My Imaginary Friend, Part 2

Operations with Complex Numbers

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Published by *K20 Center*

Grade Level	10th – 12th Grade	**Time Frame**	60-80 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	Algebra 2, Precalculus		

Essential Question

Why do we perform operations on complex numbers?

Summary

To complete this lesson, students need to know how to simplify the square root of a negative number and how to simplify i to a power.* This lesson is contained within an escape room. Students will navigate through a spaceship and, along the way, will perform operations on complex numbers so they can make it to the escape pods in time. Students will see the real-world applications of these operations through learning about electrical circuits. Students also will watch a video of an electrical engineer speaking about his career, describing how he uses electrical circuits, and explaining how to divide complex numbers using complex conjugates. *See the "My Imaginary Friend, Part 1" lesson for the prerequisite content. This is a multimodality lesson, which means it includes face-to-face, online, and hybrid versions of the lesson. The attachments also include a downloadable Common Cartridge file, which can be imported into a Learning Management System (LMS) such as Canvas or eKadence. The cartridge includes interactive student activities and teacher's notes.

Snapshot

Engage

Students read a story that sets the scene for the escape room in a spaceship. Students use their reasoning skills to turn given clues into a passcode as they learn how to navigate through the escape room.

Explore

Students are introduced to electrical circuits and perform calculations using electrical circuits. Students add and subtract complex numbers to find the total impedance, then multiply complex numbers to find the voltage.

Explain

Students watch a video of an electrical engineer speaking about his career and describing how he uses electrical circuits. Students then learn about complex conjugates and how to use them to divide complex numbers.

Extend

Students apply what they have learned about performing operations with complex numbers to progress through the escape room.

Evaluate

Students use the Fist to Five strategy to reflect on what they have learned during the lesson.

Standards

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.N.1.2: Simplify, add, subtract, multiply, and divide complex numbers.

Oklahoma Academic Standards Mathematics (Algebra 2)

PC.T.4.1: Use the relation i^2 = -1 and the mathematical properties to add, subtract, and multiply complex numbers.

Attachments

- Calibration Dial—My Imaginary Friend Part 2 - Spanish.docx
- Calibration Dial—My Imaginary Friend Part 2 - Spanish.pdf
- Calibration Dial—My Imaginary Friend Part 2.docx
- Calibration Dial—My Imaginary Friend Part 2.pdf
- Cargo Bay—My Imaginary Friend Part 2 - Spanish.docx
- Cargo Bay—My Imaginary Friend Part 2 - Spanish.pdf
- Cargo Bay—My Imaginary Friend Part 2.docx
- Cargo Bay—My Imaginary Friend Part 2.pdf
- Common Cartridge—My Imaginary Friend Part 2.zip
- Go With the Flow—My Imaginary Friend Part 2 - Spanish.docx
- Go With the Flow—My Imaginary Friend Part 2 - Spanish.pdf
- Go With the Flow—My Imaginary Friend Part 2.docx
- Go With the Flow—My Imaginary Friend Part 2.pdf
- Lesson Guide and Escape Room Script—My Imaginary Friend Part 2.docx
- Lesson Guide and Escape Room Script—My Imaginary Friend Part 2.pdf
- Lesson Slides—My Imaginary Friend Part 2.pptx
- Power Control Hub—My Imaginary Friend Part 2 - Spanish.docx
- Power Control Hub—My Imaginary Friend Part 2 - Spanish.pdf
- Power Control Hub—My Imaginary Friend Part 2.docx
- Power Control Hub—My Imaginary Friend Part 2.pdf
- User Manual Calculations Sample Responses—My Imaginary Friend Part 2.docx
- User Manual Calculations Sample Responses—My Imaginary Friend Part 2.pdf
- User Manual Calculations—My Imaginary Friend Part 2 - Spanish.docx
- User Manual Calculations—My Imaginary Friend Part 2 - Spanish.pdf
- User Manual Calculations—My Imaginary Friend Part 2.docx
- User Manual Calculations—My Imaginary Friend Part 2.pdf
- User Manual Definitions—My Imaginary Friend Part 2 - Spanish.docx
- User Manual Definitions—My Imaginary Friend Part 2 - Spanish.pdf
- User Manual Definitions—My Imaginary Friend Part 2.docx
- User Manual Definitions—My Imaginary Friend Part 2.pdf

Materials

- Lesson Slides (attached)
- Lesson Guide and Escape Room Script (attached; for teacher use; printed front/back)
- User Manual—Definitions handout (attached; one per student; printed front only)
- User Manual—Calculations handout (attached; one per student; printed front/back)
- User Manual—Calculations (Sample Responses) (attached; for teacher use)
- Power Control Hub handout (attached; one per student; printed front only)
- Cargo Bay handout (attached; one per student; printed front only)
- Pencils
- Student devices with internet access**

