



# My Imaginary Friend, Part 2

## Operations with Complex Numbers



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<b>Grade Level</b>	10th – 12th Grade	<b>Time Frame</b>	60-80 minutes
<b>Subject</b>	Mathematics	<b>Duration</b>	2-3 class periods
<b>Course</b>	Algebra 2, Precalculus		

### Essential Question

Why do we perform operations on complex numbers?

### Summary

To complete this lesson, students need to know how to simplify the square root of a negative number and how to simplify  $i$  to a power.\* This lesson is contained within an escape room. Students will navigate through a spaceship and, along the way, will perform operations on complex numbers so they can make it to the escape pods in time. Students will see the real-world applications of these operations through learning about electrical circuits. Students also will watch a video of an electrical engineer speaking about his career, describing how he uses electrical circuits, and explaining how to divide complex numbers using complex conjugates. \*See the "My Imaginary Friend, Part 1" lesson for the prerequisite content. This is a multimodality lesson, which means it includes face-to-face, online, and hybrid versions of the lesson. The attachments also include a downloadable Common Cartridge file, which can be imported into a Learning Management System (LMS) such as Canvas or eKadence. The cartridge includes interactive student activities and teacher's notes.

### Snapshot

#### Engage

Students read a story that sets the scene for the escape room in a spaceship. Students use their reasoning skills to turn given clues into a passcode as they learn how to navigate through the escape room.

#### Explore

Students are introduced to electrical circuits and perform calculations using electrical circuits. Students add and subtract complex numbers to find the total impedance, then multiply complex numbers to find the voltage.

#### Explain

Students watch a video of an electrical engineer speaking about his career and describing how he uses electrical circuits. Students then learn about complex conjugates and how to use them to divide complex numbers.

#### Extend

<https://learn.k20center.ou.edu/lesson/1539?rev=46842>

Students apply what they have learned about performing operations with complex numbers to progress through the escape room.

**Evaluate**

Students use the Fist to Five strategy to reflect on what they have learned during the lesson.

## Standards

*ACT College and Career Readiness Standards - Mathematics (6-12)*

**N 606:** Multiply two complex numbers

**N 704:** Apply properties of complex numbers and the complex number system

*Oklahoma Academic Standards Mathematics (Algebra 2)*

**A2.N.1.2:** Simplify, add, subtract, multiply, and divide complex numbers.

*Oklahoma Academic Standards Mathematics (Algebra 2)*

**PC.T.4.1:** Use the relation  $i^2 = -1$  and the mathematical properties to add, subtract, and multiply complex numbers.

## Attachments

- [Calibration Dial—My Imaginary Friend Part 2 - Spanish.docx](#)
- [Calibration Dial—My Imaginary Friend Part 2 - Spanish.pdf](#)
- [Calibration Dial—My Imaginary Friend Part 2.docx](#)
- [Calibration Dial—My Imaginary Friend Part 2.pdf](#)
- [Cargo Bay—My Imaginary Friend Part 2 - Spanish.docx](#)
- [Cargo Bay—My Imaginary Friend Part 2 - Spanish.pdf](#)
- [Cargo Bay—My Imaginary Friend Part 2.docx](#)
- [Cargo Bay—My Imaginary Friend Part 2.pdf](#)
- [Go With the Flow—My Imaginary Friend Part 2 - Spanish.docx](#)
- [Go With the Flow—My Imaginary Friend Part 2 - Spanish.pdf](#)
- [Go With the Flow—My Imaginary Friend Part 2.docx](#)
- [Go With the Flow—My Imaginary Friend Part 2.pdf](#)
- [Lesson Guide and Escape Room Script—My Imaginary Friend Part 2.docx](#)
- [Lesson Guide and Escape Room Script—My Imaginary Friend Part 2.pdf](#)
- [Lesson Slides—My Imaginary Friend Part 2.pptx](#)
- [Power Control Hub—My Imaginary Friend Part 2 - Spanish.docx](#)
- [Power Control Hub—My Imaginary Friend Part 2 - Spanish.pdf](#)
- [Power Control Hub—My Imaginary Friend Part 2.docx](#)
- [Power Control Hub—My Imaginary Friend Part 2.pdf](#)
- [User Manual Calculations Sample Responses—My Imaginary Friend Part 2.docx](#)
- [User Manual Calculations Sample Responses—My Imaginary Friend Part 2.pdf](#)
- [User Manual Calculations—My Imaginary Friend Part 2 - Spanish.docx](#)
- [User Manual Calculations—My Imaginary Friend Part 2 - Spanish.pdf](#)
- [User Manual Calculations—My Imaginary Friend Part 2.docx](#)
- [User Manual Calculations—My Imaginary Friend Part 2.pdf](#)
- [User Manual Definitions—My Imaginary Friend Part 2 - Spanish.docx](#)
- [User Manual Definitions—My Imaginary Friend Part 2 - Spanish.pdf](#)
- [User Manual Definitions—My Imaginary Friend Part 2.docx](#)
- [User Manual Definitions—My Imaginary Friend Part 2.pdf](#)

## Materials

- Common Cartridge (attached)
- User Manual—Definitions handout (attached; one per student; printed front only)
- Desmos account
- Student devices with internet access
- Paper
- Pencils

- Lesson Guide and Escape Room Script (optional; attached; for teacher use)
- User Manual—Calculations (Sample Responses) (optional; attached; for teacher use)

5 minutes

## Engage

### Teacher's Note: Desmos Activity Preparation

To use this [Desmos Classroom](#) activity, select the following link: "[My Imaginary Friend, Part 2: Synchronous](#)." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to <https://k20center.ou.edu/externalapps/using-activities/>.

