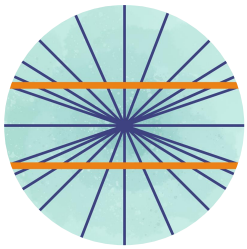

A Parallel Perspective

Writing Equations of Parallel Lines

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Grade Level	8th – 9th Grade	**Time Frame**	70–80 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Algebra 1		

Essential Question

How do you know that a line is parallel to another line?

Summary

The lesson is an extension of the slope-intercept and point-slope forms of linear equations, with a focus on parallel lines. The goal is for students to understand which characteristics make lines parallel and apply that knowledge and their pre-existing knowledge of slope-intercept and point-slope forms to solve problems. After completing the lesson, students will be able to define, recognize, and create parallel lines on graphs and with equations.

Snapshot

Engage

Students judge if they should trust their eyes to determine if lines are parallel.

Explore

Students look at one set of buildings drawn from two different perspectives and determine which lines are parallel.

Explain

Students complete guided notes with the class to confirm that parallel lines have the same slope and learn how to write the equation of a parallel line.

Extend

Students find the equation of a line parallel to their line through an I Have Who Has activity.

Evaluate

Students apply their understanding of parallel lines to find the missing vertex of a parallelogram.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G606: Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point

Oklahoma Academic Standards Mathematics (Algebra 1)

A1.A.4.2: Analyze and interpret mathematical models involving lines that are parallel, perpendicular, horizontal, and vertical.

Attachments

- Exit Ticket—A Parallel Perspective - Spanish.docx
- Exit Ticket—A Parallel Perspective - Spanish.pdf
- Exit Ticket—A Parallel Perspective.docx
- Exit Ticket—A Parallel Perspective.pdf
- Guided Notes (Model Notes)—A Parallel Perspective.docx
- Guided Notes (Model Notes)—A Parallel Perspective.pdf
- Guided Notes—A Parallel Perspective - Spanish.docx
- Guided Notes—A Parallel Perspective - Spanish.pdf
- Guided Notes—A Parallel Perspective.docx
- Guided Notes—A Parallel Perspective.pdf
- I Have Who Has Cards (Bilingual)—A Parallel Perspective.docx
- I Have Who Has Cards (Bilingual)—A Parallel Perspective.pdf
- I Have Who Has Cards—A Parallel Perspective - Spanish.docx
- I Have Who Has Cards—A Parallel Perspective - Spanish.pdf
- I Have Who Has Cards—A Parallel Perspective.docx
- I Have Who Has Cards—A Parallel Perspective.pdf
- Lesson Slides—A Parallel Perspective.pptx
- Parallel Processing—A Parallel Perspective - Spanish.docx
- Parallel Processing—A Parallel Perspective - Spanish.pdf
- Parallel Processing—A Parallel Perspective.docx
- Parallel Processing—A Parallel Perspective.pdf
- Perspective Drawings —A Parallel Perspective - Spanish.docx
- Perspective Drawings —A Parallel Perspective - Spanish.pdf
- Perspective Drawings —A Parallel Perspective.docx
- Perspective Drawings —A Parallel Perspective.pdf

Materials

- Lesson Slides (attached)
- Parallel Processing handout (attached; one per student; print one-sided)
- Perspective Drawings handout (attached; one per student; print one-sided)
- Guided Notes handout (attached; one per student; print one-sided)
- Guided Notes (Model Notes) document (attached)
- I Have Who Has Cards handout (attached; one per class; print one-sided)
- I Have Who Has Cards (Bilingual) handout (optional; attached; one per class; print two-sided)
- Exit Ticket handout (attached; one quarter page per student; print one-sided)
- Rulers
- Paper
- Pencils
- Coloring utensils (highlighters, markers, etc.; optional)

Preparation

In preparation for the Extend phase of this lesson, print the attached **I Have Who Has Cards** handout. Consider printing on card stock paper, especially if you plan to reuse these cards. This activity was designed for a class of 30 students. To meet your class size needs, each page of the document is one set of six cards, so adjust accordingly. For example, if you have a class of 24 students, just print pages 1–4. As students work through the activity, they will naturally form smaller groups of six students/cards. If you do not have even groups, pair students for extra support or give a student more than one card from the same set for a challenge.

