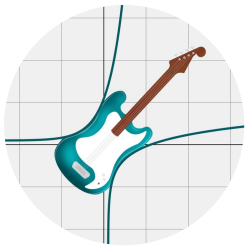

All About That Base, Part 1

Solving Exponential Equations

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Published by *K20 Center*

Grade Level	10th – 11th Grade	**Time Frame**	90 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Algebra 2		

Essential Question

How do we use logarithms to solve exponential equations?

Summary

In this lesson, students will discover how to evaluate logarithms and use that knowledge to write equations in logarithmic and exponential forms. Students will learn how to solve exponential equations with and without using logarithms and then apply their knowledge to solve real-world problems involving exponential equations. This is the first lesson in the "All About That Base" lesson duo.

Snapshot

Engage

Students observe a candy bar as it is cut in half repeatedly, modeling exponential decay, to make a prediction.

Explore

Students discover how to evaluate logarithms.

Explain

Students complete guided notes with the class and formalize their understanding of solving exponential equations.

Extend

Students apply what they have learned to solve real-world problems.

Evaluate

Students demonstrate how to solve exponential equations using logarithms.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

F702: Build functions for relations that are exponential
F707: Exhibit knowledge of logarithms

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.A.1.2: Use mathematical models to represent exponential relationships, such as compound interest, depreciation, and population growth. Solve these equations algebraically or graphically (including graphing calculator or other appropriate technology).
A2.F.2.4: Apply the inverse relationship between exponential and logarithmic functions to convert from one form to another.

Attachments

- Exit Ticket—All About That Base, Part 1 - Spanish.docx
- Exit Ticket—All About That Base, Part 1 - Spanish.pdf
- Exit Ticket—All About That Base, Part 1.docx
- Exit Ticket—All About That Base, Part 1.pdf
- Guided Notes (Teacher Guide and Model Notes)—All About That Base, Part 1.docx
- Guided Notes (Teacher Guide and Model Notes)—All About That Base, Part 1.pdf
- Guided Notes—All About That Base, Part 1 - Spanish.docx
- Guided Notes—All About That Base, Part 1 - Spanish.pdf
- Guided Notes—All About That Base, Part 1.docx
- Guided Notes—All About That Base, Part 1.pdf
- Lesson Slides—All About That Base, Part 1.pptx
- Scenario Cards (Sample Responses)—All About That Base, Part 1.docx
- Scenario Cards (Sample Responses)—All About That Base, Part 1.pdf
- Scenario Cards—All About That Base, Part 1 - Spanish.docx
- Scenario Cards—All About That Base, Part 1 - Spanish.pdf
- Scenario Cards—All About That Base, Part 1.docx
- Scenario Cards—All About That Base, Part 1.pdf
- What Power—All About That Base, Part 1 - Spanish.docx
- What Power—All About That Base, Part 1 - Spanish.pdf
- What Power—All About That Base, Part 1.docx
- What Power—All About That Base, Part 1.pdf

Materials

- Lesson Slides (attached)
- What Power? handout (attached; one per pair; print one-sided)
- Guided Notes handout (attached; one per student; print one-sided)
- Guided Notes (Teacher Guide and Model Notes) document (attached)
- Scenario Cards (attached; one per class; print one-sided)
- Scenario Cards (Sample Responses) document (attached)
- Scientific calculators
- Candy bar (one per class)
- Knife
- Cutting board
- Pencils
- Paper
- Tape or other adhesive to hang student work on the wall
- Exit Ticket handout (optional; attached; one half-sheet per student; print one-sided)

15 minutes

Engage

Teacher's Note: Activity Preparation

You need a candy bar, a knife, and a cutting board for this activity. Set up a space where students can safely watch you cut a candy bar in half repeatedly.

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question with students. Go to **slide 4** to share the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Show **slide 5** and ask students to share what they think will happen when a candy bar is cut in half repeatedly. After students have shared their ideas, place the candy bar on a cutting board and use a knife to cut it in half. Set one half aside and cut the remaining half in half. Repeat and encourage the class to continue sharing predictions. As you continue to cut the remaining piece of the candy bar in half again and again, ask students if their ideas have changed while watching this process.

