GUIDED NOTES (TEACHER GUIDE)

Notation	Comments
<u>Common Logarithm</u> $\log_{10} x = \log x$	*Students need to see that when the base is 10 on a logarithm, they don't need to write it. Consider comparing it to the index of 2 on a radical—traditionally, we just write it as a square root and do not write the 2.* *They also need to understand that the "log" button on their calculator is referring to a logarithm with a base of 10.*
<u>Natural Logarithm</u> $\log_e x = \ln x$ (where e is Euler's number: $2.7...$)	*Similarly, students need to understand that a natural logarithm is a logarithm with a base of e. They also need to understand that this is what the "ln" button on their calculator means.* *Students often ask why it is "ln" and not "nl." This is due to its translation from Latin: logarithmus naturalis. In English, we say the adjective first, so we say, "natural logarithm," but we write "ln."*

Inverse Operations	Comments
$10^{\log(x)} = x$ $2^{\log_2 x} = x$ $\ln\left(e^x\right) = x$ $\log_5\left(5^x\right) = x$	*The key takeaway here is for students to see that exponential operations are the inverse of logarithmic operations, in the same way that addition is the inverse of subtraction and squaring is the opposite of taking a square root.* *Students also need to see that the parentheses often help with readability, but they are not always required with logarithms.*

Solve each of the following equations.

1) $3^{x-7} = 27^{2x}$

Teacher	Students
Here, we see an exponential function equals an exponential function, but the bases are not the same. Could we rewrite 3 or 27 such that they would have the same base?	Yes, we can write 27 as 3 cubed.
So, let's rewrite our equation and replace 27 with 3^3.	$3^{x-7} = 27^{2x}$ $3^{x-7} = \left(3^3\right)^{2x}$
How can we simplify the right side of this equation?	Multiply the powers. $3^{x-7} = 3^{3\cdot 2x}$ $3^{x-7} = 3^{6x}$
Now, we see that the bases are equal. If the left- and right-hand sides of this equation are equal and the bases are equal, what else must be equal?	the exponents/powers
Since the equations are equal and have the same base, the powers must be equivalent. Let's write the power on the left equals the power on the right.	$x - 7 = 6x$
Now, let's solve for x.	$-7 = 5x$ $\boxed{-\frac{7}{5} = x}$
On this example, we used the property of equality to solve this exponential equation. Let's look at this problem again, but this time let's use a logarithm, which is the inverse of the exponential function.	

Teacher	Students
The first few steps are the same, no matter which approach we take, so we'll start with the step where both sides are written as 3 to a power and then simplified.	$3^{x-7} = 3^{6x}$
We know that a logarithmic function is the opposite of an exponential function, as long as the bases of each are the same. So, we're going to apply the log with a base of what to both sides?	base 3
Now, we'll take the log base 3 of both sides. See how the log with a base of 3 and the exponential with a base of 3 cancel out because they are inverses, leaving us with $x-7=6x$.	$\log_3\left(3^{x-7}\right)=\log_3\left(3^{6x}\right)$ $x-7=6x$
Let's solve for x, just like we did before.	$-7=5x$ $\boxed{-\frac{7}{5}=x}$

2) $4-2e^{x}=-23$

Teacher	Students
Here, we see e in our equation. Remember that this is Euler's number, and it's just like the 3 in the previous example—it is the base of this exponential equation. But before we worry about that exponential portion of the equation, what should be our first step to isolate this variable?	Subtract 4 from both sides. $\begin{array}{r} 4-2e^{x}=-23 \\ \underline{-4 \qquad\quad -4} \\ -2e^{x}=-27 \end{array}$
Now, we have $-2e^{x}=-27$. What should we do next?	Divide both sides by -2. $\frac{-2e^{x}}{-2}=\frac{-27}{-2}$ $e^{x}=\frac{27}{2}$
If we want to get rid of that exponential with a base of e, what should we do?	Take the log base e of both sides.
Perfect! And what is the special name for a logarithm with a base of e?	a natural logarithm
So, let's take the natural log of both sides. The $\ln\left(\frac{27}{2}\right)$ can't be simplified further, so that is our final answer.	$\ln\left(e^{x}\right)=\ln\left(\frac{27}{2}\right)$ $\boxed{x=\ln\left(\frac{27}{2}\right)}$

