GUIDED NOTES (TEACHER GUIDE)

Find the solution(s) to each of the following equations.

1) $-2(6n+1)^3+4=-6$

Teacher	Students
How do we isolate a variable?	Use inverse operations.
Remember that when we solve, we go backwards through the order of operations. Do we have any addition or subtraction **outside of the parentheses** we can undo?	Yes, we need to start by subtracting 4 from both sides. $-2(6n+1)^3+4=-6$ $-4 \quad -4$ $-2(6n+1)^3=-10$
Continuing to look outside of the parentheses, which operation should we undo next?	Divide both sides by -2. $\frac{-2(6n+1)^3}{-2}=\frac{-10}{-2}$ $(6n+1)^3=5$
What's the opposite of a power/exponent?	a radical/root
Let's take the third root of both sides, since that's the opposite of a third power.	$\sqrt[3]{(6n+1)^3}=\sqrt[3]{5}$ $6n+1=\sqrt[3]{5}$
Now, our parentheses are also gone, so we start back at the bottom of order of operations. Let's subtract 1 from both sides. Remember that we always write the radical on the end, so that minus 1 goes in front of the $\sqrt[3]{5}$.	$6n+1=\sqrt[3]{5}$ $-1 \quad -1$ $6n=-1+\sqrt[3]{5}$
Now, let's divide both sides by 6. And we get a final answer of $n=\frac{-1+\sqrt[3]{5}}{6}$.	$\frac{6n}{6}=\frac{-1+\sqrt[3]{5}}{6}$ $n=\frac{-1+\sqrt[3]{5}}{6}$

2) $3(x+1)^{\frac{4}{3}}=48$

<table>
<tr><th>Teacher</th><th>Students</th></tr>
<tr><td>What operation do we need to undo first?</td><td>Divide both sides by 3.
$\frac{3(x+1)^{\frac{4}{3}}}{3}=\frac{48}{3}$
$(x+1)^{\frac{4}{3}}=16$</td></tr>
<tr><td>Now, we have two options: We can solve this using radicals or rational exponents.
Let’s see what both options would look like. Let’s draw a T-chart under the equation and label each side.</td><td>$(x+1)^{\frac{4}{3}}=16$<table><tr><th>exponential</th><th>radical</th></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>What we’ve been given is already written with rational exponents, so we’ll copy that in the first column.</td><td>$(x+1)^{\frac{4}{3}}=16$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+1)^{\frac{4}{3}}=16$</td><td></td></tr></table></td></tr>
<tr><td>Remember that something to the four-thirds power is the third root of something to the power of 4. That power of 4 could be written inside or outside of the radical. It will make the numbers smaller and easier to work with if we write the power outside of the radical.</td><td>$(x+1)^{\frac{4}{3}}=16$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+1)^{\frac{4}{3}}=16$</td><td>$\left(\sqrt[3]{x+1}\right)^4=16$</td></tr></table></td></tr>
<tr><td>Let’s start with the left column and solve this using rational exponents, then we’ll come back and solve it again using radicals.</td><td></td></tr>
<tr><td>The only operation outside of the parentheses is the exponent of four-thirds. We need that exponent to be 1 so that we no longer need those parentheses.

Four-thirds times what is 1?</td><td>three-fourths</td></tr>
</table>

<table>
<tr><th>Teacher</th><th>Students</th></tr>
<tr><td>So, let’s raise each side to the power of three-fourths.

Remember that we need to write $\pm$ since we’ve technically taken an even root.</td><td>$(x+1)^{\frac{4}{3}}=16$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+1)^{\frac{4}{3}}=16$
$\left((x+1)^{\frac{4}{3}}\right)^{\frac{3}{4}}=(16)^{\frac{3}{4}}$
$x+1=\pm(16)^{\frac{3}{4}}$</td><td>$\left(\sqrt[3]{x+1}\right)^{4}=16$</td></tr></table></td></tr>
<tr><td>Now, let’s simplify the right-hand side of this equation by rewriting 16 as a base to a power.</td><td>$x+1=\pm(16)^{\frac{3}{4}}$
$x+1=\pm\left(2^{4}\right)^{\frac{3}{4}}$</td></tr>
<tr><td>Continuing to simplify the right-hand side…</td><td>$x+1=\pm\left(2^{4}\right)^{\frac{3}{4}}$
$x+1=\pm 2^{3}$
$x+1=\pm 8$</td></tr>
<tr><td>Now, let’s subtract 1 from both sides.

