

A Solidifying Pattern

Surface Area of Prisms and Pyramids

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Grade Level	9th – 10th Grade	**Time Frame**	120 minutes
Subject	Mathematics	**Duration**	3 class periods
Course	Geometry		

Essential Question

Why would it be useful to find the surface area?

Summary

In this lesson, students will explore how to find the surface area of prisms and pyramids. Students will discover the general formulas for finding surface area of prisms and pyramids and are introduced to the surface area of spheres and hemispheres. Students will apply what they learn to create a composite model and to a real-world situation.

Snapshot

Engage

Students use a modified Quick Write strategy to respond to the question "What would you need to know if you wanted to paint your room?" through Mentimeter.

Explore

Students discover the general formula for finding surface area of prisms and pyramids by analyzing patterns of solids they measure.

Explain

Students complete a foldable during class discussion on how to use the formulas for finding the surface area of prisms, pyramids, and other solids.

Extend

Students create a composite model and find the surface area of their model.

Evaluate

Students apply their knowledge of surface area to the real world.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G405: Use geometric formulas when all necessary information is given

Oklahoma Academic Standards Mathematics (Geometry)

G.3D.1.1: Represent, use, and apply mathematical models and other tools (e.g., nets, measuring devices, formulas) to solve problems involving surface area and volume of three-dimensional figures (prisms, cylinders, pyramids, cones, spheres, composites of these figures).

Attachments

- Creating Composites—A Solidifying Pattern - Spanish.docx
- Creating Composites—A Solidifying Pattern - Spanish.pdf
- Creating Composites—A Solidifying Pattern.docx
- Creating Composites—A Solidifying Pattern.pdf
- Exit Ticket—A Solidifying Pattern - Spanish.docx
- Exit Ticket—A Solidifying Pattern - Spanish.pdf
- Exit Ticket—A Solidifying Pattern.docx
- Exit Ticket—A Solidifying Pattern.pdf
- Foldable (Sample Responses)—A Solidifying Pattern.pdf
- Foldable—A Solidifying Pattern - Spanish.pdf
- Foldable—A Solidifying Pattern.pdf
- Lesson Slides—A Solidifying Pattern.pptx
- Nets—A Solidifying Pattern.docx
- Nets—A Solidifying Pattern.pdf
- Scratching the Surface – Blue—A Solidifying Pattern - Spanish.docx
- Scratching the Surface – Blue—A Solidifying Pattern - Spanish.pdf
- Scratching the Surface – Blue—A Solidifying Pattern.docx
- Scratching the Surface – Blue—A Solidifying Pattern.pdf
- Scratching the Surface – Red—A Solidifying Pattern - Spanish.docx
- Scratching the Surface – Red—A Solidifying Pattern - Spanish.pdf
- Scratching the Surface – Red—A Solidifying Pattern.docx
- Scratching the Surface – Red—A Solidifying Pattern.pdf

Materials

- Lesson Slides (attached)
- Scratching the Surface - Red packet (attached; one per red group; print one-sided)
- Scratching the Surface - Blue packet (attached; one per blue group; print one-sided)
- Foldable handout (attached; one per student; print two-sided)
- Model Foldable document (attached)
- Creating Composites handout (attached; one per group of 3; print one-sided)
- Exit Ticket handout (attached; one half per student; print one-sided)
- Geometric Solids, at least including: triangular, pentagonal, and hexagonal prisms; triangular, rectangular, pentagonal, and hexagonal pyramids; and cylinder
- Nets handout (attached; optional; print one-sided)
- Rulers
- Scientific calculators
- Glue (or tape)
- Scissors
- Pencils
- Paper

Preparation

Before beginning the lesson, go to mentimeter.com and create a Mentimeter presentation with an open-ended question type. Use the question "What would you need to know if you wanted to paint your room?" and use the Edit (pencil icon) option to adjust settings. Be sure to allow "multiple responses" per person. You can use the "Preview" button to get a sense of what you and your students will see during the presentation. Use the Share option (three interconnected dots icon) to generate the access code. Add the generated code to **slide 5**. Students use this code to enter their responses. Consider creating a different code for each class. Alternatively, consider resetting the results in preparation for the next class.

The Explore activity was designed to use EAI Education GeoModel Folding Shapes for the geometric solids. If you do not have access to these or similar geometric solids, use the attached **Nets** handout for students to make their own solids for this activity. Students need access to the following solids: triangular, pentagonal, and hexagonal prisms; triangular, square, pentagonal, and hexagonal pyramids; and cylinder. These solids are on pages 1–8. Each page of the document contains the net for one geometric solid. Print enough pages for each student to have one solid. There are additional nets on pages 9–11 that could be helpful for the Extend portion of the lesson.

If you do use the Nets handout, remind students with the cylinder net to not cut the circles off of the rectangle, but rather cut as close as possible. If students do accidentally cut the circle(s) off, a little tape solves this.

When printing the attached **Foldable** handout, make sure to print this handout two-sided, selecting the "flip on short edge" setting.

