

One Piece at a Time

Graphing and Analyzing Piecewise Functions

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Grade Level	10th – 11th Grade	**Time Frame**	70-90 minutes
Subject	Mathematics	**Duration**	1-2 class periods
Course	Algebra 2		

Essential Question

How do we represent a single scenario when the pattern differs for different intervals?

Summary

In this lesson, students will learn how to graph and analyze piecewise functions. The piecewise functions in this lesson will have a maximum of three branches, which could contain any combination of linear, quadratic, and exponential branches. (Note: This lesson is written so that it could be taught at the beginning of the school year or later, depending on your curriculum pacing. Interval notation is not required prerequisite content but would be helpful; calculations with exponential functions are limited to a base of one-half and two; and quadratics are written in vertex form.)

Snapshot

Engage

Students write a story to match a given distance versus time graph.

Explore

Students match graphs of piecewise functions with their corresponding domains and ranges and increasing and decreasing intervals in a Card Matching activity.

Explain

Students review and add equation cards to complete the Card Matching activity.

Extend

Students apply what they have learned to model data with piecewise functions as well as graph piecewise functions, and then analyze the graphs.

Evaluate

Students demonstrate their understanding of analyzing piecewise functions by creating their own piecewise function based on a given set of parameters.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

S506: Recognize that when a statistical model is used, model values typically differ from actual values
S705: Recognize that part of the power of statistical modeling comes from looking at regularity in the differences between actual values and model values

Oklahoma Academic Standards for Mathematics (Grades 9, 10, 11, 12)

A2.F.1.8: Graph piecewise functions with no more than three branches (including linear, quadratic, or exponential branches) and analyze the function by identifying the domain, range, intercepts, and intervals for which it is increasing, decreasing, and constant.
A2.D.1.2: Collect data and use scatterplots to analyze patterns and describe linear, exponential or quadratic relationships between two variables. Using graphing calculators or other appropriate technology, determine regression equation and correlation coefficients; use regression equations to make predictions and correlation coefficients to assess the reliability of those predictions.

Attachments

- Card Matching—One Piece at a Time.pdf
- Create a Graph—One Piece at a Time.docx
- Create a Graph—One Piece at a Time.pdf
- Graphing Piecewise Functions (Sample Responses)—One Piece at a Time.pdf
- Graphing Piecewise Functions—One Piece at a Time.docx
- Graphing Piecewise Functions—One Piece at a Time.pdf

Materials

- Graphing Piecewise Functions handout (attached; one per student; printed front only)
- Graphing Piecewise Functions (Sample Responses) document (attached; for teacher use)
- Card Matching document (optional; attached; one per pair of students; printed front only)
- Create a Graph handout (attached; one per student; printed front only)
- Desmos account
- Pencils
- Paper
- Student devices with Internet access
- Graph paper (optional)

15 minutes

Engage

Teacher's Note: Desmos Activity Preparation

To use this Desmos Classroom activity, select the following link: "One Piece at a Time." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 3. Select "Restrict to Screens 1–3" to confirm your selection. This allows students to access only screens 1–3 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to student.amplify.com/join/ and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1–2** of the Desmos Activity. **Screen 1** displays the lesson's essential question. **Screen 2** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Instruct students to find a partner or assign students partners. Direct students' attention to **screen 3** and have them work with their partner to create a story that models the given piecewise distance versus time graph.

Once students are done, ask for volunteers to share their stories with the class. Make note of students' stories and provide feedback, especially regarding the time values involving change: t = 3, 4, 5, 6, 7, and 10. Consider asking questions about why something changed in their story at one of those times. The stories should begin at t = 0 and end at t = 15. Students' stories should also contain details showing that they noticed the different slopes, like running vs. walking vs. resting.

As time allows, ask for volunteers to share how they created their story. Use these responses to help check for misunderstandings. Also listening to a peer can help others better understand how to analyze a graph.

10 minutes

Explore

Optional Preparation

Depending on the needs of your students, you may want to use the **Card Matching** document for students to match physical cards. This could be used in addition to or as an alternative to certain screens of the Desmos activity, specifically screens 4, 5, 9, 11, and 12. With screens 4–5 use the graph, domain/range, and increasing/decreasing interval cards. Screen 9 focuses on the set for Card A and introduces the equation cards. Screens 11–12 use all of the cards.

