Shape

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To develop the law of cosines, begin with . From vertex , altitude  is drawn and separates side  into segments  and .

1. Why can the segments be represented   
   in this way?
2. The altitude separates  into two right triangles. Use the Pythagorean theorem to write two equations, one relating , , and , and another relating , , and .
3. Notice that both equations contain .
   1. Why?
   2. Solve each equation for .
4. Since both of the equations in Question 3 are equal to , they can be set equal to each other.
   1. Why is this true?
   2. Set the equations equal to each other to form a new equation.
5. Notice that the equation in Question 4 involves . However,  is not a side of . Attempt to rewrite the equation in Question 4 so that it does not include . Hint, begin by expanding the quantity .
6. Now solve the equation for .
7. The equation still involves .
   1. To eliminate it from the equation, write an equivalent expression for  involving both  and .
   2. Why use ?
8. Solve the equation from Question 7 for .
   1. Why solve for ?
9. Substitute the equivalent expression for  into the equation from Question 6 and simplify. The resulting equation contains only sides and angles of . This equation is called the **Law of Cosines**.