

Function Operations, Part 2

Function Composition

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Grade Level	10th – 12th Grade	**Time Frame**	75-90 minutes
Subject	Mathematics	**Duration**	1-2 class periods
Course	Algebra 2, Precalculus		

Essential Question

What happens if the output of one function becomes the input of another?

Summary

In this lesson, students are introduced to function composition through a real-world scenario and pattern recognition. Students will learn composition notation and review domain restrictions caused by function operations. Students will use function composition to determine if functions are inverses. This lesson should be taught after students learn basic function operations and domain restrictions. Polynomial and radical functions are included. This lesson does not include exponential or logarithmic functions. Students will not be expected to perform operations on rational functions; these functions will only be the result of division. This is the second lesson in the "Function Operations" lesson duo.

Snapshot

Engage

Students create functions to represent a real-world scenario.

Explore

Students use pattern recognition to evaluate functions for numerical and non-numerical inputs.

Explain

Students complete guided notes with the class to formalize their understanding of function composition and finding domain restrictions.

Extend

Students apply what they have learned to determine whether function pairs are inverses.

Evaluate

Students demonstrate their ability to perform function operations and find domain restrictions through an Exit Ticket.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

F604: Evaluate composite functions at integer values
F708: Write an expression for the composite of two simple functions

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.F.2.2: Combine functions by composition and recognize that g(x) = f^-1 (x), the inverse function of f(x), if and only if f(g(x)) = g(f(x)) = x

Oklahoma Academic Standards Mathematics (Algebra 2)

PC.F.2.1: Model relationships through composition, and attend to the restrictions of the domain.

Attachments

- Applying Function Composition—Function Operations, Part 2 - Spanish.docx
- Applying Function Composition—Function Operations, Part 2 - Spanish.pdf
- Applying Function Composition—Function Operations, Part 2.docx
- Applying Function Composition—Function Operations, Part 2.pdf
- Coupon Conundrum—Function Operations, Part 2 - Spanish.docx
- Coupon Conundrum—Function Operations, Part 2 - Spanish.pdf
- Coupon Conundrum—Function Operations, Part 2.docx
- Coupon Conundrum—Function Operations, Part 2.pdf
- Exit Ticket—Function Operations, Part 2 - Spanish.docx
- Exit Ticket—Function Operations, Part 2 - Spanish.pdf
- Exit Ticket—Function Operations, Part 2.docx
- Exit Ticket—Function Operations, Part 2.pdf
- Exploring Function Notation—Function Operations, Part 2 - Spanish.docx
- Exploring Function Notation—Function Operations, Part 2 - Spanish.pdf
- Exploring Function Notation—Function Operations, Part 2.docx
- Exploring Function Notation—Function Operations, Part 2.pdf
- Guided Notes—Function Operations, Part 2 - Spanish.docx
- Guided Notes—Function Operations, Part 2 - Spanish.pdf
- Guided Notes—Function Operations, Part 2.docx
- Guided Notes—Function Operations, Part 2.pdf
- Lesson Slides—Function Operations, Part 2.pptx

Materials

- Lesson Slides (attached)
- Coupon Conundrum handout (attached; one half per student; printed front only)
- Exploring Function Notation handout (attached; one per pair; printed front only)
- Guided Notes handout (attached; one per student; printed front only)
- Applying Function Composition handout (attached; one per pair; printed front only)
- Exit Ticket handout (attached; one half per student; printed front only)
- Pencils
- Paper

15 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Instruct students to find a partner or assign students partners. Show **slide 5** and introduce your class to the Spotlight Questioning strategy, emphasizing that they will be working in pairs on a problem, and then you will be calling on a few individuals to share their responses with the class. This is also a great time to explain to the class why you are using the Spotlight Questioning strategy: Everyone's voice is valued.

Display **slide 6** and pass out the **Coupon Conundrum** handout to each student. Have pairs of students work through the given scenario regarding the order they think a store would apply two different coupons.

Call on your selected students to be in the "spotlight" and share the order they think a store would apply the coupons and their reasoning. After the students in the spotlight have shared their responses, ask the remaining students in your class to indicate if they agree or disagree with the responses. If the students you called on all came to the same conclusion, be sure to ask students who disagree to share their thinking too.

Show **slide 7** and direct pairs to now write an equation for each of their coupons, where $f(x)$ represents the 20% off coupon and $g(x)$ represents the $10 off coupon. As students are working, circulate the room and listen to students' conversations. Use what you hear to determine if students need a quick refresh on inputs, outputs, or writing equations for real-world scenarios.