On the Dashboard, press the orange plus sign twice to allow students to progress to **screens 4–5**. Have students work with their partner, using the Card Matching strategy, to match each graph card with a domain/range card and with an increasing/decreasing interval card. Students are provided 6 of each type of card.

Teacher's Note: Desmos Card Sort

This card matching activity may be overwhelming on small screens. Share with students the following tips to avoid frustration:

- Enter full screen: To increase the size of your workspace, select the expanding arrows in the center of the top pane of the window.
- Organize your workspace: In the same way you would arrange cards on a physical surface, organize the cards on your screen.
- Condense a stack of cards: Once you have matched a set of cards together, click the arrow.
- Enlarge the card: Click on the card to magnify the image.

Inform students that they can check their work on **screen 5**, where Desmos shows how many cards out of 18 are correctly matched. If the screen seems empty, it is because there are not yet any correct matches, whether that is from a lack of attempt, guessing, or misunderstanding.

Use student responses to determine if students need a quick refresh on domain and/or range. The idea of increasing and decreasing intervals may be new to students at this point. If students have never seen interval notation before, take a moment to explain that it is the starting and ending x-values for an interval, or segment, of the function. Otherwise, allow students to have a healthy struggle matching these cards; resist the temptation to give students the answer. If students are struggling with the idea of increasing and decreasing, consider asking guiding questions like, "*What does it mean to increase*?" or "*What do you think it means to decrease?*" or "*Which direction do we read a graph – left to right or right to left?*" Monitor student questions and address them during the Explain portion of the lesson.

15 minutes

Explain

Direct students' attention back to screen 4 and ask for volunteers to share how they matched their cards together. Use this time to make sure that the students understand the idea of increasing and decreasing and interval notation. Be sure to specifically ask about the two cards that only differ by brackets and parentheses at x = 4. Help students see the connection between open circles and parentheses and the relationship between closed circles and brackets.

Teacher's Note: Interval Notation

When you look at the graph of a parabola, for example, and write the interval of where that the function is increasing, there are options regarding whether or not to include the vertex. For example, the function $y = x^2$ for $-5 < x < 5$ is increasing to the right of the vertex, but is it increasing at the vertex? The mathematics community is divided. It is perfectly acceptable for students to write that the function is increasing on either interval: (0, 5) or (0, 5), but they must consistently use the same notation for decreasing. In other words, the graph is either **both** increasing and decreasing at x = 0 or is **neither** increasing nor decreasing at x = 0. This guidance also applies to the College Board Advanced Placement exams.

On the Dashboard, press the orange plus sign to show students the definition of a piecewise function on **screen 6**. Press the orange plus sign on the Dashboard again to show students an example of why piecewise functions have their names on **screen 7**. Use this screen to explain to students where the transition numbers are for the piecewise function: x = –2 and x = 3. These are the x-values where the function transitions from the first piece to the second piece and from the second piece to the third piece.

Teacher's Note: Guiding the Activity

On screen 11, students will be continuing their Card Match activity by adding equation cards and matching them with their already existing card sets from the Explore portion of the lesson. Since the notation of piecewise functions is new to students, walk through the process of how to match equation cards with graphs by using screens 8–10.

On the Dashboard, press the orange plus sign to allow students access to **screen 8**. Review with students that they have seen open circles on number lines when graphing inequalities. Press the orange plus sign again to allow students to access **screen 9**. Guide students to use their prior knowledge of graphing inequalities to match an equation card with this graph card and set.

Press the orange plus sign on the Dashboard and review the correct answer with the students on **screen 10**. Ask for volunteers to share how they picked the k(x) function as the correct match.

Why Open Circles?

Students will likely need a reminder about open circles, especially since they have likely not seen them in the x-y-coordinate plane. Using the k(x) function, shown on screen 10, remind students that when we graph functions, we often pick integers for x-values to make the process easier and that often works out quite well. Looking at the first piece of this piecewise function where $x < 4$, this includes x-values such as 3.5, 3.9, 3.99, 3.999, etc. But these are not x-values that anyone really wants to plug in and evaluate, especially when the function is more complicated, so instead we plug in $x = 4$ and put an open circle to indicate where the function is tending towards, $y = 5$, but does not actually equal. Similarly, on the second piece where $x > 4$, we do not traditionally plug in 4.01, 4.001, 4.0001, etc. Instead, we plug in $x = 4$ and use an open circle to indicate where the function is tending towards as x approaches 4 from the right.

