutions are extraneous. Write your solution(s) in a solution set of the form x = { , }.

$2|3(\frac{11}{3})-4|-8=6$
$2|11-4|-8=6$
$2|7|-8=6$
$2(7)-8=6$
$14-8=6$
$6=6$

$2|3(-1)-4|-8=6$
$2|-3-4|-8=6$
$2|-7|-8=6$
$2(7)-8=6$
$14-8=6$
$6=6$

$x = \{\frac{11}{3}, -1\}$

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Solving Absolute Value Equations

STEP 1:

$-3|2x+1|-3=-9$

Look at your equation.
Is the absolute value isolated?

YES ← | → NO

Isolate the absolute value.

Rewrite the equation so that the absolute value is completely isolated (all by itself).

$$-3|2x+1|-3=-9$$
$$+3 \quad +3$$
$$\frac{-3|2x+1|}{-3}=\frac{-6}{-3}$$
$$|2x+1|=2$$

STEP 2:
Is the absolute value equal to a negative number?

NO → **STEP 3:**

Split your isolated absolute value equation into two regular equations by taking the inside of the absolute value and setting it equal to the positive and negative version of the other side of the equation.

$2x+1=2$ $\qquad$ $2x+1=-2$

YES → **An absolute value is always great than or equal to zero, therefore it can never equal a negative number!**

There is NO solution to this equation! We write:

$\emptyset$

This is called the "Empty Set." It is a solution set that contains no solutions!

STEP 4:
Solve both equations.

$$2x+1=2$$
$$-1 \quad -1$$
$$\frac{2x}{2}=\frac{1}{2}$$
$$x=\frac{1}{2}$$

$$2x+1=-2$$
$$-1 \quad -1$$
$$\frac{2x}{2}=\frac{-3}{2}$$
$$x=-\frac{3}{2}$$

STEP 5:
Check to see if any of your solutions are extraneous. Write your solution(s) in a solution set of the form x = { , }.

$-3|2(\frac{1}{2})+1|-3=-9$
$-3|1+1|-3=-9$
$-3|2|-3=-9$
$-3(2)-3=-9$
$-6-3=-9$
$-9=-9$

$-3|2(-\frac{3}{2})+1|-3=-9$
$-3|-3+1|-3=-9$
$-3|-2|-3=-9$
$-3(2)-3=-9$
$-6-3=-9$
$-9=-9$

$x=\{\frac{1}{2}, -\frac{3}{2}\}$

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