5 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question with students. Move to **slide 4** to share the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Show **slide 5**. Direct students to go to menti.com, enter the generated code, and answer the following question: “What would you need to know if you wanted to paint your room?” Encourage students to write as much as they can using a modified Quick Write strategy. As students enter their responses, start the Menti presentation to display the results for students to see. Have a class discussion about student responses.

45 minutes

Explore

Teacher's Note: Activity Overview

During this phase of the lesson, students will complete three tasks with the goal of discovering the general formulas for the surface area of prisms and pyramids. Students are working with different groups throughout this activity to encourage discussion and engagement. Students will:

- **(1) Measure and calculate:** Students individually measure their solid to find its surface area. Students are in groups of four for support.
- **(2) Check their work:** Students compare their work with those who had the same solid. Students are in pairs or small groups.
- **(3) Look for a pattern:** Students work with those who have the same type of solid to find a pattern and attempt to create a formula. Students are in one of two groups.

Organizing Groups

Place students in groups of 4 and assign each group to either the red group or the blue group. Blue groups work with the more challenging solids, so be sure the advanced students are placed in these groups. Use knowledge of the students to most appropriately assign groups.

Depending on class size, this activity can adapt to a variety of scenarios.

- In a class of 12 students, consider having 3 red groups and repeating the activity later with 3 blue groups.
- In a class of 24, consider 4 red groups and 2 blue groups.

Be sure to consider the quantity of geometric solids needed for these situations. The key detail is that there is a minimum of two groups for each particular group color.

Preparing Materials

Before class, have the packets and geometric solids set out on each group's desk. Below is what is needed for each group:

- **Red Group: Scratching the Surface – Red** packet, rulers, scientific calculators, and the following solids: triangular prism, cylinder, triangular pyramid, and rectangular pyramid
- **Blue Group: Scratching the Surface – Blue** packet, rulers, scientific calculators, and the following solids: pentagonal prism, hexagonal prism, pentagonal pyramid, and hexagonal pyramid

Measure and Calculate

Explain to the students that each person will be responsible for one out of the four solids in their packet. Direct the students to work in their groups to measure their chosen solid and find the surface area. Have students identify the type of solid, measure the solid, label the measurements in centimeters, and record the measurements with work shown on the **Scratching the Surface** handout.

Display **slide 6** and direct the students to grab a ruler and pick a solid. Then have students get the paper from the **Scratching the Surface** packet that corresponds with their chosen solid.

Teacher's Note: Guiding the Activity

Encourage students to think about what they should be measuring in order to determine the surface area of the solid. If students are struggling, encourage students to imagine the nets of the solid and how to find the area of those shapes: circles, triangles, rectangles, and polygons. Students in the blue group may need a reminder of how to find the area of a polygon.

To help with comparing results, make sure that students are measuring the same parts of the geometric solids. Direct the students either to measure the outside of the solid or to take out the net to measure it, as they can differ by at least a centimeter. To save time, consider recommending that students measure the outside of the solids.

Otherwise, remember that this is an exploration and give students the opportunity to explore finding the surface area of their solids. Students formalize how to find the surface area of prisms and pyramids during the Explain portion of the lesson.

Sample Questions and Responses

Students may ask how to measure the height of a pyramid. Remind students that the height of the triangle, the measurement they need to find the area of the triangular surfaces, is not the same as the height of the pyramid. Students may recall slant height, but if not, they learn about it later in the lesson.

When measuring the cylinder, they may use the net and find that the rectangle is longer than their ruler —how would they measure that length? Remind students that that length is the circumference of the base, so they can calculate that length.

Check Your Work

Have students leave their original groups to find another person(s) who has the same solid to compare their results and work to make sure that they have the same results. If students' results significantly differ, this is the time for students to collaborate and help each other to get the same surface area results.

Display **slide 7**. Direct students to take their materials (handout, solid, and ruler) and find one or more classmates who have the same solid. Instruct students to compare their results and work. Remind them that their answers might differ slightly based on their measurements and rounding. Guide students to compare their work for how they found their surface area regardless of their final results. Students should use their solid and ruler to re-measure if needed.

Teacher's Note: Guiding the Activity

As students are comparing their work, circulate the room and listen to student conversations. Students may have found the surface area in slightly different ways, like a student individually finding the area of each base of their prism compared to a student who noticed that they were the same and doubled one area. Help students see the big picture that they had the same overall process for finding the surface area of the same solids.

Look for a Pattern: Prisms vs. Pyramids

The class will now be divided into two groups: groups of prisms and groups of pyramids. All students who measured a prism will discuss the similarities in finding the surface area of prisms to write a general formula, using words, for the surface area of any right prism, where the lateral edges of the prism are perpendicular with the bases. Similarly, all students who measured a pyramid will discuss the similarities in finding the surface area of pyramids to write a general formula, using words, for the surface area of regular pyramids.

Display **slide 8**. Direct students to take their materials (handout, solid, and ruler) and go find everyone who has their same type of solid: prism or pyramid.

Show **slide 9** and instruct students to work with their group to find the pattern to make a general formula for the surface area for their solid: prism or pyramid. Direct students to use words to write a formula for the surface area of their type of solid.