Cut out the cards. All of these cards are the same size for easy cutting. Shuffle the deck or arrange the cards in numerical order (1–30) to randomize the deck.

Bilingual Option

If you have Spanish speaking students that need extra support for this activity, you can use the attached **I Have Who Has Cards (Bilingual)** handout, printed two-sided, selecting *long edge* or *portrait* setting. This will create cards that have English on one side and Spanish on the other to support all learners.

20 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** identifies the lesson's essential question: "How do you know that a line is parallel to another line?" **Slide 4** identifies the lesson's primary learning objectives.

Go to **slide 5** and explain to students that they are about to see three images. For each image, they need to determine whether any lines are parallel, and if so, which ones. Take a moment to remind students that parallel lines are lines that do not and will never intersect. Alternatively, ask for a volunteer to remind the class what it means for two lines to be parallel. This is not yet the time to discuss slopes of parallel lines; that will be covered during the Explain phase of the lesson.

Display **slide 6**. Ask the class to quietly think about which lines they think are or are not parallel and why. Ensure that everyone has time to think about the lines in the image.

Teacher's Note: Guiding the Activity

If students ask questions, especially those about whether the lines are straight or not, respond with something like, "What do you think?" At this time, try not to answer those types of questions directly.

The images are optical illusions and the purpose of this activity is to emphasize to students that we cannot trust our eyes to determine if lines are parallel, instead we need to prove that the lines are parallel regardless of if they seem to be parallel. Again, do not yet tell students that they are viewing optical illusions.

After a couple of minutes, ask for 1–2 volunteers to share their thoughts about which lines are or are not parallel for the first image.

Transition through **slides 7–8** and repeat the process with the other two images.

Teacher's Note: Guiding the Activity

Students will have a variety of thoughts regarding which lines are parallel and which lines are not. Keep in mind that at this engaging portion of the lesson, we are asking students what they think and not yet revealing correctness. So, as students share that they think the first image has no parallel lines or the third image has many sets of parallel lines, let students use the discussion as a brainstorming process.

Now, have students find a partner or assign partners and pass out the attached **Parallel Processing** handout to each student. Here, students have the three images printed and closer to them. Tell students that they have about 5 minutes to determine which lines are parallel, if any. Allow students to use whatever resources they think would help them make a decision. Some students may ask to use a ruler to measure the distance between the lines. Some students may fold their paper along a line to make sure that the line is straight. Students can use markings that they remember from middle school geometry to indicate the pairs/sets of parallel lines, coloring utensils, or they can use any labeling system of their choice.

As students work, circulate the room. If anyone seems to finish really quickly, ask them questions about their conclusions or ask if looking at the image from a different angle (such as rotating their paper 90°) changes their opinion.

After students have had a chance to really look at the images, ask if anyone has changed their minds about which lines are parallel on any of the images. Go back through **slides 6–8** and facilitate a whole-group discussion about which lines are parallel and how we can be sure. Emphasize to students that what is really important right now is not that they are right or wrong but their reasoning.

Now transition through **slides 9–12** and answer the question, "Were we right?" for each image. These slides explain why each image is an optical illusion and why scientists think we perceive the image the way that we do.

- Image 1 is an optical illusion, published by Ewald Hering in 1861.
- Image 2 is an optical illusion, published by Johann Karl Friedrich Zöllner in 1860.
- Image 3 is known as the Café Wall Illusion and was noticed by Richard Gregory and Priscilla Heard on a café wall in Bristol, England and published in 1979.