- Go With the Flow handout (optional; attached; one half-sheet per student; printed front only)
- Calibration Dial handout (optional; attached; creates 2 dials per document; printed front only)

**See the note in the Extend portion of this lesson for alternatives to having student devices.

5 minutes

Engage

Teacher's Note: Lesson Mode Recommendation

If you have enough devices for each pair of students to have internet access, check out the hybrid or online version of this lesson. The hybrid and online versions are essentially digital synchronous and asynchronous versions, respectively. Since this is an escape room, those versions are a little easier to navigate.

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question. Display **slide 4** to go over the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Teacher's Note: Lesson Guide

Use the attached **Lesson Guide and Escape Room Script** as a quick reference guide for this lesson. It contains the directions and the escape room story for you to read aloud to the class. (The escape room story is also included throughout the Lesson Slides for students who may find it easier to read the story rather than listen to it.)

The Lesson Guide and Escape Room Script also contains Teacher's Notes for transitional points of the lesson, as well as the correct responses at the end of the document.

Display **slide 5** and read aloud the directions to the class. Explain to students that they need to assemble a crew to escape successfully. Have students work in pairs throughout the lesson.

Display **slide 6**, which begins the escape room story. Read this screen aloud to the class. Transition through **slides 7–8** and continue reading the story aloud.

Move to **slide 9** and have students work with their partners to determine the three-letter passcode.

Optional Handout

Students do not necessarily need paper for this portion of the escape room, but feel free to use the attached **Go With the Flow** handout for an optional student resource that complements slide 10.

Give students a couple minutes to figure out the passcode before moving to the next slide. Display **slide 10** to show students the correct passcode.

20 minutes

Explore

Teacher's Note: Activity Preparation and Sample Responses

Before class, decide whether you want the attached **User Manual—Calculations** handout to be guided practice or independent practice. The correct responses to the handout are on hidden slides, so if you would like students to check their work as they go, unhide slides 20, 21, 23, 24, and 29.

If you would prefer to use this handout as a graded assignment for students to turn in, see the attached **User Manual—Calculations (Sample Responses)** document for quick reference to the correct responses on those hidden slides. This document also can be used to help any visually impaired students who may have trouble reading the responses on slides 20, 21, 23, 24, and 29.

Transition through **slides 11–12** and continue reading the story aloud. Display **slide 13** and pass out the attached **User Manual—Definitions** handout. Give students a few minutes to read through the handout.

Move to **slide 14** and direct students' attention to example 1 at the bottom of the handout. Then, transition through **slides 15–17** to help students understand how to calculate the total impedance of an electrical circuit.

Next, display **slide 18**. Have students work with their partners to find the total impedance of example 2, which is also located at the bottom of the User Manual—Definitions handout.

While students discuss, pass out the attached **User Manual—Calculations** handout. Then, ask for volunteers to explain why the total impedance is (30+3*i*) ohms.

Once students understand the two example problems, display **slide 19**. Ask students to work with their partners to find the total impedance for questions 1 and 2 on the User Manual—Calculations handout.

Optional Slides

Unhide and transition through **slides 20–21** so students can check their answers for questions 1 and 2. Ask for volunteers to explain their work.

Display **slide 22** and have students work with their partners to find the voltage for questions 3 and 4 on the handout.

Optional Slides

Unhide and transition through **slides 23–24** so students can check their answers for questions 3 and 4. Ask for volunteers to explain their work.

Teacher's Note: Purpose

Finding the total impedance requires students to add and subtract complex numbers and recall their knowledge of combining like terms. Calculating voltage requires students to multiply complex numbers and recall their knowledge of expanding binomials and simplifying *i* to a power. The goal is for these real-world connections to help students understand why they are learning about operations with complex numbers.

20 minutes

Explain

Teacher's Note: Lesson Pacing

This lesson can be naturally paused and resumed at different points during the Explain portion, depending on how the class is progressing. Ideally, pause the lesson at the end of the Explain portion and resume the lesson the next day with the Extend portion.

You may find students need more time due to misconceptions with prerequisite knowledge. In this case, consider pausing the lesson after the video and question 5 and resuming the next day with question 6. You also could have students watch the video and do question 5 at home, then resume the next day with question 6 in class—just be sure to provide students with the link to the video.

Alternatively, consider allowing the lesson to take three 45-minute class periods, as follows: Day 1, Engage & Explore; Day 2, Explain; and Day 3, Extend & Evaluate.

