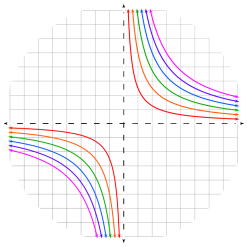

Can't Touch This, Part 2

Graphing Rational Functions

Michell Eike, Ryan Rahhal
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Grade Level	10th – 11th Grade	**Time Frame**	90-120 minutes
Subject	Mathematics	**Duration**	2-3 periods
Course	Algebra 2		

Essential Question

What can cause asymptotes?

Summary

In this lesson, students will work with rational functions that have 0, 1, or 2 vertical asymptotes and 0 or 1 horizontal asymptotes. Students will explore the relationship between the equation and graph of a rational function, learn what causes different types of asymptotes, and apply their knowledge to graph rational functions. This is the second lesson of two in the "Can't Touch This" lesson series—see "Can't Touch This, Part 1" for prerequisite content.

Snapshot

Engage

Students organize graphs of rational functions that have 0, 1, or 2 vertical asymptotes and 0 or 1 horizontal asymptotes into groups through a Card Sort activity.

Explore

Students explore the relationship between the equation and the graph of a rational function.

Explain

Students complete guided notes with the class and formalize their understanding of graphing rational functions.

Extend

Students apply what they have learned to graph rational functions.

Evaluate

Students match graphs of rational functions with asymptotes and equations in a Card Matching activity.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

F508: Find the domain of polynomial functions and rational functions
F510: Find where a rational function's graph has a vertical asymptote

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.F.1.6: Graph a rational function and identify the domain (including holes), range, x- and y-intercepts, vertical and horizontal asymptotes, using various methods and tools that may include a graphing calculator or other appropriate technology (excluding slant or oblique asymptotes).

Attachments

- Card Matching—Can't Touch This Part 2.pdf
- Card Sort—Can't Touch This Part 2.pdf
- Exploring Rational Functions—Can't Touch This Part 2 - Spanish.docx
- Exploring Rational Functions—Can't Touch This Part 2 - Spanish.pdf
- Exploring Rational Functions—Can't Touch This Part 2.docx
- Exploring Rational Functions—Can't Touch This Part 2.pdf
- Graphing With Asymptotes Part 2—Can't Touch This Part 2 - Spanish.docx
- Graphing With Asymptotes Part 2—Can't Touch This Part 2 - Spanish.pdf
- Graphing With Asymptotes Part 2—Can't Touch This Part 2.docx
- Graphing With Asymptotes Part 2—Can't Touch This Part 2.pdf
- Guided Notes Teacher Guide and Model Notes—Can't Touch This Part 2.pdf
- Guided Notes—Can't Touch This Part 2 - Spanish.docx
- Guided Notes—Can't Touch This Part 2 - Spanish.pdf
- Guided Notes—Can't Touch This Part 2.docx
- Guided Notes—Can't Touch This Part 2.pdf
- Lesson Slides—Can't Touch This Part 2.pptx

Materials

- Desmos account
- Exploring Rational Functions handout (attached; one per pair; printed front only)
- Guided Notes handout (attached; one per student; printed front/back)
- Pencil
- Student devices with internet access

10 minutes

Engage

Teacher's Note: Desmos Activity Preparation

To use this Desmos Classroom activity, select the following link: "Can't Touch This, Part 2." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 4. Select "Restrict to Screens 1–4" to confirm your selection. This allows students to access only screens 1–4 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to student.desmos.com and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1-2** of the Desmos activity. **Screen 1** shows the lesson's essential question: *What can cause asymptotes?* **Screen 2** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Ask students to find a partner or assign partners yourself. Using the Card Sort strategy, have pairs move to **screen 3** and work together to group the nine graph cards into two, three, or four groups of their choosing. After students have had a chance to organize their cards into groups, instruct students to go to **screen 4** to describe their thinking. Upon submitting their response, they should be able to see the responses of other classmates. If time allows, have students find another pair of students and discuss their thinking. Then, ask for volunteers to share with the class how they chose to organize their cards or how they would describe their groups of cards.

20 minutes

Explore

On your Desmos Dashboard, press the orange plus sign to allow students to progress to **screen 5.** Direct students to read the directions on their screen and to select the GeoGebra activity link: geogebra.org/m/d9ywzkrc. This interactive GeoGebra activity includes two GeoGebra applets: the first is focused on vertical asymptotes, while the second is focused on horizontal and slant asymptotes. Invite students to interact with both. This gives students a chance to explore rational functions that do not always have one vertical and one horizontal asymptote as they saw in the previous lesson, "Can't Touch This, Part 1."

After giving students time to explore the GeoGebra activity, pass out one of the attached **Exploring Rational Functions** handouts to each pair of students. Instruct students to work with their partner to complete the handout. In each applet, there is a reset button in the top-right corner that looks like two arrows making a circle. Encourage students to use this button if their exploration makes the graph difficult to see.

