

# Department of the Exterior Angles

## Polygons and the Exterior Angle Sum Theorem

K20 Center, Alexandra Parsons, Teresa Lansford, Michell Eike
Published by *K20 Center*

*This work is licensed under a Creative Commons CC BY-SA 4.0 License*

| | | | |
|---|---|---|---|
| **Grade Level** | 9th – 10th Grade | **Time Frame** | 80 minutes |
| **Subject** | Mathematics | **Duration** | 2 class periods |
| **Course** | Geometry | | |

## Essential Question

How do we solve problems using exterior angles?

## Summary

In this lesson, students will explore the relationships between exterior angles in regular and irregular polygons to determine the sum of exterior angles, and they will apply that knowledge to solve problems. To complete this lesson, students need to know the interior angle sum theorem. See the "Department of the Interior Angles" lesson for the prerequisite content.

## Snapshot

### Engage

Students recall properties of interior and exterior angles of a polygon to solve a problem as a group, using a modified Chain Notes strategy.

### Explore

Students discover the sum of the exterior angles of a polygon.

### Explain

Students formalize their understanding of finding the sum of the exterior angles of a polygon. Students also learn how those angles relate to interior angles and to the number of sides a polygon has.

### Extend

Students apply what they have learned to a Polygon Puzzle that requires the use of interior and exterior angle sums, results from two parallel lines cut by a transversal, and other prior knowledge of angles.

### Evaluate

Students reflect on their thinking processes for solving multi-step problems and use the Exit Ticket strategy to compare their approaches with those of their peers.

## Standards

*ACT College and Career Readiness Standards - Mathematics (6-12)*

**G402:** Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)

*Oklahoma Academic Standards Mathematics (Geometry)*

**G.2D.1.4:** Apply theorems involving the interior and exterior angle sums of polygons to solve problems using mathematical models, algebraic reasoning, and proofs.

## Attachments

- Lesson Slides (Face-to-Face)—Department of the Exterior Angles.pptx
- Lesson Slides (Hybrid)—Department of the Exterior Angles.pptx
- Pass It On—Department of the Exterior Angles - Spanish.docx
- Pass It On—Department of the Exterior Angles - Spanish.pdf
- Pass It On—Department of the Exterior Angles.docx
- Pass It On—Department of the Exterior Angles.pdf
- Perspectives—Department of the Exterior Angles - Spanish.docx
- Perspectives—Department of the Exterior Angles - Spanish.pdf
- Perspectives—Department of the Exterior Angles.docx
- Perspectives—Department of the Exterior Angles.pdf
- Playing With Polygons—Department of the Exterior Angles - Spanish.docx
- Playing With Polygons—Department of the Exterior Angles - Spanish.pdf
- Playing With Polygons—Department of the Exterior Angles.docx
- Playing With Polygons—Department of the Exterior Angles.pdf
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles - Spanish.docx
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles - Spanish.pdf
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles.docx
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles.pdf
- Polygon Patterns (Hybrid)—Department of the Exterior Angles - Spanish.docx
- Polygon Patterns (Hybrid)—Department of the Exterior Angles - Spanish.pdf
- Polygon Patterns (Hybrid)—Department of the Exterior Angles.docx
- Polygon Patterns (Hybrid)—Department of the Exterior Angles.pdf
- Polygon Puzzle (Sample Responses)—Department of the Exterior Angles.docx
- Polygon Puzzle (Sample Responses)—Department of the Exterior Angles.pdf
- Polygon Puzzle—Department of the Exterior Angles - Spanish.docx
- Polygon Puzzle—Department of the Exterior Angles - Spanish.pdf
- Polygon Puzzle—Department of the Exterior Angles.docx
- Polygon Puzzle—Department of the Exterior Angles.pdf

## Materials

- Lesson Slides (Face-to-Face) (attached)
- Pass It On handout (attached; one per group; print one-sided)
- Playing With Polygons handout (attached; one per student; print one-sided)
- Polygon Patterns (Face-to-Face) handout (attached; one per student; print two-sided)
- Polygon Puzzle handout (attached; one per pair; print one-sided)
- Polygon Puzzle (Sample Responses) document (attached)
- Perspectives handout (attached; one half page per student; print one-sided)
- Paper
- Pencil
- Colored pencils

- Straight edge
- Scissors

10 minutes

# Engage

Introduce the lesson using the attached **Lesson Slides (Face-to-Face)**. Display **slide 3** to share the lesson's essential question with students, and then move to **slide 4** to go over the lesson's learning objectives.

Display **slide 5** and place students in groups of 3–4. Give each group a copy of the attached **Pass It On** handout. Using a modified Chain Notes strategy, have students view the image on the handout and take turns identifying and writing or labeling what they know about the given polygon.

