My Imaginary Friend, Part 1
Understanding and Simplifying Imaginary Numbers

Michell Eike
Published by K20 Center

This work is licensed under a Creative Commons CC BY-SA 4.0 License

Grade Level 10th – 11th Grade    Time Frame 70 minutes
Subject Mathematics    Duration 2 class periods
Course Algebra 2

Essential Question
What are imaginary numbers?

Summary
In this lesson, students will be introduced to imaginary numbers and their history. Students will learn how to simplify the square root of a negative number and how to simplify i to a power. Before beginning this lesson, students need to (1) know how to simplify the square root of a whole number and (2) know the Product of Powers and the Power of a Power properties of exponents. This is a multimodality lesson, which means it includes face-to-face, online, and hybrid versions of the lesson. The attachments also include a downloadable Common Cartridge file, which can be imported into a Learning Management System (LMS) such as Canvas or eKadence. The cartridge includes interactive student activities and teacher's notes.

Snapshot

Engage
Students use the Not Like the Others strategy to get them in the mindset of finding patterns.

Explore
Students recall what they know about simplifying square roots and apply what they know to attempt simplifying the square root of a negative number.

Explain
Students read about the history of imaginary numbers and learn the formal definition. Students then learn how to simplify the square root of a negative number.

Extend
Students discover the pattern of simplifying i to a power and apply that pattern to larger exponents.

Evaluate
Students sort statements about real, imaginary, and complex numbers into groups using the Always, Sometimes, or Never True strategy.

https://learn.k20center.ou.edu/lesson/1477?rev=17891
Standards

Oklahoma Academic Standards for Mathematics (Grades 9, 10, 11, 12)

A2.N.1.1: Find the value of $i^n$ for any whole number n.
A2.N.1.2: Simplify, add, subtract, multiply, and divide complex numbers.

Attachments

- Common Cartridge—My Imaginary Friend Part 1.zip
- Complex Numbers Sample Responses—My Imaginary Friend Part 1.docx
- Complex Numbers Sample Responses—My Imaginary Friend Part 1.pdf
- Complex Numbers—My Imaginary Friend Part 1 - Spanish.docx
- Complex Numbers—My Imaginary Friend Part 1 - Spanish.pdf
- Complex Numbers—My Imaginary Friend Part 1.docx
- Complex Numbers—My Imaginary Friend Part 1.pdf
- History of Imaginary Numbers Infographic—My Imaginary Friend Part 1.pdf
- Lesson Slides—My Imaginary Friend Part 1.pptx

Materials

- Common Cartridge (attached)
- History of Imaginary Numbers Infographic (linked)
- Desmos account
- Student devices with internet access
Engage

Teacher's Note: Canvas Cartridge

Please note the downloadable Canvas cartridge is not yet available. However, this lesson can be done fully online without the cartridge, as students can access all necessary activities and links through Desmos.

Teacher's Note: Desmos Activity Preparation

Before starting the activity, assign student pairs. In an asynchronous setting, advise students to schedule a time to meet with their partners.

To use this Desmos Classroom activity, select the following link: "My Imaginary Friend, Part 1: Asynchronous." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

Provide students with your session code. Then, have students go to student.desmos.com and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

On screen 1, students use the Not Like the Others strategy to decide which item is not like the others, given a picture of a sandwich, a button, an orange slice, and a donut. Students must also provide their reasoning behind the selection. After submitting their answers, students are able to see their peers' responses.

On screen 2, students decide which of the following radicals is not like the others: the square roots of 20, 75, 16, -16, and 9. Again, students must explain their reasoning.

Screen 3 prompts students to think differently about that same list of radicals. Students must pick a different radical that does not belong and provide justification.
Explore

On screen 4, each student needs to collaborate with their partner. Students start by simplifying the square root of 16. After submitting the correct answer, they are prompted to share their thinking. The response they enter is then shared with their classmates.

Next, students try to simplify the square root of -16 and share their thinking.