Again, notice we’re putting the plus or minus stuff on the end to continue that good habit.</td><td>$x+1=\pm 8$
$-1 \quad -1$
$x=-1\pm 8$</td></tr>
<tr><td>We can simplify further, so we should.
We get a final answer of $x=7$ and $x=-9$.</td><td>$x=-1+8$ and $x=-1-8$
$\boxed{x=7}$ and $\boxed{x=-9}$</td></tr>
<tr><td>Now, let’s go back and solve again, but this time using radicals.</td><td>$(x+1)^{\frac{4}{3}}=16$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+1)^{\frac{4}{3}}=16$</td><td>$\left(\sqrt[3]{x+1}\right)^{4}=16$</td></tr></table></td></tr>
<tr><td>We need to undo the power of 4. What’s the opposite of a power of 4?</td><td>a fourth root</td></tr>
</table>

Teacher	Students
So, let's take the fourth root of each side. Don't forget the $\pm$ with that even root.	$(x+1)^{\frac{4}{3}}=16$ **exponential:** $(x+1)^{\frac{4}{3}}=16$ **radical:** $\left(\sqrt[3]{x+1}\right)^4=16$ $\sqrt[4]{\left(\sqrt[3]{x+1}\right)^4}=\sqrt[4]{16}$ $\sqrt[3]{x+1}=\pm 2$
Now, what is the opposite of a third root?	Take each side to the power of 3. $\sqrt[3]{x+1}=\pm 2$ $\left(\sqrt[3]{x+1}\right)^3=(\pm 2)^3$ $x+1=\pm 8$
Now, let's subtract 1 from both sides. Again, notice we're putting the plus or minus stuff on the end to continue that good habit.	$x+1=\pm 8$ $-1\ \ -1$ $x=-1\pm 8$
We can simplify further, so we should. We get a final answer of $x=7$ and $x=-9$.	$x=-1+8$ *and* $x=-1-8$ $\boxed{x=7}$ *and* $\boxed{x=-9}$
Notice that we got the same answer each time.	

3) $(x+3)^{\frac{1}{4}}-8=-6$

<table>
<tr><th>Teacher</th><th>Students</th></tr>
<tr><td>If our goal is to solve for x, what should be our first step?</td><td>Add 8 to both sides.
$(x+3)^{\frac{1}{4}}-8=-6$
$+8 \quad +8$
$(x+3)^{\frac{1}{4}}=2$</td></tr>
<tr><td>Let's draw a T-chart under the equation and label each side.</td><td>$(x+3)^{\frac{1}{4}}=2$<table><tr><th>exponential</th><th>radical</th></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>What we've been given is already written with rational exponents, so we'll copy that in the first column.</td><td>$(x+3)^{\frac{1}{4}}=2$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+3)^{\frac{1}{4}}=2$</td><td></td></tr></table></td></tr>
<tr><td>A one-fourth power is what kind of root?</td><td>a fourth root</td></tr>
<tr><td>So, let's rewrite the equation with a radical and put it in the second column.</td><td>$(x+3)^{\frac{1}{4}}=2$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+3)^{\frac{1}{4}}=2$</td><td>$\sqrt[4]{x+3}=2$</td></tr></table></td></tr>
<tr><td>Looking at the rational exponents column, what is the opposite of a one-fourth power?</td><td>Take both sides to the power of 4.
$(x+3)^{\frac{1}{4}}=2$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$(x+3)^{\frac{1}{4}}=2$
$\left((x+3)^{\frac{1}{4}}\right)^4=(2)^4$
$x+3=16$</td><td>$\sqrt[4]{x+3}=2$</td></tr></table></td></tr>
</table>

