Department of the Exterior Angles

Polygons and the Exterior Angle Sum Theorem

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Grade Level	9th – 10th Grade	**Time Frame**	80 minutes
Subject	Mathematics	**Duration**	1–2 class period(s)
Course	Geometry		

Essential Question

How do we solve problems using exterior angles?

Summary

Students will explore the relationships between exterior angles in regular and irregular polygons to determine the sum of exterior angles, and they will apply that knowledge to solve problems. To complete this lesson, students need to know the interior angle sum theorem. See the "Department of the Interior Angles" lesson for the prerequisite content.

Snapshot

Engage

Students recall properties of interior and exterior angles of a polygon to solve a problem as a group, using a modified Chain Notes strategy.

Explore

Students discover the sum of the exterior angles of a polygon.

Explain

Students formalize their understanding of finding the sum of the exterior angles of a polygon. Students also learn how those angles relate to interior angles and to the number of sides a polygon has.

Extend

Students apply what they have learned to a Polygon Puzzle that requires the use of interior and exterior angle sums, results from two parallel lines cut by a transversal, and other prior knowledge of angles.

Evaluate

Students reflect on their thinking processes for solving multi-step problems and use the Exit Ticket strategy to compare their approaches with those of their peers.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G401: Use properties of parallel lines to find the measure of an angle

Oklahoma Academic Standards Mathematics (Geometry)

G.2D.1.4: Apply theorems involving the interior and exterior angle sums of polygons to solve problems using mathematical models, algebraic reasoning, and proofs.

Attachments

- Lesson Slides (Face-to-Face)—Department of the Exterior Angles.pptx
- Lesson Slides (Hybrid)—Department of the Exterior Angles.pptx
- Pass It On—Department of the Exterior Angles - Spanish.docx
- Pass It On—Department of the Exterior Angles - Spanish.pdf
- Pass It On—Department of the Exterior Angles .docx
- Pass It On—Department of the Exterior Angles .pdf
- Perspectives—Department of the Exterior Angles - Spanish.docx
- Perspectives—Department of the Exterior Angles - Spanish.pdf
- Perspectives—Department of the Exterior Angles.docx
- Perspectives—Department of the Exterior Angles.pdf
- Playing With Polygons—Department of the Exterior Angles - Spanish.docx
- Playing With Polygons—Department of the Exterior Angles - Spanish.pdf
- Playing With Polygons—Department of the Exterior Angles.docx
- Playing With Polygons—Department of the Exterior Angles.pdf
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles - Spanish.docx
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles - Spanish.pdf
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles.docx
- Polygon Patterns (Face-to-Face)—Department of the Exterior Angles.pdf
- Polygon Patterns (Hybrid)–Department of the Exterior Angles.docx
- Polygon Patterns (Hybrid)–Department of the Exterior Angles.pdf
- Polygon Patterns (Hybrid)—Department of the Exterior Angles - Spanish.docx
- Polygon Patterns (Hybrid)—Department of the Exterior Angles - Spanish.pdf
- Polygon Puzzle Sample Responses—Department of the Exterior Angles.docx
- Polygon Puzzle Sample Responses—Department of the Exterior Angles.pdf
- Polygon Puzzle–Department of the Exterior Angles - Spanish.docx
- Polygon Puzzle–Department of the Exterior Angles - Spanish.pdf
- Polygon Puzzle—Department of the Exterior Angles.docx
- Polygon Puzzle—Department of the Exterior Angles.pdf

Materials

- Lesson Slides (Hybrid) (attached)
- Pass It On handout (attached; one per group; printed front only)
- Polygon Patterns (Hybrid) handout (attached; one per student; printed front/back)
- Polygon Puzzle handout (attached; one per student pair; printed front only)
- Polygon Puzzle (Sample Responses) (attached; for teacher use)
- Perspectives handout (attached; one half-sheet per student; printed front only)
- Pencil
- Student devices with internet access

10 minutes

Engage

Optional Technology Integration

This lesson can be integrated into NearPod, Desmos Classroom, discussion boards, or any other learning platform you choose to use with students.

If you wish, you may give students digital copies of the attached **Pass It On** handout for the following activity. Alternatively, you may upload the image and instructions from the handout to your learning platform of choice and have students participate in a discussion board.

Introduce the lesson using the attached **Lesson Slides (Hybrid)**. Display **slide 3** to share the lesson's essential question with students, and move to **slide 4** to go over the lesson's learning objectives.

Display **slide 5** and place students in groups of 3–4. Give each group a copy of the attached **Pass It On** handout. Using a modified Chain Notes strategy, have students view the image on the handout and take turns identifying and writing or labeling what they know about the given polygon.

Ask students to select one member of each group to go first—this person should record one observation on the paper and pass it to the person on their right, repeating until all group members have recorded their observations. Remind students that they need to justify what they write.

Teacher's Note: Guiding the Lesson

Students are likely to create some type of naming system for the angles to help them communicate as they write. Do not tell students exactly how to label their unnamed angles. Students are welcome to be creative with their labeling methods.

Have students continue passing the paper and adding information until they have recorded everything they can think of related to the given polygon.

Once all groups have finished their lists, give them time to discuss what they have written. Bring the class back together and have students share out what their groups observed about the polygon.

Sample Student Responses

There are many ways for students to phrase their justifications. Push students to use academic, mathematical vocabulary.

