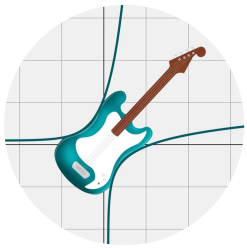

All About That Base, Part 1

Solving Exponential Equations

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Grade Level	10th – 11th Grade	**Time Frame**	90 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Algebra 2		

Essential Question

How do we use logarithms to solve exponential equations?

Summary

In this lesson, students will discover how to evaluate logarithms and use that knowledge to write equations in logarithmic and exponential forms. Students will learn how to solve exponential equations with and without using logarithms and then apply their knowledge to solve real-world problems involving exponential equations. This is the first lesson in the "All About That Base" lesson duo.

Snapshot

Engage

Students observe a candy bar as it is cut in half repeatedly, modeling exponential decay. Students see that the remaining amount of candy bar is approaching zero, the asymptote.

Explore

Students discover how to evaluate logarithms.

Explain

Students complete guided notes with the class and formalize their understanding of solving exponential equations.

Extend

Students apply what they have learned to solve real-world problems and complete a Gallery Walk. Students then summarize the process of solving exponential equations.

Evaluate

Students demonstrate how to solve exponential equations using logarithms.

Standards

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.A.1.2: Use mathematical models to represent exponential relationships, such as compound interest, depreciation, and population growth. Solve these equations algebraically or graphically (including graphing calculator or other appropriate technology).
A2.F.2.4: Apply the inverse relationship between exponential and logarithmic functions to convert from one form to another.

Attachments

- Exit Ticket—All About That Base, Part 1 - Spanish.docx
- Exit Ticket—All About That Base, Part 1 - Spanish.pdf
- Exit Ticket—All About That Base, Part 1.docx
- Exit Ticket—All About That Base, Part 1.pdf
- Guided Notes (Teacher Guide and Model Notes)—All About That Base, Part 1.docx
- Guided Notes (Teacher Guide and Model Notes)—All About That Base, Part 1.pdf
- Guided Notes—All About That Base, Part 1 - Spanish.docx
- Guided Notes—All About That Base, Part 1 - Spanish.pdf
- Guided Notes—All About That Base, Part 1.docx
- Guided Notes—All About That Base, Part 1.pdf
- Lesson Slides—All About That Base, Part 1.pptx
- Scenario Cards (Sample Responses)—All About That Base, Part 1.docx
- Scenario Cards (Sample Responses)—All About That Base, Part 1.pdf
- Scenario Cards—All About That Base, Part 1 - Spanish.docx
- Scenario Cards—All About That Base, Part 1 - Spanish.pdf
- Scenario Cards—All About That Base, Part 1.docx
- Scenario Cards—All About That Base, Part 1.pdf
- What Power—All About That Base, Part 1 - Spanish.docx
- What Power—All About That Base, Part 1 - Spanish.pdf
- What Power—All About That Base, Part 1.docx
- What Power—All About That Base, Part 1.pdf

Materials

- Desmos account
- Guided Notes handout (attached; one per student; printed front only)
- Guided Notes (Teacher Guide and Model Notes) (attached; for teacher use)
- Scenario Cards (Sample Responses) (attached; for teacher use)
- Scientific calculators
- Candy bar (one per class)
- Knife
- Cutting board
- Pencils
- Paper
- Student devices with internet access

15 minutes

Engage

Teacher's Note: Activity Preparation

You need a candy bar, a knife, and a cutting board for the Engage activity. Set up a space where students can safely watch you cut a candy bar in half repeatedly.

Teacher's Note: Desmos Activity Preparation

To use this Desmos Classroom activity, select the following link: "All About That Base, Part 1." Create an account or sign in under the "Activity Sessions'' heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code'' and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 3. Select "Restrict to Screens 1–3" to confirm your selection. This allows students to access only screens 1–3 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to https://student.desmos.com and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1–2** of the Desmos activity. **Screen 1** displays the lesson's essential question. **Screen 2** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Have students go to **screen 3** and ask them to share what they think will happen when a candy bar is cut in half repeatedly. After students have shared their ideas, place the candy bar on a cutting board and use a knife to cut it in half. Set one half aside and cut the remaining half in half. Repeat and encourage the class to continue sharing predictions. As you continue to cut the remaining piece of candy bar in half again and again, ask students if their ideas have changed while watching this process.