Stop when you have reached the point where the remaining piece of the candy bar is too small to cut in half safely. Ask students to imagine you have the ability to continue cutting the candy bar piece in half for days, years, and even centuries into the future. Again, ask students to share their predictions about what will happen.

Teacher's Note: Scaffolding Discussion

Students might say you can't cut molecules, atoms, particles, etc., in half. In this case, remind them that the question is "theoretical," as no one can live long enough to cut the candy bar in half repeatedly for centuries. Other students might say there will be nothing left. Challenge these students with the idea that, every time you cut the candy bar in half, there will always be a remaining half to cut again.

After sufficient class discussion, connect this activity to exponential decay. Take the concept of "someday" not having any candy bar remaining and connect it to the horizontal asymptote of an exponential function: As time moves toward infinity, the remaining amount of candy bar approaches zero, but it never equals zero.

Optional Addition

After the candy bar demonstration and discussion, consider giving students miniature candy bars and asking them to model exponential decay by eating only half of the remaining candy bar with each bite.

10 minutes

Explore

Ask students to find partners or assign partners. Display **slide 6** and pass out the attached **What Power?** handout to each pair.

Have students use questions 1 and 2, which have been completed for them, to help them answer the remaining questions. Inform students that the *WhatPower* function is "just some function" and that they should treat this handout like a puzzle or a riddle.

Teacher's Note: Guiding the Activity

If students get stuck, encourage them to justify the work for questions 1 and 2. Ask students to write out how they think someone got those answers. Remind them that their justification needs to work for both questions 1 and 2.

Do not give students the answers or tell them what the *WhatPower* function is. Give students time and allow them to have a healthy struggle. The purpose is for students to figure out how a logarithmic function works on their own. If students can explain the rule for the *WhatPower* function in their own words, the transition to using logarithms will be much smoother.

Once students have completed the handout, transition through **slides 7–8** so they can check their work.

20 minutes

Explain

Teacher's Note: Activity Preparation

Before walking students through the **Guided Notes** handout, be aware of what type of calculators students are expected to use for the real-world examples during the Extend portion of the lesson. Do students' calculators allow logarithms of any base? Or do they have only common logarithm and natural logarithm buttons?

If students' calculators have only the common and natural logarithm buttons, then have them use the Guided Notes as is—the change of base formula is at the bottom of the page, and students need it to get decimal approximations for the Extend portion of the lesson. However, if students can enter logarithms of any base into their calculators, then you may consider removing the "Change of Base" portion of the Guided Notes handout, including example 4.

Display **slide 9** and explain to students that the *WhatPower* function is actually the logarithm function. Have students write and evaluate the two expressions on slide 9. Students can write these expressions on the back of the What Power? handout or on a piece of scratch paper. After a few minutes, ask for volunteers to share how they arrived at their answers.

Show **slide 10** and pass out the attached **Guided Notes** handout to each student.

Before working through the front side of the Guided Notes, have students flip to the back of the handout and copy the table shown on the slide. Have students work with their partners to complete the table. Again, ask for volunteers to share their thinking. Be sure that the class understands the equivalent forms of logarithmic and exponential equations before moving on.

Go to **slide 11** and have students return to the front side of their Guided Notes. Complete the handout as a class.

Teacher's Note: Guiding the Activity

On example 1, show students how to solve that exponential equation with and without the use of a logarithm. This style of question can be seen on the ACT and is intended to be solved without the use of logarithms to save time. However, students also need to see how to solve exponential equations using logarithms to prepare them for example 2, which requires the use of logarithms to solve.

For more support and recommendations as you walk students through the Guided Notes, see the attached **Guided Notes (Teacher Guide and Model Notes)** document.

Have students add their completed Guided Notes to their math notebooks if that is a classroom norm.

40 minutes

Extend

Teacher's Note: Activity Preparation

Before beginning this activity, print the attached **Scenario Cards** and cut out the eight scenario cards. Each group of 3–4 students will work with one scenario, so depending on your class, you may not need to use all eight scenario cards. Keep in mind that scenarios 1 and 2 are easier, and scenario 8 is more challenging.