For more detailed information about Desmos features and how-to tips, go to <https://k20center.ou.edu/externalapps/desmos-home-page/>.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 3. Select "Restrict to Screens 1–3" to confirm your selection. This allows students to access only screens 1–3 at this time. For more information about teacher pacing, go to <https://k20center.ou.edu/externalapps/pacing-activities/>.

Provide students with your session code. Then, have students go to [student.desmos.com](https://student.desmos.com) and enter the session code.

### Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1–2** of the Desmos activity. **Screen 1** displays the lesson's essential question. **Screen 2** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

On the Desmos dashboard, click the orange "Edit" button and select screens 3 and 7. This allows students to access **screens 3–7**. Beginning on **screen 3**, have students work in pairs to progress to the next check-in point, which is screen 7.

The Desmos activity's design requires students to enter the correct answer(s) on each screen in order to progress to the next. For example, if students do not enter their names on screen 3, then screens 4–6 show a message that says "Access Denied," with directions to go back to enter the names of their crew.

Students must solve their first riddle on **screen 6**, where they are given a circuit diagram, a list of symbols with corresponding letters, and a clue that says, "Go with the flow." Students are expected to use the three symbols in the diagram, find the corresponding letters, and enter those three letters in the table to unlock the next screen. Later in the lesson, students learn what a circuit diagram is and what those symbols mean.

If students enter the code backwards, they see an error message (orange triangle) with a hint to go the opposite direction. If students enter a correct letter but do not capitalize the letter, they see an error message with a hint to capitalize the letter. Once students enter the correct three-letter code, they can progress to the next screen.

Check the Desmos dashboard to view student progress. As students reach screen 7, pass out the attached **User Manual—Definitions** handout.

20 minutes

## Explore

On the Desmos dashboard, click the orange "Edit" button and select screens 3 and 14. This allows students to access **screens 3–14**. Have students work in pairs to progress to the next check-in point, which is screen 14. Students may find it helpful or even necessary to review previous screens as they work, so be careful to ensure the pacing does not limit students' ability to look at previous screens.

On **screen 8**, students are asked to read the User Manual—Definitions handout, which contains definitions and examples regarding electrical circuits. Learning about computations with electrical circuits is a real-world application of operations with complex numbers.

On **screens 9–10**, students walk through examples 1 and 2 from the handout with a more thorough explanation of how to calculate the total impedance of a circuit. Going through each example unlocks the following screens.

Students on **screen 11** are asked to calculate the total impedance of a circuit on their own. Submitting the correct answer unlocks the next screen. On **screen 12**, students must find the total impedance of a second circuit. Again, submitting the correct answer unlocks the following screen.

On **screens 13–14**, students are given the voltage formula:  $\text{Voltage} = (\text{Current}) \cdot (\text{Impedance})$ . Students then are asked to calculate the voltage, given the current and impedance. Submitting a correct answer on each screen unlocks the next.

### Optional Teacher Resource

Use the attached **User Manual—Calculations (Sample Responses)** document as a teacher resource for questions 1–4, if needed. This document is also a teacher resource for questions 5–6 that are done during the Explain portion of this lesson.

### Teacher's Note: Purpose

Finding the total impedance requires students to add and subtract complex numbers and recall their knowledge of combining like terms. Calculating voltage requires students to multiply complex numbers and recall their knowledge of expanding binomials and simplifying  $i$  to a power. The goal is for these real-world connections to help students understand why they are learning about operations with complex numbers.

20 minutes

## Explain

### Teacher's Note: Lesson Pacing

This lesson can be naturally paused and resumed at different points during the Explain portion, depending on how the class is progressing. Ideally, pause the lesson at the end of the Explain portion and resume the next day with the Extend portion.

You may find students need more time due to misconceptions with prerequisite knowledge. In this case, consider pausing the lesson after the video and question 5 and resuming the next day with question 6. You also could have students watch the video and do question 5 at home and resume the next day with question 6 in class. Alternatively, consider allowing the lesson to take three 45-minute class periods, as follows: Day 1, Engage & Explore; Day 2, Explain; and Day 3, Extend & Evaluate.

You do not actually need to pause the student pacing in Desmos.