As students work, display **slide 10**. Use these guiding questions to help students reach this goal.

- When finding the surface area, is there something that all prisms or all pyramids have in common?
- Does your formula hold true for everyone in your group (for all prisms or for all pyramids)?

Teacher's Note: Guiding the Activity

As students work towards writing a general formula, students in the prisms group should be noticing that they all found the area of the bases, which were the same, and they found the area of the sides, which was the number of sides of the base times the side length of the base times the height of the prism. Students may notice that the number of sides of the base times the side length of the base is the perimeter of the base.

- (Prism Surface Area) = 2·(area of the base) + (number of sides of the base)·(side length of base)·(height of the prism)

Students in the pyramid group should be noticing that they all found the area of the base and the area of each triangle times the number of sides of the base. The area of each triangle is one-half of the side length of the base times the height of the triangle (slant height).

- (Pyramid Surface Area) = (area of the base) + (number of sides of base)·(½)·(side length of base)·(height of triangle)

30 minutes

Explain

Show **slide 11** for students to see the connection between their formulas in words to the algebraic formulas. Have students compare what they have on their handout and what they see on the slide.

Give each student a copy of the attached **Foldable** handout and one pair of scissors. Transition through **slides 12–18** to show students how to prepare their foldable.

Show **slide 19** for students to see what the variables mean in the surface area of a prism formula. Direct students to find an Elbow Partner to fill out the "Surface Area of Prisms" section of the foldable. Have students fill in the blanks on the left and complete the example on the right using the formula they just learned.

After a few minutes, have the class come together to discuss what they wrote and compare results for the surface area of the given prism. Use the attached **Foldable (Sample Responses)** document as needed.

Show **slide 20** for students to see what the variables mean in the surface area of a pyramid formula. Have students work with their elbow partner to fill out the "Surface Area of Regular Pyramids" section of the foldable. Have students fill in the blanks on the left and complete the example on the right using the formula they just learned.

After a few minutes, have the class come together to discuss what they wrote and compare results for the surface area of the given pyramid.

Show **slide 21** for students to see what the variable means in the surface area formulas for a sphere and hemisphere. Have students work with their elbow partner to fill out the "Surface Area of Other Solids" section of the foldable. Have students fill in the blanks on the left and complete the example on the right using the formula they just learned.

After a few minutes, have the class come together to discuss what they wrote and compare results for the surface area of the given solid.

Sample Questions and Responses

Students might ask why a sphere has a coefficient of 4 while a hemisphere has 3. Have students imagine the stitching of a baseball and if they could undo the stitching and lay out the leather—how many circles would they estimate? There are 4 circles. That is why the coefficient for spheres is 4. A hemisphere is half of the sphere, so there are 2 circles plus the circle of the base, which equals 3 circles.

Now direct students to glue their Foldable into their math notebook, if that is a classroom norm.

30 minutes

Extend

Display **slide 22** and direct students to get into groups of 3 or assign groups. Pass out a piece of blank paper and one copy of the attached **Creating Composites** handout to each group of students. Instruct students to select one solid per column (for columns 1–4). Tell students to circle their selected solids on their handout and then create a model using their selected solids. Challenge students to also use a solid from the Extra Challenge column in their models.

Show **slide 23** and instruct students to sketch only their model on a blank sheet of paper and find the surface area on the back of their Creating Composites handout.

Optional Model Representations

Consider giving students options for how they display/share their models. Instead of sketching their model, they could:

- Use the Nets handout to build a model by cutting them out and taping the solids together.
- Design a digital 3D model. If a 3D printer is available, also print the model.

Teacher's Note: Guiding the Activity

Some students may find it helpful to use the geometric solids to build their model or to help decide which solids to select. Since students potentially could all select a cylinder, for example, have a plan for what to do if you do not have enough solids.

Help students see that when they stack solids, they subtract that touching area twice. For example, imagine a triangular prism stacked on the cube, such that the bases of the prisms are touching. If students use the formulas to find the surface area of each solid individually, then they need to subtract the area of the base of the triangular prism once to remove the area from that solid, then again to remove that area from the cube, leaving only the visible area.

Optional Additional Activity

If time allows, have students display their sketches of their models around the classroom. Instruct groups of students to select another group's model then find its surface area. Have students check their work with the group who designed the model and compare surface area results as well as the process of finding the surface area.

10 minutes

Evaluate

Use the Exit Ticket strategy to individually assess what students have learned from the lesson. Display **slide 24** and pass out a half-page copy of the attached **Exit Ticket** handout to each student. Students are given a scenario of painting a room and are asked to find the area to be painted and the number of gallons of paint needed.

Collect the handout and use responses to see what misconceptions still exist.

Optional Slides

If time allows, unhide **slide 25** and review the process of finding the surface area of the room with the class.

Resources

- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Elbow partners. Strategies. https://learn.k20center.ou.edu/strategy/116
- K20 Center. (n.d.). Mentimeter. Tech Tools. https://learn.k20center.ou.edu/tech-tool/645
- K20 Center. (n.d.). Quick write. Strategies. https://learn.k20center.ou.edu/strategy/1127