3) $10^{-12x} + 6 = 100$

Teacher	Students
What should be our first step?	Subtract 6 from both sides. $$\begin{array}{r} 10^{-12x} + 6 = 100 \\ -6 \quad -6 \\ \hline 10^{-12x} = 94 \end{array}$$
What should we do next?	Take the log base 10 of both sides. $$\log\left(10^{-12x}\right) = \log(94)$$
Now, how does that simplify?	The log and exponential, both with a base of 10, are inverses, so the left is now just $-12x$. And the right side doesn't reduce. $$-12x = \log(94)$$
How do we isolate our variable?	Divide both sides by -12. $$\frac{-12x}{-12} = \frac{\log(94)}{-12}$$ $$\boxed{x = \frac{\log(94)}{-12}}$$

Consideration

Depending on the type of calculator available to your students, they may or may not need to learn about the change of base formula during this lesson. After the Guided Notes, students will apply what they have learned to solve real-world problems that need a decimal approximation. Let the available resources determine if the change of base formula should be taught during this lesson or the next: "All About That Base, Part 2."

Change of Base Formula	Comments
$\log_b a = \frac{\log a}{\log b}$ $\log_b a = \frac{\ln a}{\ln b}$ *In the next lesson, this formula will be proven.*	*The change of base formula also can be written as* $\log_m n = \frac{\log_b n}{\log_b m}$ *for any base* b*. However, the formula versions provided in the Guided Notes are typically the most useful, as they allow students to get a decimal approximation easily using a scientific calculator.* *Consider helping students remember the formula this way: "The* ***base*** *goes on the* ***bottom****."* $\log_b a = \frac{\log a}{\log b} \begin{matrix} \leftarrow \log(original\ input) \\ \leftarrow \log(original\ base) \end{matrix}$

Use the change of base formula to rewrite the logarithmic expression below.

4) $\log_3 10 =$

Teacher	Students
Using the common log version of this formula, how would we rewrite the expression $\log_3 10 = ?$	We write the log of 10 over the log of 3. $\log_3 10 = \boxed{\frac{\log 10}{\log 3}}$
Using the natural log version of this formula, how would we rewrite the expression $\log_3 10 = ?$	We write the natural log of 10 over the natural log of 3. $\log_3 10 = \boxed{\frac{\ln 10}{\ln 3}}$

Encourage students to type each fraction into a scientific calculator to see that they get the same result, $2.095...$*, whether they use a common log or a natural log.*

GUIDED NOTES (MODEL NOTES)

Notation

Common Logarithm

$\log_{10} x = \log x$

Natural Logarithm

$\log_e x = \ln x$

(where e is Euler's number: $2.7\ldots$)

Inverse Operations: Exponential and Logarithmic

$10^{\log(x)} = x$

$\ln\left(e^x\right) = x$

$2^{\log_2 x} = x$

$\log_5\left(5^x\right) = x$

Examples

Solve each of the following equations.

1) $3^{x-7} = 27^{2x}$

$3^{x-7} = \left(3^3\right)^{2x}$

$3^{x-7} = 3^{6x}$

$x - 7 = 6x$

$-7 = 5x$

$\boxed{-\frac{7}{5} = x}$

$\log_3\left(3^{x-7}\right) = \log_3\left(3^{6x}\right)$

$x - 7 = 6x$

$-7 = 5x$

$\boxed{-\frac{7}{5} = x}$

2) $4 - 2e^x = -23$

$-2e^x = -27$

$e^x = \frac{27}{2}$

$\ln\left(e^x\right) = \ln\left(\frac{27}{2}\right)$

$\boxed{x = \ln\left(\frac{27}{2}\right)}$

3) $10^{-12x} + 6 = 100$

$10^{-12x} = 94$

$\log\left(10^{-12x}\right) = \log(94)$

$-12x = \log(94)$

$\boxed{x = \frac{\log(94)}{-12}}$

Change of Base

$\log_b a = \frac{\log a}{\log b}$

$\log_b a = \frac{\ln a}{\ln b}$

Use the change of base formula to rewrite the logarithmic expression below.

4) $\log_3 10 = \boxed{\frac{\log 10}{\log 3}}$ or $\boxed{\frac{\ln 10}{\ln 3}}$