GUIDED NOTES (TEACHER GUIDE)

Find the solution(s) to each of the following equations. Check for extraneous solutions.

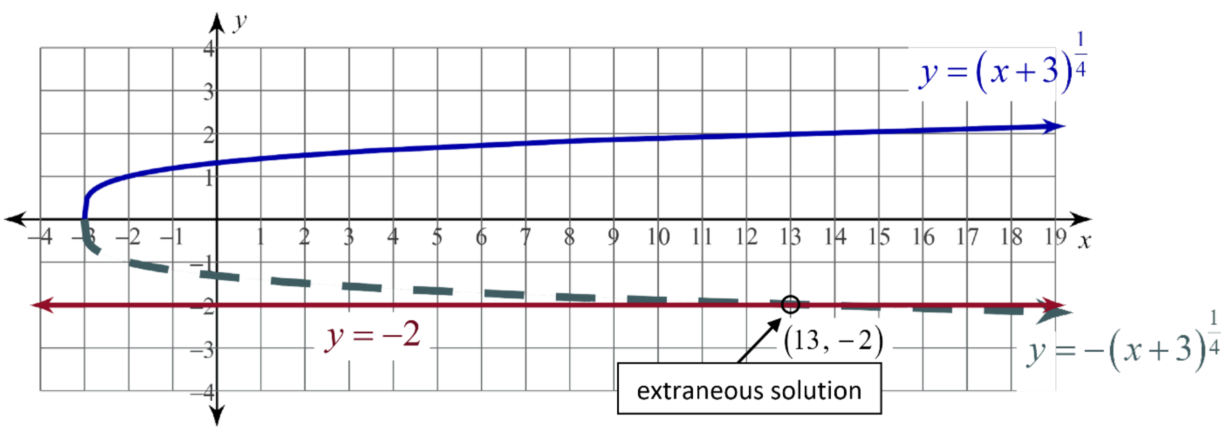
1) $(x+3)^{\frac{1}{4}}+8=6$

Why do we have extraneous solutions? Where do they come from?

Extraneous solutions often arise when we take both sides to an even power. Consider the equation $2=-2$. We know that 2 does not equal -2; that is not a true statement.

But let's square both sides: $(2)^2=(-2)^2$. Now, it is a true statement: $4=4$.

Basically, that is how extraneous solutions "sneak" into our work. Think about example 1 once we isolated the expression to the rational power: $(x+3)^{\frac{1}{4}}=-2$. Let's graph each side of that equation: y = (left side) and y = (right side). Below, the blue curve is the left side of our equation, and the red line is the right side. The dashed curve is the negative of our blue equation—the opposite of what we actually have. However, if it were taken to the power of 4—an even power—it would have the same result as taking what we do have to the power of 4.



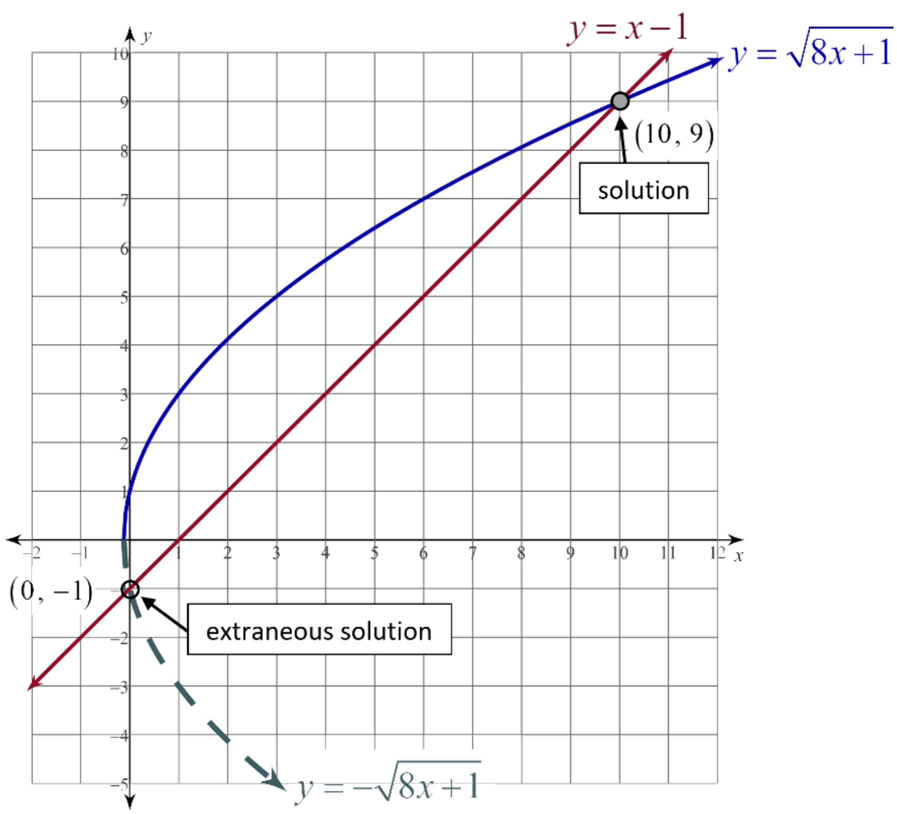
Notice that the blue curve and red line never intersect, but the dashed curve and red line do. Where the dashed curve and red line intersect is the extraneous solution, and since the blue curve and red line never intersect, there is **no solution**.