If students struggle with this idea, consider making a table with x-values: 3, 3.5, 3.9, 3.99, 3.999, 4, 4.001, 4.01, 4.1, 4.5, and 5 and use the piecewise function to get the corresponding y-values: 4, 4.5, 4.9, 4.99, 4.999, DNE, 4.999, 4.99, 4.9, 4.5, and 4. Plot these points. Show students how they would get the same result and how much easier it is to just use $x = 3$, 4, and 5 with an open circle at $x = 4$.

On the Dashboard, press the orange plus sign twice to allow students to progress to **screens 11–12**. Direct students to work with their partner to match the equation cards with the card sets. If students do not see cards on this screen, then they did not correctly complete the Card Matching activity on screen 4 and need to go back and correct their work.

Inform students that they can check their work on **screen 12**, where Desmos will show how many cards out of 12 are correctly matched. If the screen seems empty, it is because there are not yet any correct matches, whether that is from a lack of attempt, guessing, or misunderstanding.

30 minutes

Extend

Teacher's Note: Activity Preparation

Students will be finding the domain and range of functions and typing their answer into Desmos. Decide which notation you prefer students to use for domain and range: set notation, interval notation, etc. Take a moment to input the notation you prefer into the Desmos activity yourself to make sure you can help students as needed and to make sure the request of students is easily achieved. From the Desmos activity page, click on screen 15 or click the "Student Preview" button and navigate to screen 15, and type in the answers as if you were a student. Use the "Type Math" button and "Open Keyboard" buttons as needed.

On the Dashboard, click the orange "Stop" button; now students can complete the Desmos activity at their own pace.

Instruct students to continue working with their partner to complete **screens 13–18**. Here they will use two scatter plots with provided data on Oklahoma population and median home values. Here students identify the correct shape of each interval of the data as linear, quadratic, or exponential. Then they analyze the graphs to determine when the function is increasing or decreasing and to determine the domain and range of the function. Questions 1–4 (on screens 11–12 and 14–15) provide students with feedback for their responses. Questions 5–6 (on screens 13 and 16) ask about domain and range and do not provide feedback. Tell students the notation that you prefer them to use for domain and range.

As students finish screen 18, pass out the **Graphing Piecewise Functions** handout to each student.

Alternative Handout

The functions on the Graphing Piecewise Functions handout are on **screen 19** of the Desmos activity. Instead of using the provided handout, you could have students use graph paper.

Direct students to graph the given piecewise functions, label the intercepts, identify the domain and range, and write the intervals where the function is increasing, decreasing, and constant. Remind students to pay extra attention to the open and closed circles at the transition numbers.

5 minutes

Evaluate

Give each student the **Create a Graph** handout and direct students to work independently, so you can individually assess what students have learned from this lesson.

Alternative Handout

The criteria on the Create a Graph handout are on **screen 20** of the Desmos activity. Instead of using the provided handout, you could have students use graph paper.

Teacher's Note: Differentiation

If time allows, challenge students to write the piecewise function of their graph. Keep in mind the curriculum sequencing when adding this challenge. If this lesson is taught at the beginning of the year, students may not yet know how to write the equation of quadratic or more complicated functions. Consider telling students that the pieces need to be linear, so that they can successfully write the equation.

Teacher's Note: Big Picture

Remember that absolute value functions are a special case of piecewise functions where the two pieces of the function have reflective symmetry. Depending on your students' needs, or where you use this lesson in your curriculum pacing, consider adding how to graph absolute functions and how to algebraically write an absolute value function as a piecewise function to this .

Resources

- Bureau, U. S. C. (2021, October 8). *Historical Population Change Data (1910-2020)*. Census.gov. https://www.census.gov/data/tables/time-series/dec/popchange-data-text.html
- *Historical home prices: US monthly median from 1953-2021*. DQYDJ. (2021, November 6). https://dqydj.com/historical-home-prices/
- K20 Center. (n.d.). Card matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- PublicDomainPictures. (2012, March 1). Hands [Photograph]. Pixabay. https://pixabay.com/photos/hands-puzzle-pieces-connect-20333/
- K20 Center. (n.d.). Desmos classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081