Teacher	Students
What should our next step be? And we get a final answer of $x=13$.	Subtract 3 from both sides. $x+3=16$ $-3 \quad -3$ $\boxed{x=13}$
Now, let's solve this again, but this time using radicals.	$(x+3)^{\frac{1}{4}}=2$ exponential: $(x+3)^{\frac{1}{4}}=2$ radical: $\sqrt[4]{x+3}=2$
What's the opposite of a fourth root?	Take both sides to the power of 4. $(x+3)^{\frac{1}{4}}=2$ exponential: $(x+3)^{\frac{1}{4}}=2$ radical: $\sqrt[4]{x+3}=2$ $\left(\sqrt[4]{x+3}\right)^4=(2)^4$ $x+3=16$
What should our next step be? And we get a final answer of $x=13$.	Subtract 3 from both sides. $x+3=16$ $-3 \quad -3$ $\boxed{x=13}$
Wait a minute. I saw an even root in our problem—why does our answer not have a plus or minus symbol?	The fourth root was already in the problem—we did not take an even root. We were given an even root, so there is only one solution.

4) $-31=-4(3m)^{\frac{2}{3}}+5$

<table>
<tr><th>Teacher</th><th>Students</th></tr>
<tr><td>Where should we start?</td><td>Subtract 5 from both sides.
$-31=-4(3m)^{\frac{2}{3}}+5$
$-5 \qquad\qquad -5$
$-36=-4(3m)^{\frac{2}{3}}$</td></tr>
<tr><td>What operation should we perform next?</td><td>Divide both sides by -4.
$\frac{-36}{-4}=\frac{-4(3m)^{\frac{2}{3}}}{-4}$
$9=(3m)^{\frac{2}{3}}$</td></tr>
<tr><td>Now, we see that rational exponent, so for our notes, we're going to make a table to show both methods.</td><td>$9=(3m)^{\frac{2}{3}}$<table><tr><th>exponential</th><th>radical</th></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>Fill in each column with that first line.

Consider rewriting 9 as a base to a power in the rational exponents column.

When we write that radical, should the power go inside or outside of the radical?</td><td>The power should go on the outside.
$9=(3m)^{\frac{2}{3}}$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$3^2=(3m)^{\frac{2}{3}}$</td><td>$9=\left(\sqrt[3]{3m}\right)^2$</td></tr></table></td></tr>
<tr><td>Let's solve the equation using rational exponents. What is the opposite of a two-thirds power?

Don't forget the $\pm$; we did take an even root.</td><td>Take both sides to the power of three-halves.
$9=(3m)^{\frac{2}{3}}$<table><tr><th>exponential</th><th>radical</th></tr><tr><td>$3^2=(3m)^{\frac{2}{3}}$
$\left(3^2\right)^{\frac{3}{2}}=\left((3m)^{\frac{2}{3}}\right)^{\frac{3}{2}}$
$\pm 3^3=3m$
$\pm 27=3m$</td><td>$9=\left(\sqrt[3]{3m}\right)^2$</td></tr></table></td></tr>
</table>

Teacher	Students
Now, let's divide both sides by 3 and get a final answer of positive 9 and negative 9.	$\frac{\pm 27}{3} = \frac{3m}{3}$ $\boxed{\pm 9 = m}$
One more time, but this time with radicals.	$9 = (3m)^{\frac{2}{3}}$ exponential: $3^2 = (3m)^{\frac{2}{3}}$ radical: $9 = \left(\sqrt[3]{3m}\right)^2$
What is the opposite of a second power?	Take the second root (or square root) of both sides. $9 = (3m)^{\frac{2}{3}}$ exponential: $3^2 = (3m)^{\frac{2}{3}}$ radical: $9 = \left(\sqrt[3]{3m}\right)^2$ $\sqrt{9} = \sqrt{\left(\sqrt[3]{3m}\right)^2}$ $\pm 3 = \sqrt[3]{3m}$
What is the opposite of a third root?	Take both sides to the power of 3. $\pm 3 = \sqrt[3]{3m}$ $(\pm 3)^3 = \left(\sqrt[3]{3m}\right)^3$ $\pm 27 = 3m$
Now, let's divide both sides by 3. We get a final answer of plus or minus 9.	$\frac{\pm 27}{3} = \frac{3m}{3}$ $\boxed{\pm 9 = m}$

GUIDED NOTES (MODEL NOTES)

Find the solution(s) to each of the following equations.