For example, $-(x+3)^{\frac{1}{4}}=-2$ would have a solution of $x=13$. But for $(x+3)^{\frac{1}{4}}=-2$, our actual equation, $x=13$ is **extraneous**.

2) $\sqrt{8x+1} = x-1$

Why did we get both a solution and an extraneous solution?

Think about example 2: $\sqrt{8x+1} = x-1$. Again, let's graph each side of that equation: y = (left side) and y = (right side). Below, the blue curve is the left side of our equation, and the red line is the right side. The dashed curve is the negative of our blue equation—the opposite of what we actually have. However, if it were squared—in other words, taken to an even power—it would have the same result as squaring what we do have.



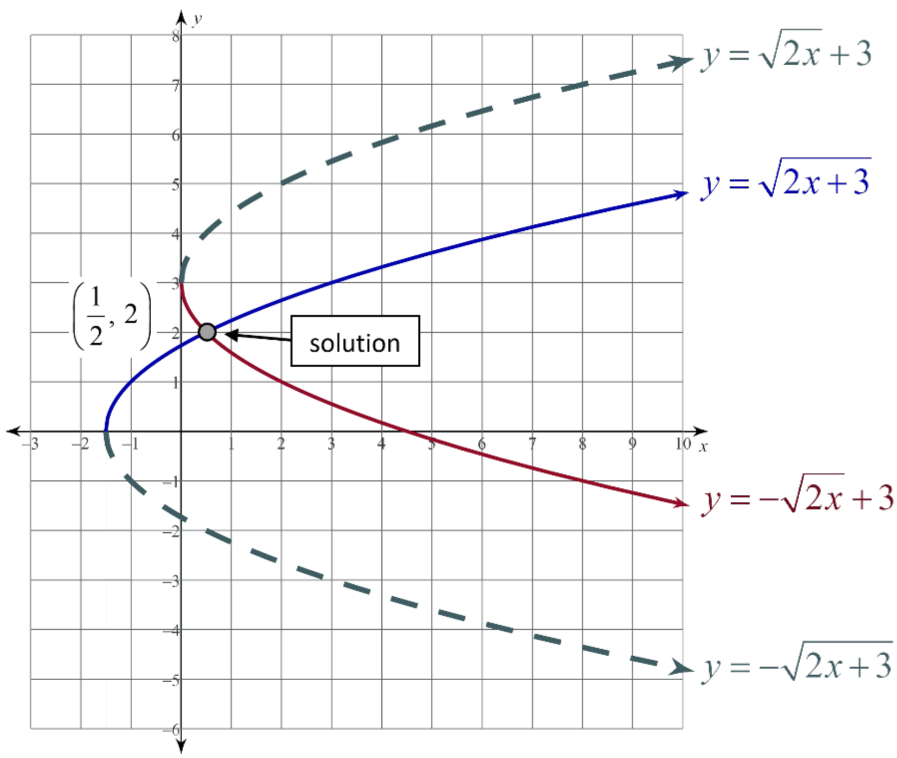
Notice that the blue curve and red line intersect when $x = 10$. This is the **1 solution** to our actual equation. Meanwhile, the dashed curve and red line intersect when $x = 0$, so this is our **extraneous solution**.