Stop when you have reached the point where the remaining piece of candy bar is too small to cut in half safely. Ask students to imagine you have the ability to continue cutting the candy bar piece in half for days, years, and even centuries into the future. Again, ask students to share their predictions about what will happen.

Teacher's Note: Scaffolding Discussion

Students might say you can't cut molecules, atoms, particles, etc. in half. In this case, remind them the question is "theoretical," as no one can live long enough to cut the candy bar in half repeatedly for centuries. Other students might say there will be nothing left. Challenge these students with the idea that, every time you cut the candy bar in half, there always will be a remaining half to cut again.

After sufficient class discussion, connect this activity to exponential decay. Take the concept of "someday" not having any candy bar remaining and connect it to the horizontal asymptote of an exponential function: As time moves toward infinity, the remaining amount of candy bar approaches zero—but it never equals zero.

Optional Addition

After the candy bar demonstration and discussion, consider giving students miniature candy bars and asking them to model exponential decay by eating only half of the remaining candy bar with each bite.

10 minutes

Explore

Ask students to find partners or assign student pairs. On the Desmos dashboard, click the orange plus sign to allow students to progress to **screen 4**.

Have students work with their partners to evaluate each expression. Students should use the first two expressions, which have been completed for them, to help them answer the remaining questions. Inform students that the *WhatPower* function is "just some function" and that they should treat this activity like a puzzle or a riddle.

Desmos provides students with immediate feedback on each response.

Teacher's Note: Guiding the Activity

If students get stuck, encourage them to justify the work for the first two expressions. Ask students to write out how they think someone got those answers. Remind them that their justification needs to work for both expressions.

Do not give students the answers or tell them what the *WhatPower* function is. Give students time and allow them to have a healthy struggle. The purpose is for students to figure out how a logarithmic function works on their own. If students can explain the rule for the *WhatPower* function in their own words, the transition to using logarithms will be much smoother.

20 minutes

Explain

Teacher's Note: Activity Preparation

Before walking students through the Guided Notes handout, be aware of what type of calculators students are expected to use for the real-world examples during the Extend portion of the lesson. Do students' calculators allow logarithms of any base? Or do they have only common logarithm and natural logarithm buttons?

If students' calculators have only the common and natural logarithm buttons, then have them use the Guided Notes as is—the change of base formula is at the bottom of the page, and students need it to get decimal approximations for the Extend portion of the lesson. However, if students can enter logarithms of any base into their calculators, then you may consider removing the "Change of Base" portion of the Guided Notes handout, including example 4.

In Desmos, example 4 is on screen 12. If you do not need screen 12, you can edit the activity or just instruct students to skip that screen.

On the dashboard, click the orange plus sign to allow students to progress to **screen 5**. Here, students are told that the *WhatPower* function is actually the logarithm function. Students then use the same reasoning to evaluate two logarithmic expressions, this time using logarithmic notation. Again, Desmos provides students with immediate feedback on each response. After a few minutes, ask for volunteers to share how they arrived at their answers.

Click the orange plus sign on the dashboard to allow students to progress to **screen 6**. Here, the lesson pauses to focus on how to type a logarithm of any base into Desmos. Students are not doing any calculations on this screen; they are focusing solely on syntax. If your students already have experience with this, consider having them skip screen 6.

Click the orange plus sign on the dashboard to allow students to progress to **screen 7**. Here, students use their reasoning for evaluating logarithmic expressions to rewrite logarithmic equations into exponential form and vice versa. Again, ask for volunteers to share their thinking. Be sure that the class understands the equivalent forms of logarithmic and exponential equations before moving on.

Give each student a copy of the attached **Guided Notes** handout. On the Desmos dashboard, click the orange plus sign to allow students to progress to **screen 8**. Explain to students the shorthand notation for common and natural logarithms. Point out to students that inverse operations have the same base.

On the dashboard, click the orange plus sign four times to allow students to progress to **screens 9–12**. Have students use Desmos to complete the handout.

Teacher's Note: Guiding the Activity

On example 1, show students how to solve that exponential equation with and without the use of a logarithm. This style of question can be seen on the ACT and is intended to be solved without the use of logarithms to save time. However, students also need to see how to solve exponential equations using logarithms to prepare them for example 2, which requires the use of logarithms to solve.

For more support and recommendations as you walk students through the Guided Notes, see the attached **Guided Notes (Teacher Guide and Model Notes)** document.

Have students add their completed Guided Notes to their math notebooks if that is a classroom norm.

