

Function Operations, Part 2

Function Composition

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Grade Level	10th – 12th Grade	**Time Frame**	75-90 minutes
Subject	Mathematics	**Duration**	1-2 class periods
Course	Algebra 2, Precalculus		

Essential Question

What happens if the output of one function becomes the input of another?

Summary

In this lesson, students are introduced to function composition through a real-world scenario and pattern recognition. Students will learn composition notation and review domain restrictions caused by function operations. Students will use function composition to determine if functions are inverses. This lesson should be taught after students learn basic function operations and domain restrictions. Polynomial and radical functions are included. This lesson does not include exponential or logarithmic functions. Students will not be expected to perform operations on rational functions; these functions will only be the result of division. This is the second lesson in the "Function Operations" lesson duo.

Snapshot

Engage

Students create functions to represent a real-world scenario.

Explore

Students use pattern recognition to evaluate functions for numerical and non-numerical inputs.

Explain

Students complete guided notes with the class to formalize their understanding of function composition and finding domain restrictions.

Extend

Students apply what they have learned to determine whether function pairs are inverses.

Evaluate

Students demonstrate their ability to perform function operations and find domain restrictions through an Exit Ticket.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

F604: Evaluate composite functions at integer values
F708: Write an expression for the composite of two simple functions

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.F.2.2: Combine functions by composition and recognize that g(x) = f^-1 (x), the inverse function of f(x), if and only if f(g(x)) = g(f(x)) = x

Oklahoma Academic Standards Mathematics (Algebra 2)

PC.F.2.1: Model relationships through composition, and attend to the restrictions of the domain.

Attachments

- Applying Function Composition—Function Operations, Part 2 - Spanish.docx
- Applying Function Composition—Function Operations, Part 2 - Spanish.pdf
- Applying Function Composition—Function Operations, Part 2.docx
- Applying Function Composition—Function Operations, Part 2.pdf
- Coupon Conundrum—Function Operations, Part 2 - Spanish.docx
- Coupon Conundrum—Function Operations, Part 2 - Spanish.pdf
- Coupon Conundrum—Function Operations, Part 2.docx
- Coupon Conundrum—Function Operations, Part 2.pdf
- Exit Ticket—Function Operations, Part 2 - Spanish.docx
- Exit Ticket—Function Operations, Part 2 - Spanish.pdf
- Exit Ticket—Function Operations, Part 2.docx
- Exit Ticket—Function Operations, Part 2.pdf
- Exploring Function Notation—Function Operations, Part 2 - Spanish.docx
- Exploring Function Notation—Function Operations, Part 2 - Spanish.pdf
- Exploring Function Notation—Function Operations, Part 2.docx
- Exploring Function Notation—Function Operations, Part 2.pdf
- Guided Notes—Function Operations, Part 2 - Spanish.docx
- Guided Notes—Function Operations, Part 2 - Spanish.pdf
- Guided Notes—Function Operations, Part 2.docx
- Guided Notes—Function Operations, Part 2.pdf
- Lesson Slides—Function Operations, Part 2.pptx

Materials

- Desmos account
- Guided Notes handout (attached; one per student; printed front only)
- Pencils
- Paper
- Student devices with internet access

15 minutes

Engage

Teacher's Note: Desmos Activity Preparation

To use this Desmos Classroom activity, select the following link: "Function Operations, Part 2." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 3. Select "Restrict to Screens 1–3" to confirm your selection. This allows students to access only screens 1–3 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to student.desmos.com and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1–2** of the Desmos activity. **Screen 1** displays the lesson's essential question. **Screen 2** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Instruct students to find a partner or assign students partners. Direct students' attention to **screen 3** and introduce your class to the Spotlight Questioning strategy, emphasizing that they will be working in pairs on a problem, and then you will be calling on a few individuals to share their responses with the class. This is also a great time to explain to the class why you are using the Spotlight Questioning strategy: Everyone's voice is valued.

On the Desmos dashboard, click the orange plus sign twice to allow students to progress to **screens 4–5**. Have pairs of students work through the given scenario regarding the order they think a store would apply two different coupons. After students have had a chance to discuss, guide students to submit their thoughts on screen 5. Remind students that it is okay if they are unsure but that you would like to read their thinking.

Call on your selected students to be in the "spotlight" and share the order they think a store would apply the coupons and their reasoning. After the students in the spotlight have shared their responses, ask the remaining students in your class to indicate if they agree or disagree with the responses. If the students you called on all came to the same conclusion, be sure to ask students who disagree to share their thinking too.

Click the orange plus sign on the dashboard to allow students to progress to **screen 6**. Direct pairs to write an equation for each of their coupons, where $f(x)$ represents the 20% off coupon and $g(x)$ represents the $10 off coupon. As students are working, circulate the room and listen to students' conversations. Use what you hear to determine if students need a quick refresh on inputs, outputs, or writing equations for real-world scenarios.