15 minutes

Explore

Ask students, “Should we trust our eyes to determine if lines are parallel?” Facilitate a short discussion and push the students toward the conclusion that math or even graph paper might be helpful.

Teacher's Note: Discovery Is a Process

Remind students that this is a judgment-free environment where everyone is working together to discover more about parallel lines. Reiterate that the expectation is not for each statement to be completely correct right now. Rather, this activity allows you to see where students are in their thought process so that you can help them continue to build and refine their understanding as the lesson progresses.

Show **slide 13** and give each student a copy of the attached **Perspective Drawings** handout. Have students use coloring utensils or other labels to indicate sets of parallel lines. Here, students see the same set of three buildings from two different perspectives: an aerial perspective and a drafting perspective.

Encourage students to try a different approach than what they tried during the Engage portion of the lesson. Remind them that we should not trust our eyes and ask if they can think of a different way to determine if two lines are parallel. Direct students to write their justifications regarding which lines are parallel.

Teacher's Note: Guiding the Activity

Again, this is not yet the time to bring up slopes of parallel lines. However, students will likely make this observation on their own: if the lines never cross, then they must rise and run together (at the same rate). Encourage students to discuss with their partner. Ask guiding questions to lead students to consider comparing the slopes without directly telling them to find the slopes.

Use the hidden **slides 14–15** to gauge what to expect from students. The intention is for students to focus on the main structure or edges of the buildings, not the details of the buildings. However, if some pairs finish too quickly, encourage them to use a ruler to extend some of the lines to find more slopes or more sets of parallel lines.

To end this portion of the lesson, bring the class together for a whole-class discussion about what they noticed. Did they notice that the roof and floors of a building should be parallel? Did they notice that the perspective determined if this was graphically true? Also, help students understand that being parallel in three dimensions does not guarantee being parallel in two dimensions, but that graphing in three dimensions is a topic for a later course. Again, emphasize the importance of not trusting their eyes and the importance of needing the precision that mathematics offers.

15 minutes

Explain

Teacher's Note: Guiding the Lesson

The content on **slides 16–20** is intended to help students eliminate any misconceptions and master the standard at hand. Consider these slides as starting points, and feel free to add, modify, or delete information to best meet your students' needs.

Slide 18 shows a reminder of when to use point-slope form as well as the equation. If using point-slope form is a struggle for your students, unhide this slide and use it to review point-slope form before completing examples 3–4.

Display **slide 16** and give each student a copy of the attached **Guided Notes** handout. Ask pairs to determine whether the lines on the graph are parallel. Then ask for volunteers to share their thoughts. Discuss the characteristics that make up parallel lines.

Teacher's Note: Guiding the Lesson

This is now the time in the lesson to resolve any misunderstandings. At this point, students should be drawing the conclusion that they need to compare the slopes of the lines to determine if the lines are parallel. If students are unsure about this, facilitate a class discussion about why parallel lines must have the same slope: they rise and run at the same rate and therefore never intersect. Contrast this with the idea of lines having different slopes implies that the lines at some point must intersect.

After students feel comfortable with the first example, show **slide 17** and have students independently determine if the lines are parallel. Call on a student to explain their thinking for the second example.

Display **slide 19** and direct students' attention to the third example on their handout. Ask guiding questions and walk through the example of how to write the equation of a line that passes through a given point, (2, 1), and is parallel to a given line, $y = 3x + 5$. Walk through the steps together to solve the problem.

Teacher's Note: Guiding the Lesson

Students should know how to write the equation for examples 3 and 4 from their prior knowledge of writing a linear equation in point-slope form and the knowledge about the slopes of parallel lines from earlier in this lesson. Push students to tell you how to write the equation for example 3 instead of you telling them how to write the equation.

Again, this is the time in the lesson to correct any misconceptions.

Once you feel as though students are comfortable with the concepts, move to **slide 20**. Have students work with a partner or individually to write the equation for the fourth example. Move around the room to make sure everyone is on the same page.