The top angle is 90° since the square implies that the angle is 90°. The interior angles corresponding to those angles are also 90° because they are supplementary angles. The sum of the interior angles of a pentagon is (5-2)·180° = 540° because of the sum of interior angles formula. If we know that three of the interior angles are 90°, then 540° - 3(90°) = 540° - 270° = 270°; so the two unknown interior angles are half of 270°, which is 135°. If those interior angles are 135°, then their supplementary angle, x°, must be 45° because 135° + 45° = 180°.

30 minutes

Explore

Teacher's Note: Activity Preparation

For this activity, each student needs a device with internet access to explore the GeoGebra activity and interact with the applets.

Optional Technology Integration

If you wish, you may give students digital copies of the attached **Polygon Patterns (Hybrid)** handout for the following activity.

Display **slide 6** and give each student a copy of the attached **Polygon Patterns (Hybrid)** handout. Provide students with the link to the GeoGebra activity: geogebra.org/m/we6ww7cz. Have them follow the directions for each applet.

The GeoGebra activity is a series of interactive applets in which students can explore the relationships between interior and exterior angles and the number of sides a polygon has. Students also can adjust the lengths of the sides and the interior and exterior angles of the polygons. They can then see how their adjustments affect the sum of the exterior angles.

Remind students to move the points at least three times in each applet before drawing any conclusions. Ask students to record their observations of each polygon in the tables on their handouts.

Teacher's Note: Formative Assessment

The GeoGebra activity is useful for visualizing the concepts, but it does not guide students or provide the correct answers. Monitor students' learning by checking in, seeing their answers, and remediating if needed.

Once students have completed the GeoGebra activity and have recorded their observations, have them discuss their findings with their groups from the previous activity.

10 minutes

Explain

As groups wrap up their discussions from the GeoGebra activity, ask students to answer questions 1–5 on the Polygon Patterns (Hybrid) handout. Have students write their own definitions of exterior angles based on:

- Their observations,
- The relationship between the number of sides a polygon has and the number of exterior angles it has,
- The relationship between corresponding exterior and interior angles, and
- The relationship between the number of sides a polygon has and the sum of its exterior angles.

Teacher's Note: Guiding the Lesson

Based on their interaction with the GeoGebra activity and group discussions, students should conclude that there are the same number of exterior angles as there are sides of a polygon, and that no matter how many sides a polygon has, the sum of its exterior angles is always 360°.

When students are done writing, ask for volunteers to share their conclusions from the handout.

Display **slide 7** and play the following video, titled "Exterior angles, by magic pi - math animations," so that students can see why a polygon's exterior angles always have a sum of 360°.

Embedded video
https://youtube.com/watch?v=VYbiE9sDXXk

Optional Word Wall

If you have a classroom word wall—or if you're having each student maintain a personal dictionary in their interactive notebook—this lesson has some great words to add:

- Alternate interior angles
- Alternate exterior angles
- Corresponding angles
- Supplementary angles
- Vertical angles
- Triangle sum theorem
- Polygon interior angle sum theorem
- Polygon exterior angle sum theorem
- Parallel lines
- Transversal

25 minutes

Extend

Have students work in pairs for this activity.

Display **slide 8** and pass out one copy of the attached **Polygon Puzzle** handout to each pair of students.

Teacher's Note: Guiding the Lesson

Give students time to struggle with this problem—it is meant to be challenging. Encourage students to use all their previous learning to solve the problem. Ask them to think about the strategies they have learned to find unknown angles. Solving this puzzle requires students to recall their prior knowledge of interior and exterior angle sums, parallel lines cut by a transversal, etc.

Sample Student Responses

If needed, use the attached **Polygon Puzzle (Sample Responses)** document as a teaching resource.

5 minutes

Evaluate

Optional Technology Integration

If you wish, you may give students digital copies of the attached **Perspectives** handout for the following activity.

Display **slide 9** and have each student find a new partner (not the person they worked with on the Polygon Puzzle).

Inform students they are going to use the Exit Ticket strategy to assess their understanding of the lesson. Give each student a half-sheet from the attached **Perspectives** handout.

With their new partners, ask students to discuss how they each solved the Polygon Puzzle and compare their strategies for solving it. Have students record the similarities and differences between their approaches on the handout.

Teacher's Note: Checking for Understanding

As you collect the handouts, look at student responses to see which misunderstandings persist. Additionally, consider using student responses to showcase a unique strategy someone used to solve the puzzle. Students need to see the value in having more than one correct way to solve a problem.

Resources

- Brzezinski, T. and Eike, M. (n.d.). Department of the exterior angles. GeoGebra. https://www.geogebra.org/m/we6ww7cz
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Chain notes. Strategies. https://learn.k20center.ou.edu/strategy/52
- K20 Center. (n.d.). Gallery walk / carousel. Strategies. https://learn.k20center.ou.edu/strategy/118
- K20 Center. (n.d.). Desmos classroom. Tech Tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). GeoGebra. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2352
- Odintsov, R. (2020, September 16). Black and brown floral glass ceiling [Photograph]. Pexels. https://www.pexels.com/photo/art-pattern-architecture-window-5668546/
- VanHattum, S. (2019, April 14). Exterior angles, by magic pi - math animations [Video]. YouTube. https://www.youtube.com/watch?v=VYbiE9sDXXk