Do not be tempted to give students the answers for writing their equations. Instead encourage them to try their best and let them know that they will see these functions again later in the lesson. Have students set aside their coupon functions until the Explain portion of the lesson.

10 minutes

Explore

Display **slide 8** and give each pair of students a copy of the attached **Exploring Function Notation** handout. Direct students to work with their partner to answer each question in the order it was given. As students work, circulate the room and monitor students' discussions, but remember that this is the time for students to try these problems on their own and later receive more guidance from you.

Teacher's Note: Guiding the Activity

The purpose of this activity is for students to see the pattern of evaluating functions and apply the pattern to interesting situations. Specifically, students are asked to plug in numbers, symbols, and words into the given function in preparation for plugging in a function.

Allow students to have a healthy struggle with this activity but use guiding questions to keep students encouraged. If a student is stuck on a question, ask the student to explain how they did the previous problem. Really listen to how they approached the previous problem. If their explanation is missing some key details, ask them to explain a different previous problem. Then ask why they feel like the current problem needs to be done differently.

As students finish the last few problems, show **slide 9** and encourage the class to check their work for questions 1–5.

Because some students need to hear a pattern more than see a pattern, use slide 8 to read the question and first step aloud, ("*f* of 3 equals 3 squared minus 5; *f* of –1 is –1 squared minus 5; ...; *f* of a star is a star squared minus 5."). Then show **slide 10** and continue reading aloud through question 9. Ask the class what they think *f* of *g* of *x* equals. Click to display the answer for question 10, again reading it aloud. Guide the class to check their work, discuss with their partner, and ask any questions.

20 minutes

Explain

Show **slide 11** and give each student a copy of the attached **Guided Notes** handout. Have students consider that the coupon scenario from earlier is actually an example of a composition of functions. When the first coupon is applied, that is the application of a function. The result of that function is what the second coupon is applied to. In other words, the output of the first function becomes the input of the second function. Have students find the composition of g and f, using the functions they created on their Coupon Conundrum handout. This is also a great time for students to check their functions before finding the composition of them. Click to display the result on slide 11.

If time allows, have students plug in $x = 100$ and compare this method to how they approached the coupon problem initially.

Transition to **slide 12** and ask students to find the composition of f and g. Click to display the result. Then ask for volunteers to explain what this means in the context of their coupon scenario.

Sample Student Responses

Students should notice that the order of the composition represents the order of the coupons being applied. In other words, $f(g(x))$ represents the \$10 off coupon being used before the 20% off coupon.

Display **slide 13** and use the Guided Notes to explain the notation used for function composition. Emphasize that the composition symbol between the letters is not a multiplication symbol, which is a common misconception. Complete the examples on the Guided Notes handout with the class. Have students add this to their math notebook if that is a classroom norm.

20 minutes

Extend

Show **slide 14** and pass out the **Applying Function Composition** handout to each pair of students.

Direct students' attention to the first part of the handout: Verifying Inverses. Have students work with their partner to determine if the pair of functions are inverses by finding the composition of f and g and the composition of g and f.

As students finish question 1, begin transitioning through **slides 15–17** so students can check their work for questions 1–3. Be sure that slides are displayed after the class has finished a problem.

Show **slide 18** and remind students of the Spotlight Questioning strategy, again emphasizing the value of hearing responses from everyone, and direct their attention to the second part of their handout: Extending Your Knowledge. Tell the class that they are to work with their partner to answer questions 4–7 and to be ready for the spotlight.

Display **slide 19** for students to use to help with their explanations. Using the Spotlight Questioning strategy, call on one student to explain how they found their result on question 4. Repeat this with the remaining questions 5–7.

Use the hidden **slide 20** for quick reference for questions 4–7.

10 minutes

Evaluate

Display **slide 21** and use the Exit Ticket strategy to assess what students have learned individually. Distribute the **Exit Ticket** handout to each student. Direct students to use the given functions to perform the indicated operations and remind them to write the domain restrictions if there are any.

Alternative Pacing

After collecting the Exit Ticket handout, and if there is enough time, consider unhiding and reviewing the solutions on **slides 22–25**. You may choose to assign the Exit Ticket as homework and review the solutions on slides 22–25 as bellwork during the next class period.

Use student responses to determine if students need additional practice of function composition, a review of function operations or domain restrictions from "Function Operations, Part 1," or if students are ready to move to the next topic.

Resources

- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Function Operations: Part 1. Lessons. https://learn.k20center.ou.edu/lesson/2177
- K20 Center. (n.d.). Spotlight questioning. Strategies. https://learn.k20center.ou.edu/strategy/2229
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081