

My Imaginary Friend, Part 1

Understanding and Simplifying Imaginary Numbers

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Grade Level	10th – 11th Grade	**Time Frame**	70 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Algebra 2		

Essential Question

What are imaginary numbers?

Summary

In this lesson, students will be introduced to imaginary numbers and their history. Students will learn how to simplify the square root of a negative number and how to simplify i to a power. Before beginning this lesson, students need to (1) know how to simplify the square root of a whole number and (2) know the Product of Powers and the Power of a Power properties of exponents. This is a multimodality lesson, which means it includes face-to-face, online, and hybrid versions of the lesson. The attachments also include a downloadable Common Cartridge file, which can be imported into a Learning Management System (LMS) such as Canvas or eKadence. The cartridge includes interactive student activities and teacher's notes.

Snapshot

Engage

Students use the Not Like the Others strategy to get them in the mindset of finding patterns.

Explore

Students recall what they know about simplifying square roots and apply what they know to attempt simplifying the square root of a negative number.

Explain

Students read about the history of imaginary numbers and learn the formal definition. Students then learn how to simplify the square root of a negative number.

Extend

Students discover the pattern of simplifying i to a power and apply that pattern to larger exponents.

Evaluate

Students sort statements about real, imaginary, and complex numbers into groups using the Always, Sometimes, or Never True strategy.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

N 606: Multiply two complex numbers
N 704: Apply properties of complex numbers and the complex number system

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.N.1.1: Find the value of i^n for any whole number n.
A2.N.1.2: Simplify, add, subtract, multiply, and divide complex numbers.

Attachments

- Complex Numbers Sample Responses—My Imaginary Friend Part 1.docx
- Complex Numbers Sample Responses—My Imaginary Friend Part 1.pdf
- Complex Numbers—My Imaginary Friend Part 1 - Spanish.docx
- Complex Numbers—My Imaginary Friend Part 1 - Spanish.pdf
- Complex Numbers—My Imaginary Friend Part 1.docx
- Complex Numbers—My Imaginary Friend Part 1.pdf
- History of Imaginary Numbers Infographic—My Imaginary Friend Part 1.pdf
- Lesson Slides—My Imaginary Friend Part 1.pptx

Materials

- Common Cartridge (attached)
- History of Imaginary Numbers Infographic (linked)
- Desmos account
- Student devices with internet access

10 minutes

Engage

Teacher's Note: Desmos Activity Preparation

To use this Desmos Classroom activity, select the following link: "My Imaginary Friend, Part 1: Synchronous." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 3. Select "Restrict to Screens 1–3" to confirm your selection. This allows students to access only screens 1–3 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to student.desmos.com and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1–2** of the Desmos activity. **Screen 1** displays the lesson's essential question. **Screen 2** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

On **screen 3**, have students use the Not Like the Others strategy to decide which item is not like the others, given a picture of a sandwich, a button, an orange slice, and a donut. Then, have students volunteer to share what they selected and why they made their selection.

On the Desmos dashboard, click the orange plus sign to allow students to progress to **screen 4**. Using the same strategy, have students decide which of the following radicals is not like the others: the square roots of 20, 75, 16, -16, and 9. Again, ask students to share what they selected and why they made their selection.

10 minutes

Explore

On the Desmos dashboard, click the orange "Edit" button and select screens 5 and 7. This allows students to access **screens 5–7**.

On **screen 5**, have students work in pairs to simplify the square root of 16. After submitting the correct answer, they are prompted to share their thinking.

Next, ask students to try to simplify the square root of -16.

Teacher's Note: Purpose

The purpose here is for students to activate their prior knowledge of simplifying radicals. Students are likely to notice -4 is not the answer to the square root of -16. Students then should realize there is not a real number that can be squared and have a negative result. Remember, at this point, real numbers are the only numbers most students have had experience with.

After a few minutes, encourage students to move to **screen 6** and simplify the square root of 20. After submitting the correct answer, they are prompted to share their thinking.

Then, ask students to try again to simplify the square root of -16. After a few minutes, have students go to **screen 7** and use the I Notice, I Wonder strategy to reflect on what they just did.

Teacher's Note: Process

Use this time to check for misunderstandings of prior knowledge. Students should apply the process they used to simplify the square root of 20 to simplify the square root of -16, as follows: sqrt(-16) = sqrt(16)·sqrt(-1) = 4·sqrt(-1). The goal is for students to observe that they do not have a way to simplify the square root of -1.

25 minutes

Explain

As a class, invite students to discuss what they noticed and wondered in the Explore portion. Guide students to come to an agreement that there is not a real number squared that equals a negative number.

Teacher's Note: Oklahoma Process Standards

The content in the infographic linked below helps support the Oklahoma *Mathematical Actions and Processes: Develop a Productive Mathematic Disposition* by showing students that, with perseverance, new mathematics can be developed and be useful. During the Extend portion of the lesson, this standard is supported again by having students look for and apply patterns.

On the Desmos dashboard, click the orange "Edit" button and select **screen 8**. This screen prompts students to leave the Desmos activity and go to the provided infographic link to read about the history of imaginary numbers. Give students 5 quiet minutes to read about the history of imaginary numbers.

At the end of the infographic, have students watch the "My Imaginary Friend, Part 1" video. The video shows students how to simplify the square roots of -16, -20, and -18.