Inform students that they will not receive feedback on this screen and that you want them to try their best. Let them know that they will see these functions again later in the lesson (during the Explain portion).

10 minutes

Explore

On the Desmos dashboard, click the orange plus sign to allow students to progress to **screen 7**. Direct students to work with their partner to answer each question in the order it was given. Have students get out a piece of paper to show their work for questions 1–10. They are to only type in the answer for question 10 on the screen. As students work, circulate the room and monitor students' discussions, but remember that this is the time for students to try these problems on their own and later receive more guidance from you.

Teacher's Note: Guiding the Activity

The purpose of this activity is for students to see the pattern of evaluating functions and apply the pattern to interesting situations. Specifically, students are asked to plug in numbers, symbols, and words into the given function in preparation for plugging in a function.

Allow students to have a healthy struggle with this activity but use guiding questions to keep students encouraged. If a student is stuck on a question, ask the student to explain how they did the previous problem. Really listen to how they approached the previous problem. If their explanation is missing some key details, ask them to explain a different previous problem. Then ask why they feel like the current problem needs to be done differently.

As students finish the last question, click the orange plus sign to allow students to access **screen 8**. If a student sees an error message on screen 8, then they have yet to complete question 10 from the previous screen. Guide students to type what they think $f(g(x))$ equals.

Direct students' attention to screen 8. Because some students need to hear a pattern more than see a pattern, use screen 8 to read the question and first step aloud, ("*f* of 3 equals 3 squared minus 5; *f* of –1 is –1 squared minus 5; ...; *f* of a star is a star squared minus 5; ..."). Guide the class to check their work, discuss with their partner, and ask any questions.

20 minutes

Explain

Give each student a copy of the attached **Guided Notes** handout. On the dashboard, click the orange plus sign twice to allow students to progress to **screens 9–10**.

Have students consider that the coupon scenario from earlier is actually an example of a composition of functions. When the first coupon is applied, that is the application of a function. The result of that function is what the second coupon is applied to. In other words, the output of the first function becomes the input of the second function. Have students find the composition of g and f, using the functions they created earlier, which are shown on this screen. This is also a great time for students to check their functions before finding the composition of them. Have pairs find the composition of g and f.

If time allows, have students plug in $x = 100$ and compare this method to how they approached the coupon problem initially.

Direct students' attention to screen 10 and ask students to find the composition of f and g and explain its meaning in the context of their coupon scenario.

Students receive immediate feedback for their algebraic responses on both screens 9 and 10. Screen 10 includes a place for students to type the meaning in their own words.

As students finish screen 10, ask for volunteers to explain what $f(g(x))$ means as it relates to their coupons.

Sample Student Responses

Students should notice that the order of the composition represents the order of the coupons being applied. In other words, $f(g(x))$ represents the \$10 off coupon being used before the 20% off coupon.

On the dashboard, click the orange plus sign to allow students to progress to **screen 11**. Direct students' attention to screen 11 and explain the notation used for function composition. Emphasize that the composition symbol between the letters is not a multiplication symbol, which is a common misconception.

On the dashboard, click the orange plus sign four times to allow students to progress to **screens 12–15**. Direct students to use the examples and explanations in Desmos to complete the Guided Notes handout.

Have students add their completed Guided Notes to their math notebook if that is a classroom norm.

20 minutes

Extend

On the dashboard, click the orange plus sign three times to allow students to progress to **screens 16–18**. Have students work with their partner to determine if the pair of functions are inverses by finding the composition of *f* and *g* and the composition of *g* and *f*. Students will receive immediate feedback based on their entered responses.

Remind students of the Spotlight Questioning strategy, again emphasizing the value of hearing responses from everyone. Click the orange plus sign four times to allow students to progress to **screens 19–22**. Tell the class that they are to work with their partner to answer questions 1–4 and to be ready for the spotlight.

Using the Spotlight Questioning strategy, call on one student to explain how they found their result on question 1 from screen 19. Repeat this with questions 2–4 on screens 20–22.

10 minutes

Evaluate

On the dashboard, click the orange "Stop" button; now students can complete the Desmos activity at their own pace. Direct their attention to **screens 23–24**. Using the Exit Ticket strategy, individually assess what students have learned. Instruct students to use the given functions to perform the indicated operations and to not forget the domain restrictions if there are any.

Use student responses to determine if students need additional practice of function composition, a review of function operations or domain restrictions from "Function Operations, Part 1," or if students are ready to move to the next topic.

Resources

- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Function Operations: Part 1. Lessons. https://learn.k20center.ou.edu/lesson/2177
- K20 Center. (n.d.). Spotlight questioning. Strategies. https://learn.k20center.ou.edu/strategy/2229
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081