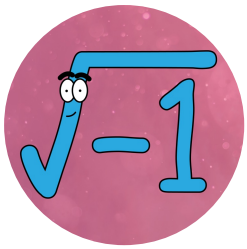

My Imaginary Friend, Part 1

Understanding and Simplifying Imaginary Numbers

Michell Eike
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Grade Level	10th – 11th Grade	**Time Frame**	70 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Algebra 2		

Essential Question

What are imaginary numbers?

Summary

In this lesson, students will be introduced to imaginary numbers and their history. Students will learn how to simplify the square root of a negative number and how to simplify i to a power. Before beginning this lesson, students need to (1) know how to simplify the square root of a whole number and (2) know the Product of Powers and the Power of a Power properties of exponents. This is a multimodality lesson, which means it includes face-to-face, online, and hybrid versions of the lesson. The attachments also include a downloadable Common Cartridge file, which can be imported into a Learning Management System (LMS) such as Canvas or eKadence. The cartridge includes interactive student activities and teacher's notes.

Snapshot

Engage

Students use the Not Like the Others strategy to get them in the mindset of finding patterns.

Explore

Students recall what they know about simplifying square roots and apply what they know to attempt simplifying the square root of a negative number.

Explain

Students read about the history of imaginary numbers and learn the formal definition. Students then learn how to simplify the square root of a negative number.

Extend

Students discover the pattern of simplifying i to a power and apply that pattern to larger exponents.

Evaluate

Students sort statements about real, imaginary, and complex numbers into groups using the Always, Sometimes, or Never True strategy.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

N 606: Multiply two complex numbers
N 704: Apply properties of complex numbers and the complex number system

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.N.1.1: Find the value of i^n for any whole number n.
A2.N.1.2: Simplify, add, subtract, multiply, and divide complex numbers.

Attachments

- Complex Numbers Sample Responses—My Imaginary Friend Part 1.docx
- Complex Numbers Sample Responses—My Imaginary Friend Part 1.pdf
- Complex Numbers—My Imaginary Friend Part 1 - Spanish.docx
- Complex Numbers—My Imaginary Friend Part 1 - Spanish.pdf
- Complex Numbers—My Imaginary Friend Part 1.docx
- Complex Numbers—My Imaginary Friend Part 1.pdf
- History of Imaginary Numbers Infographic—My Imaginary Friend Part 1.pdf
- Lesson Slides—My Imaginary Friend Part 1.pptx

Materials

- Lesson Slides (attached)
- Complex Numbers handout (attached; one per student; printed front/back)
- Complex Numbers (Sample Responses) (attached; for teacher use)
- History of Imaginary Numbers Infographic (attached; one per student; printed front only)
- Pencils
- Paper

10 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question. Display **slide 4** to go over the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Go to **slide 5** and pass out the **Complex Numbers** handout. Inform students they are going to use this handout throughout the lesson. First, have students use the Not Like the Others strategy to decide which item is not like the others, given a picture of a sandwich, a button, an orange slice, and a donut. Then, have students volunteer to share what they selected and why they made their selection.

Using the same strategy, go to **slide 6** and have students decide which of the following radicals is not like the others: the square roots of 20, 75, 16, -16, and 9. Again, ask students to share what they selected and why they made their selection.

Possible Student Responses

To see possible student responses for activities throughout the lesson, refer to the attached **Complex Numbers (Sample Responses)** document. This is a filled-out version of students' Complex Numbers handout.

10 minutes

Explore

Display **slide 7**. Using the Complex Numbers handout, have students work in pairs to simplify the square root of 16 and justify their answer. Next, ask students to try to simplify the square root of -16.

Teacher's Note: Purpose

The purpose here is for students to activate their prior knowledge of simplifying radicals. Students are likely to notice -4 is not the answer to the square root of -16. Students then should realize there is not a real number that can be squared and have a negative result. Remember, at this point, real numbers are the only numbers most students have had experience with.

After a few minutes, encourage students to move forward and simplify the square root of 20, justifying their answer. Then, ask students to try again to simplify the square root of -16.

Go to **slide 8**. Have students use the I Notice, I Wonder strategy to reflect on what they just did. Ask students to write what they notice and what they wonder on the handout.

Teacher's Note: Process

Use this time to check for misunderstandings of prior knowledge. Students should apply the process they used to simplify the square root of 20 to simplify the square root of -16, as follows: sqrt(-16) = sqrt(16)·sqrt(-1) = 4·sqrt(-1). The goal is for students to observe that they do not have a way to simplify the square root of -1.

25 minutes

Explain

Display **slide 9**. As a class, invite students to discuss what they noticed and wondered in the Explore portion. Guide students to come to an agreement that there is not a real number squared that equals a negative number. Display **slide 10** to help resolve any misunderstandings about simplifying square roots.

Go to **slide 11**. Explain to students that there are more numbers than just real numbers. Pass out the attached **History of Imaginary Numbers Infographic** and give students 5 quiet minutes to read it.

Teacher's Note: Oklahoma Process Standards

The content in the attached infographic helps support the Oklahoma *Mathematical Actions and Processes: Develop a Productive Mathematic Disposition* by showing students that, with perseverance, new mathematics can be developed and be useful. During the Extend portion of the lesson, this standard is supported again by having students look for and apply patterns.