Embedded video
https://youtube.com/watch?v=pj3S6X-b5Yc

On the dashboard, click the orange plus sign to allow students to progress to **screen 9**, then click again for **screen 10** as students practice what they've just learned.

Students should work in pairs to simplify the square root of -48 and the negative square root of -36. The Desmos activity has a built-in self-check on these screens, so students receive immediate feedback about their responses. After submitting the correct answer, students are asked to explain their thinking, which is then shared with their classmates.

20 minutes

Extend

On the Desmos dashboard, click the orange "Edit" button and select screens 11 and 14. This allows students to access **screens 11–14**.

Beginning on **screen 11**, students should work with their partners to complete the tables and then progress to **screens 12–13**. Encourage students to use properties of exponents and the given information, $i^1 = i$ and $i^2 = -1$, to complete the table. The Desmos activity has a built-in self-check on these screens, so students receive immediate feedback about their completed tables.

Teacher's Note: Scaffolding

The tables are scaffolded by giving students a little less guidance as they progress through the table. It begins by showing the work and correct results for i^1 and i^2. Then, it provides the work for i^3 and i^4 and, if wrong answers are submitted, hints. For i^5 and i^6, only hints are given after incorrect answers. After that, students are given only a "Try Again" hint for any following incorrect results.

On **screen 14**, students see all of their responses collected in one table. Students then reflect on the table, again using the I Notice, I Wonder strategy.

Next, ask students to work with their partners to generalize a pattern and develop "rules" or observations they could use to quickly simplify i to a much larger power. They may also develop and record their ideas on screen 14.

As a class, have students volunteer to share what they noticed and wondered. Use these observations and questions to develop strategies for simplifying i to any whole number power.

Teacher's Note: Guiding the Lesson

If students are struggling to see a pattern, suggest that they compare each row or have students try to predict the results of the next column, if the table were to continue. Many of students' generalizations are likely to come from how they approached completing the table. Guide students to see the benefits of different approaches.

Some students may notice a cyclical pattern; some may notice the last row is always +1. Some may observe that when i is taken to a multiple of four power, $i^{(\text{multiple of } 4)}$, the result is +1. Some students may look at the exponent and write it with as many factors of i^2 as possible.

For example, a student might write: $i^{23} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11} \cdot i = -i$; in that case, a student only needs to remember the math fact of $i^2 = -1$.

On the dashboard, click the orange plus sign to allow students to progress to **screen 15**. On this screen, students apply the patterns they noticed to larger exponents. Have students work with their partners to simplify i^{100}, i^{45}, and i^{67}. The Desmos activity has a built-in self-check on this screen, so students receive immediate feedback about their results.

Teacher's Note: ACT Prep

Simplifying i to a large power is a common and quick question on the ACT. Help students be mindful of the fact that these types of questions are on the ACT and that there is no need to memorize the whole table to quickly arrive at the answer.

Alternative Method

If students struggle with using properties of exponents, one alternative approach is to look at the remainder after dividing the exponent by 4. Write the remainder as the new exponent for i and simplify. This method works because of the properties of exponents and the fact that $i^4 = 1$. For example, i^{11} => 11÷4 = 2R3 => $i^{11} = i^3 = -i$, and i^{26} => 26÷4 = 6R2 => $i^{26} = i^2 = -1$.

5 minutes

Evaluate

Use the Always, Sometimes, or Never True strategy to assess what students have learned during the lesson. On the Desmos dashboard, click the orange "Edit" button and select screen 16. This displays **screen 16** for students.

Explain to students that they must decide whether "always," "sometimes," or "never" is the most appropriate word to describe how often each of the following statements is true. Students also must be able to explain their reasoning.

Teacher's Note: Activity Preparation

Decide if you want to assess students individually, in pairs, or as a whole class, and consider how you want students to explain their thinking. The card sort is designed to assess individual students or student pairs. If using the card sort, students may share their reasoning by writing it on paper.

Alternatively, you may assign each statement to a pair of students, then have each pair explain the word they chose for their assigned statement and their reasoning to the whole class.

If you prefer to assess the class as a whole, you may ask students to vote on the most appropriate word for each statement, discuss, then have the class come to an agreement. Discussing as a class takes more time, but it can help students deepen their understanding.

As students decide whether each statement is "always," "sometimes," or "never" true, have them sort the statement cards into those three groups by dragging the cards on top of one another.

- i is a real number.
- i to an even power is -1.
- i to an odd power is +i or -i.
- i to a multiple of 4 power is +1.
- i to a power simplifies to a complex number.
- i to a power simplifies to a real number.
- i to a power simplifies to an imaginary number.
- The square root of a positive real number is an imaginary number.
- A real number squared is a negative number.

Use student responses to see which misconceptions persist.

Resources

- K20 Center. (n.d.). Always, Sometimes, or Never True. Strategies. https://learn.k20center.ou.edu/strategy/145
- K20 Center. (n.d.). I Notice, I Wonder. Strategies. https://learn.k20center.ou.edu/strategy/180
- K20 Center. (n.d.). My Imaginary Friend, Part 1 [Video]. YouTube. https://www.youtube.com/watch?v=pj3S6X-b5Yc
- K20 Center. (n.d.). Not Like the Others. Strategies. https://learn.k20center.ou.edu/strategy/77
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081