40 minutes

Extend

Teacher's Note: Activity Preparation

During this portion of the lesson, students work in groups to solve real-world problems that involve exponential equations. Students then post their work on the wall in the form of mini-posters and complete a Gallery Walk. Decide beforehand what paper you would like students to use: lined paper, standard copy paper, etc. Provide students with this material before beginning the activity. Also decide where you would like students to hang their mini-posters and designate this space as the "presentation gallery."

Optional Digital Gallery Walk

If students need to present virtually, consider having them use a digital platform like Google Slides. Keep in mind that it is not recommended to ask students to type out their math work, as that is often a difficult task. Instead, students could solve the problem on paper, take a picture of their work, and upload it to the chosen platform. Then, students could label and design their digital mini-posters using the tools of that platform.

On the Desmos dashboard, click the orange "Stop" button to allow students to complete the Desmos activity at their own pace.

Have students form groups of 3–4 or assign groups. Introduce students to the Gallery Walk strategy and give them an overview of this activity:

- Work with your group to solve the problem on your given scenario card.
- Create a mini-poster to display your work.
- Split your group into presenters and analysts.
- Presenters: You stay with the mini-poster and answer questions about your scenario.
- Analysts: You go to each mini-poster and analyze your peers' work.
- Switch roles and repeat.
- Summarize the process of solving exponential equations.

In their groups, have students go to **screen 13** and select a topic on the screen. Consider creating a process to ensure that different groups select different topics. For example, when one group announces a chosen topic, no other group is allowed to select it.

Teacher's Note: Selecting Scenarios

Depending on your class, you may not need to use all eight scenarios on the screen. Keep in mind that scenario 1 (bacteria growth) and scenario 2 (truck depreciation) are easier, and scenario 8 (archaeology) is more challenging. If you would like to tweak the scenario options, you can edit the Desmos activity or just inform students of which scenarios they cannot choose.

Once all groups have selected a topic, ask students to work together to solve the given problem before creating a mini-poster to show their work.

Have students go to **screen 14** and review the expectations for the mini-poster.

Sample Responses

See the attached **Scenario Cards (Sample Responses)** document for examples of how students' mini-posters might show the given information, the identified unknown, the steps for solving, and the final result written as a complete sentence for each scenario.

Teacher's Note: ACT Prep

The types of questions on the scenario screen can be seen on the ACT. Students often struggle to understand what variables represent, what a question is asking, or the time since a certain date, like in scenario 3 (radioactive decay), scenario 5 (U.S. population growth), and scenario 8 (archaeology).

To check student understanding, consider joining the Gallery Walk as an analyst and asking the presenters about the common struggles listed above.

After groups complete their mini-posters, have them go to **screen 15**. Have groups divide in half and decide who will be presenters and who will be analysts. Groups of three can have one presenter and two analysts or vice versa. Assure students it does not matter who presents or analyzes first, since they are to switch roles later in the activity.

Guide presenters to the "presentation gallery" and have them hang their mini-posters. After reminding students of their roles' responsibilities, have them begin the Gallery Walk.

Once students have had enough time to visit 3–4 mini-posters, have them go to **screen 16**. Ask students to pause and switch roles. Again remind students of their roles' responsibilities and have them begin the Gallery Walk.

After giving this group of analysts about the same amount of time to visit mini-posters, have all students go back and sit with their groups. Instruct students to go to **screen 17** and type a short paragraph explaining the general process of solving exponential equations.

5 minutes

Evaluate

Use the Exit Ticket strategy to assess what students have learned individually. The question on **screen 18** does not require the use of a logarithm, while the question on **screen 19** does. Here, Desmos provides feedback on student responses only to the teacher.

Alternative Pacing

You may choose to assign the Exit Ticket activity as homework and review the solutions as bellwork the next class period. If time allows, consider starting the next class period with a 3–5 minute review of these two questions.

Use students' feedback to determine if remediation is needed or if they are ready for the next lesson: "All About That Base, Part 2."

Resources

- Eureka Math. (2014). Algebra II module 3, topic B, lesson 8: The "WhatPower" function [Handout]. NYS Common Core Mathematics Curriculum. https://www.engageny.org/resource/algebra-ii-module-3-topic-b-lesson-8/file/108756
- K20 Center. (n.d.). Gallery Walk / Carousel. Strategies. https://learn.k20center.ou.edu/strategy/118
- K20 Center. (n.d.). Bell Ringers and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081