Once students understand why imaginary numbers exist, go to **slide 12** and demonstrate how to simplify the square root of a negative number.

Optional Video

If you wish, you may have students also watch the "My Imaginary Friend, Part 1" video on **slide 13**. To make this slide visible to the class, right-click on the slide in the left-hand navigation panel and deselect "Hide Slide" in the dropdown menu. The video shows students how to simplify the square roots of -16, -20, and -18.

Embedded video

https://youtube.com/watch?v=pj3S6X-b5Yc

Display **slide 14** and have students practice what they've just learned. Ask students to work with their partners to simplify the square root of -48 and the negative square root of -36 on the Complex Numbers handout.

20 minutes

Extend

Display **slide 15** and have students work with their partners to complete the table on the handout. Encourage students to use properties of exponents and the given information, $i^1 = i$ and $i^2 = -1$, to complete the table.

Teacher's Note: Scaffolding

If students are struggling to get started, recommend that students rewrite i^3 as $i^2 \cdot i$ and i^4 as $i^2 \cdot i^2$ and use the given information to replace i^2 with -1 and simplify. Students should continue using properties of exponents and their previous results to complete the table.

Show **slide 16** and have students check their tables. Be sure to allow students time to find their mistakes and refine their understanding.

Display **slide 17**. Ask students to write what they notice and wonder regarding the table on the handout.

After students are finished writing, have them work with their partners to generalize a pattern and develop "rules" or observations they could use to quickly simplify i to a much larger power. They may also develop and record their ideas in the Extend: I Notice, I Wonder portion of the handout.

Display **slide 18**. As a class, have students volunteer to share what they noticed and wondered. Use these observations and questions to develop strategies for simplifying i to any whole number power.

Teacher's Note: Guiding the Lesson

If students are struggling to see a pattern, suggest that they compare each row or have students try to predict the results of the next column if the table were to continue. Many of students' generalizations are likely to come from how they approached completing the table. Guide students to see the benefits of different approaches.

Some students may notice a cyclical pattern; some may notice the last row is always +1. Some may observe that when i is taken to a multiple of four power, $i^{(\text{multiple of } 4)}$, the result is +1. Some students may look at the exponent and write it with as many factors of i^2 as possible.

For example, a student might write: $i^{23} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11} \cdot i = -i$; in that case, a student only needs to remember the math fact of $i^2 = -1$.

Display **slide 19** and have students apply the patterns they noticed to larger exponents. Ask students to work with their partners to simplify i^{100}, i^{45}, and i^{67} on the handout.

When students are finished, display **slide 20** so students can check their work. Be sure to allow students time to find their mistakes and refine their understanding.

Teacher's Note: ACT Prep

Simplifying i to a large power is a common and quick question on the ACT. Help students be mindful of the fact that these types of questions are on the ACT and that there is no need to memorize the whole table to quickly arrive at the answer.

Alternative Method

If students struggle with using properties of exponents, one alternative approach is to look at the remainder after dividing the exponent by 4. Write the remainder as the new exponent for i and simplify. This method works because of the properties of exponents and the fact that $i^4 = 1$. For example, i^{11} => 11÷4 = 2R3 => $i^{11} = i^3 = -i$, and i^{26} => 26÷4 = 6R2 => $i^{26} = i^2 = -1$.

5 minutes

Evaluate

Display **slide 21**. Use the Always, Sometimes, or Never True strategy to assess what students have learned during the lesson. Explain to students that they must decide whether "always," "sometimes," or "never" is the most appropriate word to describe how often each statement on the following slide is true. Students also must be able to explain their reasoning.

Teacher's Note: Activity Preparation

Decide if you want to assess students individually, in pairs, or as a whole class. If you want to assess students individually, have each student number their paper 1–7 and then write their answer and reasoning for each statement. You may do the same thing with student pairs.

Alternatively, you may assign each statement to a pair of students, then have each pair explain the word they chose for their assigned statement and their reasoning to the whole class.

If you prefer to assess the class as a whole, you may ask students to vote on the most appropriate word for each statement, discuss, then have the class come to an agreement. Discussing as a class takes more time, but it can help students deepen their understanding.

Show **slide 22** and have students decide whether each statement is "always," "sometimes," or "never" true:

1. i is a real number.
2. i to an even power is -1.
3. i to an odd power is +i or -i.
4. i to a multiple of 4 power is +1.
5. i to a power simplifies to a complex number.
6. i to a power simplifies to a real number.
7. i to a power simplifies to an imaginary number.

Optional Slide

If you want to display the answers, you may choose to show **slide 23** to the class by right-clicking on the slide in the left-hand navigation panel and deselecting "Hide Slide" in the dropdown menu.

Resources

- K20 Center. (n.d.). Always, Sometimes, or Never True. Strategies. https://learn.k20center.ou.edu/strategy/145
- K20 Center. (n.d.). I Notice, I Wonder. Strategies. https://learn.k20center.ou.edu/strategy/180
- K20 Center. (n.d.). My Imaginary Friend, Part 1 [Video]. YouTube. https://www.youtube.com/watch?v=pj3S6X-b5Yc
- K20 Center. (n.d.). Not Like the Others. Strategies. https://learn.k20center.ou.edu/strategy/77
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081