**Teacher's Note: Purpose**

The purpose here is for students to activate their prior knowledge of simplifying radicals. Students are likely to notice -4 is not the answer to the square root of -16. Students then should realize there is not a real number that can be squared and have a negative result. Remember, at this point, real numbers are the only numbers most students have had experience with.

On screen 5, students start by simplifying the square root of 20. After submitting the correct answer, they are prompted to share their thinking. The response they enter is shared with their classmates.

Then, students must try again to simplify the square root of -16 and share their thinking.

**Teacher's Note: Process**

Students should apply the process they used to simplify the square root of 20 to simplify the square root of -16, as follows: \( \sqrt{-16} = \sqrt{16} \cdot \sqrt{-1} = 4 \cdot \sqrt{-1} \). The goal is for students to observe that they do not have a way to simplify the square root of -1.

On screen 6, students use the [I Notice, I Wonder](https://www.nsta.org) strategy to reflect on what they just did. Their submitted responses are then shared with their classmates.
**Teacher's Note: Oklahoma Process Standards**

The content in the infographic linked below helps support the Oklahoma *Mathematical Actions and Processes: Develop a Productive Mathematic Disposition* by showing students that, with perseverance, new mathematics can be developed and be useful. During the Extend portion of the lesson, this standard is supported again by having students look for and apply patterns.

**Screen 7** prompts students to leave the Desmos activity and go to the [provided infographic link](https://youtube.com/watch?v=pj3S6X-b5Yc) to read about the history of imaginary numbers.

At the end of the infographic, students should watch the "My Imaginary Friend, Part 1" video. The video shows them how to simplify the square roots of -16, -20, and -18.

**Embedded video**

https://youtube.com/watch?v=pj3S6X-b5Yc

On **screens 8-9**, students practice what they've just learned. Students should work with their partners to simplify the square root of -48 and the negative square root of -36.

The Desmos activity has a built-in self-check on these screens, so students receive immediate feedback about their responses. After submitting the correct answer, students are asked to explain their thinking, which is then shared with their classmates.
20 minutes

Extend

On **screens 10–12**, students must work through a series of tables to discover patterns for simplifying \( i \) to a power. The Desmos activity has a built-in self-check on these screens, so students receive immediate feedback about their completed tables.

**Teacher's Note: Scaffolding**

The tables are scaffolded by giving students a little less guidance as they progress through the table. It begins by showing the work and correct results for \( i^1 \) and \( i^2 \). Then, it provides the work for \( i^3 \) and \( i^4 \) and, if wrong answers are submitted, hints. For \( i^5 \) and \( i^6 \), only hints are given after incorrect answers. After that, students are given only a "Try Again" hint for any following incorrect results.

On **screen 13**, students see all of their responses collected in one table. Students then reflect on the table, again using the **I Notice, I Wonder** strategy. Their submitted responses are shared with their classmates.

On **screen 14**, students apply the patterns they noticed to simplify \( i^{100}, i^{45}, \) and \( i^{67} \). The Desmos activity has a built-in self-check on this screen, so students receive immediate feedback about their results.

**Teacher's Note: ACT Prep**

Simplifying \( i \) to a large power is a common and quick question on the ACT. Help students be mindful of the fact that these types of questions are on the ACT and that there is no need to memorize the whole table to quickly arrive at the answer.
5 minutes

**Evaluate**

Using the *Always, Sometimes, or Never True* strategy on **screen 15**, students reflect and are assessed on what they have learned during the lesson.

Students must decide whether each of the following statements is always true, sometimes true, or never true. They then sort the cards into those three groups by dragging the cards on top of one another.

- i is a real number.
- i to an even power is -1.
- i to an odd power is +i or -i.
- i to a multiple of 4 power is +1.
- i to a power simplifies to a complex number.
- i to a power simplifies to a real number.
- i to a power simplifies to an imaginary number.
- The square root of a positive real number is an imaginary number.
- A real number squared is a negative number.

Use student responses to see which misconceptions persist.
Resources