Ask students to select one member of each group to go first—this person should record one observation on the paper and pass it to the person on their right, repeating until all group members have recorded their observations. Remind students that they need to justify what they write.

### Teacher's Note: Guiding the Lesson

Students are likely to create some type of naming system for the angles to help them communicate as they write. Do not tell students exactly how to label their unnamed angles. Students are welcome to be creative with their labeling methods.

Have students continue passing the paper and adding information until they have recorded everything they can think of related to the given polygon.

Once all groups have finished their lists, give them time to discuss what they have written. Bring the class back together and have students share what their groups observed about the polygon.

### Sample Student Responses

There are many ways for students to phrase their justifications. Push students to use academic, mathematical vocabulary.

The top angle is 90° since the square implies that the angle is 90°. The interior angles corresponding to those angles are also 90°, because they are supplementary angles. The sum of the interior angles of a pentagon is $(5 - 2) \cdot 180° = 540°$ because of the sum of interior angles formula. If we know that three of the interior angles are 90°, then $540° - 3(90°) = 540° - 270° = 270°$; so the two unknown interior angles are half of 270°, which is 135°. If those interior angles are 135°, then their supplementary angle, $x°$, must be 45° because $135° + 45° = 180°$.

30 minutes

# Explore

**Teacher's Note: Activity Preparation**

Each student needs scissors, colored pencils, and a straight edge for this activity.

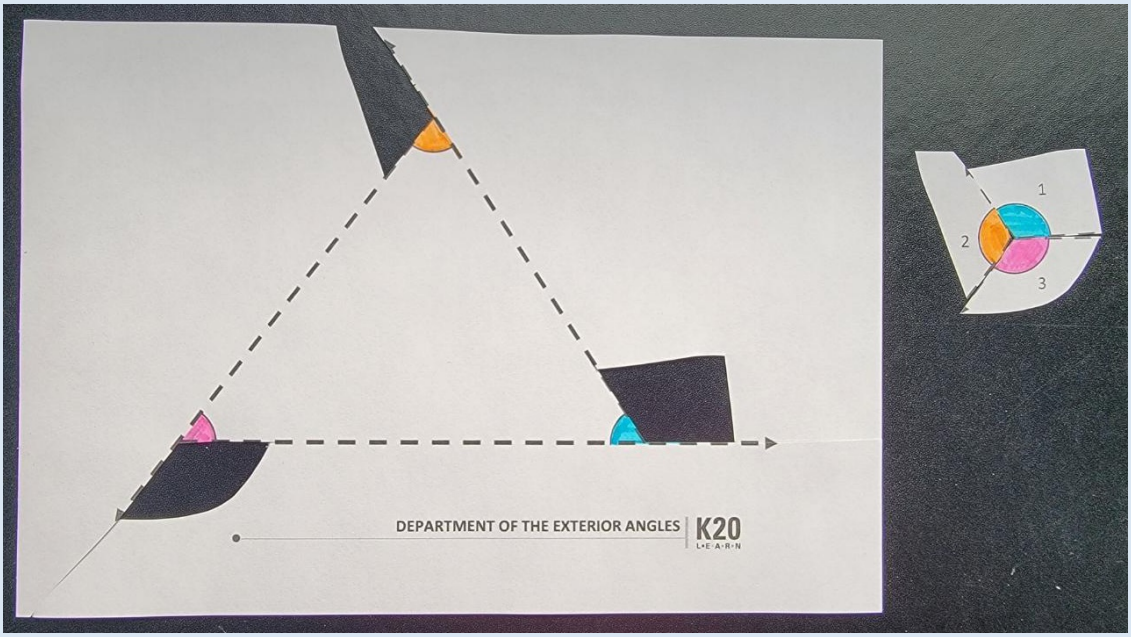
Students end this portion of the lesson with a Gallery Walk. Students can display their final products in the center of their work stations. Alternatively, students could display their final products elsewhere in your classroom. Make a decision ahead of time so that students know where to put their work.

Display **slide 6** and give each student a copy of the attached **Playing with Polygons** handout, scissors, colored pencils, and a straight edge.

Remind students to carefully read all the directions on the handout *before using scissors*. Then, have students follow the step-by-step directions for cutting out a polygon's exterior angles and putting their vertices together.

Once students have completed the activity using the triangle on the handout, have them share what shape the exterior angles created.

**Sample Student Response**




After discussing the shape formed by the triangle's exterior angles, show **slide 7** and ask students, "How could this change if the number of sides changes?" Have students discuss their predictions with their groups from the previous activity. After students have taken time to discuss, ask for volunteers to share their predictions.