1)

$$-2(6n+1)^3+4=-6$$
$$-4 \quad -4$$
$$\frac{-2(6n+1)^3}{-2}=\frac{-10}{-2}$$
$$(6n+1)^3=5$$
$$\sqrt[3]{(6n+1)^3}=\sqrt[3]{5}$$
$$6n+1=\sqrt[3]{5}$$
$$-1 \quad -1$$
$$6n=-1+\sqrt[3]{5}$$
$$\frac{6n}{6}=\frac{-1+\sqrt[3]{5}}{6}$$
$$\boxed{n=\frac{-1+\sqrt[3]{5}}{6}}$$

2)

$$3(x+1)^{\frac{4}{3}}=48$$
$$\frac{3(x+1)^{\frac{4}{3}}}{3}=\frac{48}{3}$$
$$(x+1)^{\frac{4}{3}}=16$$

exponential	*radical*
$(x+1)^{\frac{4}{3}}=16$	$\left(\sqrt[3]{x+1}\right)^4=16$
$\left((x+1)^{\frac{4}{3}}\right)^{\frac{3}{4}}=(16)^{\frac{3}{4}}$	$\sqrt[4]{\left(\sqrt[3]{x+1}\right)^4}=\sqrt[4]{16}$
$x+1=\pm(16)^{\frac{3}{4}}$	$\sqrt[3]{x+1}=\pm 2$
$x+1=\pm\left(2^4\right)^{\frac{3}{4}}$	$\left(\sqrt[3]{x+1}\right)^3=(\pm 2)^3$
$x+1=\pm 2^3$	$x+1=\pm 8$
$x+1=\pm 8$	$x=-1+8$ *and* $x=-1-8$
$-1 \quad -1$	$\boxed{x=7}$ *and* $\boxed{x=-9}$
$x=-1\pm 8$	
$x=-1+8$ *and* $x=-1-8$	
$\boxed{x=7}$ *and* $\boxed{x=-9}$	

even root

Write the power outside of the radical to keep the numbers smaller (so it's easier).

3)

$$(x+3)^{\frac{1}{4}}-8=-6$$
$$+8 \quad +8$$
$$(x+3)^{\frac{1}{4}}=2$$

exponential	*radical*
$(x+3)^{\frac{1}{4}}=2$	$\sqrt[4]{x+3}=2$
$\left((x+3)^{\frac{1}{4}}\right)^4=(2)^4$	$\left(\sqrt[4]{x+3}\right)^4=(2)^4$
$x+3=16$	$x+3=16$
$-3 \quad -3$	$-3 \quad -3$
$\boxed{x=13}$	$\boxed{x=13}$

4)

$$-31=-4(3m)^{\frac{2}{3}}+5$$
$$-5 \qquad -5$$
$$\frac{-36}{-4}=\frac{-4(3m)^{\frac{2}{3}}}{-4}$$
$$9=(3m)^{\frac{2}{3}}$$

exponential	*radical*
$9=(3m)^{\frac{2}{3}}$	$9=\left(\sqrt[3]{3m}\right)^2$
$\left(3^2\right)^{\frac{3}{2}}=\left((3m)^{\frac{2}{3}}\right)^{\frac{3}{2}}$	$\sqrt{9}=\sqrt{\left(\sqrt[3]{3m}\right)^2}$
$\pm 3^3=3m$	$\pm 3=\sqrt[3]{3m}$
$\pm 27=3m$	$(\pm 3)^3=\left(\sqrt[3]{3m}\right)^3$
$\boxed{\pm 9=m}$	$\pm 27=3m$
	$\boxed{\pm 9=m}$

even root