Sample Student Responses

As students explore vertical asymptotes, they may observe the following:

- When m and n are both 1, b_0 shifts the vertical asymptote left and right.
- When m and n are both 2 and b_0 is positive, there are two vertical asymptotes. The graph is broken into three pieces.
- When m and n are both 2 and b_0 is negative, there are zero vertical asymptotes. The graph looks like a bump.

As students explore horizontal and slant asymptotes, they may observe the following:

- When n = 1 and m = 0 or 1, the graph looks like what we saw in the previous lesson—one vertical and one horizontal asymptote.
- When n = 1 and m = 2, the horizontal asymptote is not horizontal—maybe it's a slant asymptote. When the graph is zoomed out, it looks like some of the graphs in part 1 of this lesson, but the asymptote is a line with a positive slope.
- When n = 2 and m = 0 or 2, there are two vertical asymptotes.
- When n = 2 and m = 1 or 3, the curve crosses an asymptote—one was horizontal and one wasn't.
- When n = 1 and m = 3, the graph looks weird, like a non-linear asymptote and a vertical asymptote.

After students complete the GeoGebra activity, ask for volunteers to share their observations with the class.

Teacher's Note: Guiding the Activity

Students may make observations that significantly differ from the sample responses above. Use their responses to check for misunderstandings. Encourage students to do their best to use academic language to describe what they see.

Regarding the non-linear asymptote, students are not expected to find the equations of non-linear asymptotes or graph equations with non-linear asymptotes in high school. The purpose here is exposure and to emphasize that slant asymptotes exist when the degree of the numerator is only one greater than the degree of the denominator. Students are often expected to write the equations for and graph slant asymptotes in precalculus courses. In Algebra 2, students only need to recognize that they exist and that not every rational function has a horizontal asymptote. This Desmos activity is designed to show students that slant asymptotes exist without asking students to find the equation of one.

25 minutes

Explain

Give each student a copy of the attached **Guided Notes** handout, then press the orange plus sign on your Desmos Dashboard three times to allow students to progress to **screens 6–8**. Direct students to use Desmos to complete the handout.

Have students read the definitions on screen 6 and complete example 1 on screen 7. Next, have the class come together to discuss and ask questions. Ask for volunteers to explain how to find vertical and horizontal asymptotes in their own words. Ask students to explain why the example 1 graph had two vertical asymptotes. Ask students about the "DNE" in the y-column. Ask students to describe what the graph is doing at x = 0, and why it is possible that the curve crossed the horizontal asymptote. Use student responses to help clarify any misunderstandings.

Have students continue working through example 2 on screen 8. Then, have the class come together to discuss and ask questions once again. Ask for volunteers—preferably different volunteers than those who explained example 1—to share in their own words the process of graphing rational functions. Use student responses to help clarify any misunderstandings.

Once finished, have students add their completed Guided Notes to their math notebook if that is a classroom norm.

25 minutes

Extend

Have each pair of students find another pair and partner up to create groups of four students. On your Dashboard, press the orange plus sign to allow students to progress to **screen 9**. Instruct students to work with their group to graph the first rational function.

Teacher's Note: Feedback From Desmos

Note that the Desmos activity is designed to withhold feedback until the student has entered values for the vertical and horizontal asymptotes and the x-column. Once the student has attempted these, Desmos provides specific feedback to direct the student toward what, if anything, needs to be corrected. If they have entered the correct values, then the graph shows the "connecting dots" animation.

When groups finish question 1, press the orange plus sign on the Dashboard to allow students to progress to **screen 10**. Instruct students to work with just one person from their original group of four to complete question 2.

When students complete question 2, press the orange plus sign on the Dashboard to allow students to progress to **screen 11**. Challenge students to now work independently to complete question 3. Remind students that this is a great opportunity to reflect on what they know and what questions they may have.

10 minutes

Evaluate

On the Dashboard, click the orange "Stop" button; this allows students to complete the rest of the Desmos activity at their own pace.

Direct each student to find their original partner. Go to **screen 12**. Instruct students to complete their Card Matching activity by matching a graph card, an asymptotes card, and an equation card together. Remind students that there will be six sets of cards, so three of the graphs will not have matching asymptotes or equation cards. These three cards should not be matched together.

While students are working, walk around the classroom and ask students to share their thinking about certain matches you see they've made. Remember, again, to encourage academic vocabulary. While circulating the room, use what you hear students talking about and what questions they ask to see what misconceptions still exist. Inform students that they can check their work on **screen 13**, where Desmos shows how many cards out of 21 are correctly matched (including the three unmatchable graphs).

Resources

- K20 Center. (n.d.). Card Matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- K20 Center. (n.d.). Card Sort. Strategies. https://learn.k20center.ou.edu/strategy/147
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). GeoGebra. Tech tools. https://learn.k20center.ou.edu/tech-tool/2352