Display **slide 25** and have students try to answer question 5 from the User Manual—Calculations handout. Give students a few minutes to struggle with the problem. Then, ask the class what operation they need to use with complex numbers in this scenario. Ask for volunteers to explain why they need to divide complex numbers.

Move to **slide 26**. Introduce the "My Imaginary Friend, Part 2" video, which features an electrical engineer talking about his career, describing how he uses complex numbers, and explaining how to divide complex numbers. Play the video.

Embedded video

https://youtube.com/watch?v=b4egCAQtzzk

The video begins with the electrical engineer interview, which ends at the 12:45 mark. The rest of the video demonstrates how to divide complex numbers using a complex conjugate.

After the video, display **slide 27** so students can see the entire worked-out solution for question 5. Ask students to discuss the problem with their partners and ask one another questions.

As a class, ask students what questions they still have. After addressing any questions, move to **slide 28** and have students find the current for question 6 on the handout.

Optional Slide

Unhide and display **slide 29** so students can check their answer for question 6. Ask for volunteers to explain their work.

Teacher's Note: ACT Prep

There is often one question on the ACT about complex conjugates, and it's usually as simple as this: Students are given a complex number and asked to find its complex conjugate.

Use this lesson to help students understand that a conjugate is *not* the opposite of the number, which is a common misconception. If students can grasp why a complex conjugate is used to divide complex numbers, they may become more comfortable with this concept without feeling like they need to memorize its formal definition.

30 minutes

Extend

Technology Alternatives and Optional Handout

This is the portion where each pair of students needs one device with internet access, as there are two GeoGebra activities that students must navigate through. However, if technology is limited, the escape room still can be completed with the use of a SMART Board to display the webpage for the Cargo Bay handout.

The calibration dial for the Power Control Hub handout can be physically constructed before class using the attached **Calibration Dial** handout. This is a two-page document that ideally should be printed on cardstock or heavier paper and could be reused. To ensure students have all necessary layers for the calibration dial, each student pair should cut out a half-sheet from the first page and a half-sheet from the second page of the document. Constructing the dial also requires a brad or other paper fastener—even a paper clip could work. Directions for how to cut and assemble the calibration dial are included in the document.

This portion also could be done with stations if only a few online devices are available and not enough for each pair to have access. If this is the case, consider passing out the Cargo Bay handout to half the class while the other half works on the Power Control Hub handout to help with any potential congestion at the stations.

Transition through **slides 30–31** and continue reading the story aloud. Then, display **slide 32** and pass out the attached **Power Control Hub** handout. Remind students to read everything on the handout before they begin working on it with their partners.

While students work, monitor student progress while walking around the room with the Lesson Guide and Escape Room Script on hand. Be sure not to immediately check answers or give assistance. It is important for students to have a little time to struggle as they work through these challenging problems.

As students start the third question on the handout, consider staying at your desk or a location in which students can easily and comfortably reach you to check their three-letter code. As noted in the Lesson Guide and Escape Room Script, if students have the correct Transfer Power Code, then you should exchange students' Power Control Hub handout for the attached **Cargo Bay** handout.

Students may finish these handouts at staggering times, so having quick reference to the Lesson Guide and Escape Room Script can help you ensure the activity goes smoothly. When students finish the Cargo Bay handout and come to you to check their Release Code, let students who are correct know that they've escaped successfully (and that they must keep the code a secret).

Use the Power Control Hub and Cargo Bay handouts to determine what misconceptions students may have regarding operations with complex numbers.

Once the whole class has escaped successfully, display **slide 33**.

5 minutes

Evaluate

Display **slide 34** and have students use the Fist to Five strategy to reflect on what they've learned during the lesson. On slide 34, ask students to focus their reflection on their understanding of adding and subtracting complex numbers.

Then, display **slide 35** and have students reflect on their understanding of multiplying complex numbers. Finally, display **slide 36** and have students reflect on their understanding of dividing complex numbers.

Resources

- K20 Center. (n.d.). Fist to Five. Strategies. https://learn.k20center.ou.edu/strategy/68
- K20 Center. (n.d.). GeoGebra. Tech tools. https://learn.k20center.ou.edu/tech-tool/2352
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). My Imaginary Friend, Part 2 cover image [Composite image]. K20 Center.
- Vecteezy. (n.d.). Rocket Ship Flying [Image]. Vecteezy. https://www.vecteezy.com/vector-art/1871713-happy-successful-businessman-holding-goal-flag-standing-on-rocket-ship-flying-through-starry-sky-business-startup-concept
- Vecteezy. (n.d.). Round Spaceship on White Background [Image]. Vecteezy. https://www.vecteezy.com/vector-art/359100-round-spaceship-on-white-background