On the Desmos dashboard, click the orange plus sign to allow students to progress to **screen 15**. On this screen, students are asked to use the voltage formula to calculate the current, given the voltage and impedance. This calculation requires students to divide complex numbers, which likely is not something they have done before.

In the upper-left corner of your screen, click the icon above the word "Pause." Then, ask the class what operation they need to use with complex numbers in this scenario. Ask for volunteers to explain why they need to divide complex numbers.

After the class discussion, introduce the "[My Imaginary Friend, Part 2](#)" video, which features an electrical engineer talking about his career, describing how he uses electrical circuits, and explaining how to divide complex numbers. Direct students' attention to the classroom screen and play the video.

### Embedded video

<https://youtube.com/watch?v=b4egCAQtzzk>

The video begins with the electrical engineer interview, which ends at the 12:45 mark. The rest of the video demonstrates how to divide complex numbers using a complex conjugate.

Alternatively, you may have students watch the video on their own through Desmos. In this case, ask students to use the link on screen 15. The question on screen 15 is the example provided in the video.

To allow students to resume the Desmos activity, click the icon above the word "Pause." Watching the video provides students with the correct answer to submit, which unlocks the following screen.

On the Desmos dashboard, click the orange plus sign to allow students to progress to **screen 16**. On this screen, students are asked to calculate the current on their own. Again, submitting a correct answer unlocks the next screen.

**Teacher's Note: ACT Prep**

There is often one question on the ACT about complex conjugates, and it's usually as simple as this: Students are given a complex number and asked to find its complex conjugate.

Use this lesson to help students understand that a conjugate is *not* the opposite of the number, which is a common misconception. If students can grasp why a complex conjugate is used to divide complex numbers, they may become more comfortable with this concept without feeling like they need to memorize its formal definition.

30 minutes

## Extend

On the Desmos dashboard, click the orange "Stop" button to allow students to progress through the rest of the screens at their own pace.

As students work with their partners, monitor student progress while walking around the room. Be sure not to immediately check answers or give assistance. It is important for students to have a little time to struggle as they work through these challenging problems.

### Optional Teacher Resource

You may use the last page of the attached **Lesson Guide and Escape Room Script** as a resource for checking codes and grading, if needed.

On **screen 17**, students continue to read the story. **Screen 18** prompts students to leave the Desmos activity and go to the provided link to the "Power Control Hub" [GeoGebra](https://www.geogebra.org/m/rwwffvkr) activity: [geogebra.org/m/rwwffvkr](https://www.geogebra.org/m/rwwffvkr).

In this activity, students must answer a series of questions about complex conjugates, dividing complex numbers, and simplifying radicals. Students should put their answers in the calibration dial (modified cipher wheel) within the GeoGebra activity to find a three-letter code.

Once they have the three-letter code, students should return to the Desmos activity to enter the code and unlock the next screen. If students enter a correct letter but do not capitalize the letter, they see an error message with a hint to capitalize the letter.

On **screen 19**, students continue reading the story. **Screen 20** prompts students to leave the Desmos activity and go to the provided link to the "Cargo Bay" GeoGebra activity: [geogebra.org/m/whtn2brj](https://www.geogebra.org/m/whtn2brj).

In this activity, students must answer a series of questions about adding and subtracting complex numbers, multiplying complex numbers, and simplifying to a power. Using the given flashlight tool, students should input their answers to find a three-letter code.

Once they have the three-letter code, students should return to the Desmos activity to enter the code and unlock the next screen. If students enter a correct letter but do not capitalize the letter, they see an error message with a hint to capitalize the letter.

On **screen 21**, students read the conclusion of their escape room story and learn they have escaped successfully.

5 minutes

## Evaluate

On **screen 22**, students are asked to use a modified version of the [Fist to Five](#) strategy to reflect on what they've learned during the lesson. The screen shows a thumbs-down icon and a thumbs-up icon at the ends of a scale. Students should move a point along the scale to show how confident they feel about each of the operations on complex numbers: addition and subtraction, multiplication, and division.

Students also have the option to type and elaborate on how they feel. There is not a submit button on this screen since the written responses are optional.

## Resources

- K20 Center. (n.d.). Fist to Five. Strategies. <https://learn.k20center.ou.edu/strategy/68>
- K20 Center. (n.d.). GeoGebra. Tech tools. <https://learn.k20center.ou.edu/tech-tool/2352>
- K20 Center. (n.d.). Desmos Classroom. Tech tools. <https://learn.k20center.ou.edu/tech-tool/1081>
- K20 Center. (n.d.). My Imaginary Friend, Part 2 cover image [Composite image]. K20 Center.
- Vecteezy. (n.d.). Rocket Ship Flying [Image]. Vecteezy. <https://www.vecteezy.com/vector-art/1871713-happy-successful-businessman-holding-goal-flag-standing-on-rocket-ship-flying-through-starry-sky-business-startup-concept>
- Vecteezy. (n.d.). Round Spaceship on White Background [Image]. Vecteezy. <https://www.vecteezy.com/vector-art/359100-round-spaceship-on-white-background>