GRAPHING RATIONAL FUNCTIONS: GUIDED NOTES

Definitions

Rational function: $\dfrac{p(x)}{q(x)} = \dfrac{a_m x^m + \ldots + a_0}{b_n x^n + \ldots + b_0}$

Vertical asymptote: a line that a curve approaches and **never** crosses, because we can't divide by zero

Horizontal asymptote: a line with a slope of zero that the curve approaches and **sometimes** crosses*

Slant (oblique) asymptote: a line with a slope other than zero (and is not undefined) that the curve approaches and **sometimes** crosses*

vertical asymptote(s): denominator $\neq 0$

slant asymptote: when $n + 1 = m$
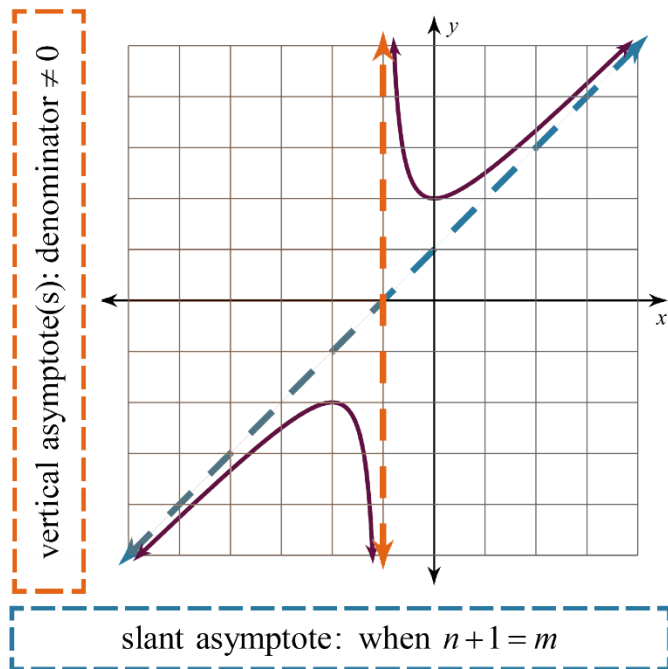
The curve is allowed to cross horizontal and slant asymptotes.

- Rational functions can have 0, 1, 2, … vertical asymptotes.
- Rational functions can have 0 or 1 horizontal asymptotes.
- Rational functions can have 0 or 1 slant asymptotes.

How to Graph a Rational Function

Step 1) Find the asymptote(s).

- If the degree on the top is greater than the degree on the bottom, then the ratio for a horizontal asymptote would be a number over zero, which is undefined. Because of this, there is **no horizontal asymptote** when $m > n$.

- If the degree on the top is only 1 greater than the degree on the bottom, then you have a **slant asymptote**.

Step 2) Sketch the asymptote(s) with dashed lines.

- Do not worry about sketching slant asymptotes at this time.

Step 3) Make a table.

- Pick x-values based on the vertical asymptote(s).

- If there is no vertical asymptote, then let $x = 0$ be the middle number in your table.
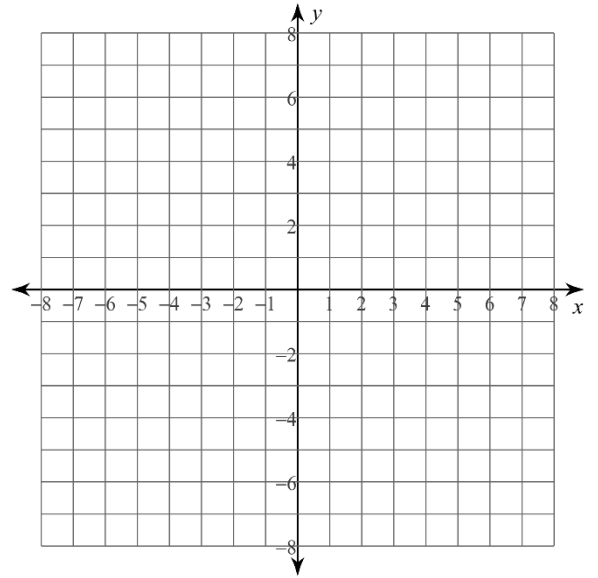
Step 4) Plot points and connect dots.

Examples

Graph each function. Be sure to label the asymptote(s).

1) $y = \dfrac{8x}{x^2 - 9}$

x	y

2) $y = \dfrac{x^2 + 4x}{2x - 1}$

x	y