Write the answer and work for example 4 on the board for students to check their work or ask for a volunteer to explain their work depending on the needs of your class. Use the attached **Guided Notes (Model Notes)** document as needed.

15 minutes

Extend

Show **slide 21** and give each student one card from the **I Have Who Has Card** deck. Introduce students to the I Have Who Has strategy and explain what is on the card and the procedure for the activity. Students' cards have an equation at the top of their card: "I have [this equation]," which is the answer to someone else's card. At the bottom of their card, there is the question, "Who has a line parallel to mine that goes through the point (x, y)?"

Students are done when they have assembled into five groups of six students, where each group is a circle of students whose cards chain-link together. For example, Card 29 says, "I have $y + 3 = 2(x - 1)$. Who has a line parallel to mine that goes through $(-1, 3)$?" The person with the answer to that question is the person holding Card 1, which says, "I have $y - 3 = 2(x + 1)$...." This continues until the fifth person in the group reads Card 14, which says, "...Who has a line parallel to mine that goes through $(-1, 3)$?" The answer is the person with Card 29, who completes the circle.

Use the hidden **slide 22** to check students' work. Consider unhiding that slide to let students self-check at the end of the activity.

Teacher's Note: Guiding the Activity

Give students a few minutes to figure out a strategy before giving assistance. Students may initially ask how they are supposed to know who should be together in a group or even how to start. Let students have a healthy struggle to encourage critical thinking. Encourage them to find a classmate or small group to talk through a process or compare cards.

After a few minutes, if students are still struggling, consider asking some of the following guiding questions:

- What did we learn earlier in the lesson about what makes parallel lines parallel?
- What part of the equation was the most important when determining if the lines were parallel?
- What if you started by finding other people with parallel lines first? Then you could worry about the ordered pair next.

Based on your observations, consider giving the class the hint that each group should have six cards.

Once the class has sorted themselves into small groups with the same slopes (parallel lines), again challenge students to recall what they learned during the Explain portion of the lesson. Consider having students get their Guided Notes for reference. However, encourage students to not use a writing utensil or paper for this activity. The cards are designed for students to use point-slope form unless the given point is the y-intercept, then they are expected to use slope-intercept form. Students should be becoming more and more familiar with these different versions of the equation of a line. So, even if students think this is challenging without paper to write on, remind them that this is the one time that we do not want them to write down their thoughts and that this challenge is good for them. That healthy struggle strengthens their knowledge.

5 minutes

Evaluate

Display **slide 23** and use the Exit Ticket strategy to assess what students have learned individually. Give each student a copy of the attached **Exit Ticket** quarter-page handout. Students are asked to determine the fourth vertex of a parallelogram given three vertices.

Use student responses to determine if the class is ready for the next topic or if remediation is needed. Use the hidden **slide 24** to check students' work. Consider unhiding the slide and using it as bellwork the following day.

Teacher's Note: ACT Prep

Using properties of parallel (and perpendicular) lines to write an equation or find the coordinates of a point is skill needed for the ACT exam. The Exit Ticket question is similar to what is seen on the ACT, aside from the multiple-choice distractors. You can search online for more ACT-style questions by using key words such as "parallel line equations" and "ACT prep." Searching "parallel lines" will often give results for parallel lines cut by a transversal, which is regularly seen in a Geometry class.

Resources

- Fibonacci. (2007). Café wall illusion [Image]. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Caf%C3%A9_wall.svg
- Hans-Werner34. (2005). CAFEWALL ANIM.gif [Image]. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:CAFEWALL_ANIM.gif
- Fibonacci. (2007). Zöllner illusion [Image]. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Zollner_illusion.svg
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). I have who has. Strategies. https://learn.k20center.ou.edu/strategy/1497
- Wolfram MathWorld. (2022). Hering illusion [Image]. Wolfram. https://mathworld.wolfram.com/HeringIllusion.html