During this portion of the lesson, students work in groups to solve real-world problems that involve exponential equations. Students then post their work on the wall in the form of mini-posters and complete a Gallery Walk. Decide beforehand what paper you would like students to use: lined paper, standard copy paper, etc. Provide students with this material before beginning the activity. Also, decide where you would like students to hang their mini-posters and designate this space as the "presentation gallery."

Have students form groups of 3–4 or assign groups. Introduce students to the Gallery Walk strategy and give them an overview of this activity:

- Work with your group to solve the problem on your given scenario card.
- Create a mini-poster to display your work.
- Split your group into presenters and analysts.
- Presenters: You stay with the mini-poster and answer questions about your scenario.
- Analysts: You go to each mini-poster and analyze your peers' work.
- Switch roles and repeat.
- Summarize the process of solving exponential equations.

Display **slide 12** and distribute one scenario card to each group. Ask students to work together to solve the given problem before creating a mini-poster to show their work.

Show **slide 13** and review the expectations for the mini-poster.

Sample Responses

See the attached **Scenario Cards (Sample Responses)** document for examples of how students' mini-posters might show the given information, the identified unknown, the steps for solving, and the final result written as a complete sentence for each scenario.

Teacher's Note: ACT Prep

The types of questions on the scenario cards can be seen on the ACT. Students often struggle to understand what variables represent, what a question is asking, or the time since a certain date, like in scenarios 3, 5, and 8.

To check student understanding, consider joining the Gallery Walk as an analyst and asking the presenters about the common struggles listed above.

After groups complete their mini-posters, display **slide 14**. Have groups divide in half and decide who will be presenters and who will be analysts. Groups of three can have one presenter and two analysts or vice versa. Assure students that it does not matter who presents or analyzes first, since they are to switch roles later in the activity.

Guide presenters to the "presentation gallery" and have them hang their mini-posters. After reminding students of their roles' responsibilities, have them begin the Gallery Walk.

Once students have had enough time to visit 3–4 mini-posters, show **slide 15**. Ask students to pause and switch roles. Remind students of their roles' responsibilities and have them begin the Gallery Walk.

After giving this group of analysts about the same amount of time to visit mini-posters, move to **slide 16** and ask all students to go back and sit with their groups. Have students get out a piece of notebook paper and write a short paragraph explaining the general process of solving exponential equations.

Optional Handout

In the Evaluate portion of the lesson, you have the option to pass out the attached **Exit Ticket** handout or have students use a piece of notebook paper to copy and solve the equations on the Lesson Slides. If you decide to use the Exit Ticket handout for the Evaluate, consider passing it out at the end of the Gallery Walk so each student can write their paragraph on the back of the handout.

5 minutes

Evaluate

Display **slide 17** and use the Exit Ticket strategy to assess what students have learned individually. Have students take out a piece of notebook paper or give each student a half-page of the attached **Exit Ticket** handout. Then have students write and solve the two exponential equations on the slide.

Alternative Pacing

You may choose to assign the Exit Ticket as homework and review the solutions on **slide 18** as bellwork during the next class period. If time allows, consider starting the next class period with a 3–5 minute review of slides 17–18.

After you have collected students' responses, use their feedback to determine if remediation is needed or if they are ready for the next lesson: All About That Base, Part 2.

Resources

- Eureka Math. (2014). Algebra II module 3, topic B, lesson 8: The "WhatPower" function [Handout]. NYS Common Core Mathematics Curriculum. https://www.engageny.org/resource/algebra-ii-module-3-topic-b-lesson-8/file/108756
- K20 Center. (n.d.). Amplify classroom. Tech Tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Canva. Tech Tools. https://learn.k20center.ou.edu/tech-tool/612
- K20 Center. (n.d.). Gallery walk / carousel. Strategies. https://learn.k20center.ou.edu/strategy/118
- K20 Center. (n.d.). Google slides. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2335