Display **slide 8** and assign each group a type of polygon: a quadrilateral, a pentagon, a hexagon, etc. Pass out paper for each student to create their own regular or irregular polygon. (It is unlikely that two students in one group create the exact same polygon, but you may encourage students to vary their drawings if you wish.)

Have students follow the handout's step-by-step directions for creating a polygon, cutting out its exterior angles, and putting their vertices together. Steps 1–3 are on slide 8. Once groups are done with their straight edges, move to **slide 9** to show steps 4–6. Use these slides for easy reference and to give students a sense of pacing for the activity.

### Teacher's Note: Differentiation

Consider assigning polygons with more sides to students who need a challenge. This also could help with lesson pacing.

### Potential Student Question

Students are likely to ask if it matters how they extend the edges of their polygons—clockwise or counterclockwise. It does not matter as long as they are consistent. Consider having half of each group extend the legs of their polygons clockwise and having the other half of each group extend the legs of their polygons counterclockwise; then, compare the results.

Once students have completed the activity using their assigned polygon types, ask students to display their final products at the center of their work stations or in another location you have previously designated. Have groups title their collection of work with the name of their assigned polygon type.

Show **slide 10** and give each student a copy of the attached **Polygon Patterns (Face-to-Face)** handout. Have students participate in a Gallery Walk to look at four other groups' sets of exterior angles. Ask students to record their observations in the tables on their handouts.

When students have finished the Gallery Walk, have them rejoin their own groups to discuss their observations and notes.

10 minutes

# Explain

As groups wrap up their discussions from the Gallery Walk, display **slide 11** and ask students to answer questions 1–4 on the back of the Polygon Patterns handout. Have students write their own definitions of *exterior angles* based on:

- Their observations,
- The relationship between the number of sides a polygon has and the number of exterior angles it has,
- The relationship between corresponding exterior and interior angles, and
- The relationship between the number of sides a polygon has and the sum of its exterior angles.

### Teacher's Note: Guiding the Lesson

Based on the Gallery Walk and group discussions, students should conclude that there are the same number of exterior angles as there are sides of a polygon, and that no matter how many sides a polygon has, the sum of its exterior angles is always 360°.

When students are finished writing, ask for volunteers to share their conclusions from the handout.

Show **slide 12** and play the *Exterior Angles* video on the slide so that students can see why a polygon's exterior angles always have a sum of 360°.

**Embedded video**
https://youtube.com/watch?v=VYbiE9sDXXk

### Optional Word Wall

If you have a classroom word wall—or if you are having each student maintain a personal dictionary in their interactive notebook—this lesson has some great words to add:

- Alternate interior angles
- Alternate exterior angles
- Corresponding angles
- Supplementary angles
- Vertical angles
- Triangle sum theorem
- Polygon interior angle sum theorem
- Polygon exterior angle sum theorem
- Parallel lines
- Transversal

25 minutes

# Extend

Have students find a partner or assign partners. Display **slide 13** and give each pair a copy of the attached **Polygon Puzzle** handout.

### Teacher's Note: Guiding the Lesson

Give students time to struggle with this problem—it is meant to be challenging. Encourage students to use all their previous learning to solve the problem. Ask them to think about the strategies they have learned to find unknown angles. Solving this puzzle requires students to recall their prior knowledge of interior and exterior angle sums, parallel lines cut by a transversal, etc.

### Sample Student Responses

Use the attached **Polygon Puzzle (Sample Responses)** document as needed.

5 minutes

# Evaluate

Show **slide 14** and have each student find a new partner (not the person they worked with on the Polygon Puzzle).

Use the Exit Ticket strategy to assess students' understanding of the lesson. Give each student one half-sheet of the attached **Perspectives** handout.

With their new partners, ask students to discuss how they each solved the Polygon Puzzle and compare their strategies for solving it. Have students record the similarities and differences between their approaches on the handout.

### Teacher's Note: Checking for Understanding

As you collect the handouts, look at student responses to see which misunderstandings persist. Additionally, consider using student responses to showcase a unique strategy someone used to solve the puzzle. Students need to see the value in having more than one correct way to solve a problem.

## Resources


- Brzezinski, T. and Eike, M. (n.d.). Department of the exterior angles. GeoGebra. https://www.geogebra.org/m/we6ww7cz
- K20 Center. (n.d.). Amplify classroom. Tech Tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Chain notes. Strategies. https://learn.k20center.ou.edu/strategy/52
- K20 Center. (n.d.). Gallery walk / carousel. Strategies. https://learn.k20center.ou.edu/strategy/118
- K20 Center. (n.d.). GeoGebra. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2352
- VanHattum, S. (2019, April 14). *Exterior angles, by magic pi - math animations* [Video]. YouTube. https://www.youtube.com/watch?v=VYbiE9sDXXk