For example, $-\sqrt{8x+1} = x-1$ would have a solution of $x = 0$. But for $\sqrt{8x+1} = x-1$, our actual equation, $x = 10$ is the solution, while $x = 0$ is **extraneous**.

3) $\sqrt{2x+3}+\sqrt{2x}=3$

When is there not an extraneous solution?

Think about example 3 once we isolated the radical: $\sqrt{2x+3}=3-\sqrt{2x}$. Again, let's graph each side of that equation: y = (left side) and y = (right side). Below, the blue curve is the left side of our equation, and the red curve is the right side. The dashed curves are the negatives of our blue and red equations—the opposites of what we actually have.



Notice that the blue and red curves intersect when $x=\frac{1}{2}$. This is the **1 solution** to our actual equation. Nowhere else do any of the curves intersect, which is why there is **no extraneous solution**. There is only one point of intersection, so $x=\frac{1}{2}$ is the solution, and there is not an extraneous solution.

GUIDED NOTES (MODEL NOTES)

Definition

Extraneous solution: an extra result that does not make the original equation true

Example: Solve: $\sqrt{m} = -1$

$$\left(\sqrt{m}\right)^2 = (-1)^2$$

But $m = 1$ does not work: $m = 1$ $\quad \sqrt{(1)} \neq -1$

Example Problems

Find the solution(s) to each of the following equations. Check for extraneous solutions.

1) $(x+3)^{\frac{1}{4}} + 8 = 6$

$$-8 \quad -8$$

$$(x+3)^{\frac{1}{4}} = 2$$

$$\left((x+3)^{\frac{1}{4}}\right)^4 = (2)^4$$

$$x + 3 = 16$$

$$-3 \quad -3$$

$$x = 13$$

$\boxed{\text{no solution}}$

$x = 13$ is an extraneous solution

Check: $((13)+3)^{\frac{1}{4}} + 8 = 6$?

$$(16)^{\frac{1}{4}} + 8 = 6 \text{ ?}$$

$$2 + 8 \neq 6$$

2) $\sqrt{8x+1} = x - 1$

$$\left(\sqrt{8x+1}\right)^2 = (x-1)^2$$

$$8x + 1 = x^2 - 2x + 1$$

$$-8x - 1 \qquad -8x - 1$$

$$0 = x^2 - 10x$$

$$0 = x(x-10)$$

$$x = 0 \text{ and } x - 10 = 0$$

$$x = 0 \text{ and } x = 10$$

$\boxed{x = 10}$

$x = 0$ is an extraneous solution

Check: $\sqrt{8(0)+1} = (0) - 1$? $\qquad \sqrt{8(10)+1} = (10) - 1$?

$$\sqrt{1} = -1 \text{ ?} \qquad \sqrt{81} = 9 \text{ ?}$$

$$1 \neq -1 \qquad 9 = 9$$

Hint: *When there are multiple radicals, isolate one radical, then take both sides to the power. Repeat as needed.*

3) $\sqrt{2x+3} + \sqrt{2x} = 3$

$$-\sqrt{2x} \quad -\sqrt{2x}$$

$$\sqrt{2x+3} = 3 - \sqrt{2x}$$

$$\left(\sqrt{2x+3}\right)^2 = \left(3 - \sqrt{2x}\right)^2$$

$$2x + 3 = 9 - 3\sqrt{2x} - 3\sqrt{2x} + 2x$$

$$2x + 3 = 2x + 9 - 6\sqrt{2x}$$

$$-2x - 9 \quad -2x - 9$$

$$-6 = -6\sqrt{2x}$$

$$\frac{-6}{-6} = \frac{-6\sqrt{2x}}{-6}$$

$$1 = \sqrt{2x}$$

$$(1)^2 = \left(\sqrt{2x}\right)^2$$

$$1 = 2x$$

$$\frac{1}{2} = \frac{2x}{2}$$

$$\frac{1}{2} = x$$

Check:

$$\sqrt{2\left(\frac{1}{2}\right) + 3} + \sqrt{2\left(\frac{1}{2}\right)} = 3 \text{ ?}$$

$$\sqrt{1+3} + \sqrt{1} = 3 \text{ ?}$$

$$\sqrt{4} + 1 = 3 \text{ ?}$$

$$2 + 1 = 3$$

$$\boxed{x = \frac{1}{2}}$$