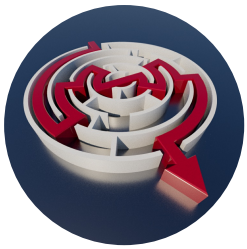

# Finding Factors, Part 2

## Factoring Polynomials

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|---|---|---|---|
| **Grade Level** | 10th – 11th Grade | **Time Frame** | 105-130 minutes |
| **Subject** | Mathematics | **Duration** | 2-3 class periods |
| **Course** | Algebra 2 | | |

## Essential Question

How are polynomial equations solved?

## Summary

In this lesson, students will recall expanding polynomials and factoring quadratics. Students will learn how to factor polynomials with two, three, or four terms: difference of two squares, sum or difference of two cubes, trinomials of the quadratic form that are not quadratics, and grouping. Students will use this knowledge to factor and solve (factorable) polynomials. This lesson is not intended to be taught immediately after the Finding Factors, Part 1 lesson, but rather after students finish learning about quadratic functions.

## Snapshot

**Engage**

Students recall the relationship between *x*-intercepts of a graph and the intercept form of a quadratic.

**Explore**

Students match expanded and factored forms of polynomial expressions to solve a diamond puzzle.

**Explain 1**

Students complete guided notes with the class and formalize their understanding of factoring two-term polynomials.

**Extend 1**

Students apply what they have learned to factor two-term polynomials.

**Explain 2**

Students complete guided notes with the class and solidify their understanding of factoring polynomials that are of the quadratic form or use grouping.

**Extend 2**

Students apply what they have learned to factor polynomial expressions and solve polynomial equations through a Choice Board.

**Evaluate**

Students reflect on their learning and demonstrate their understanding by creating a flowchart about the process of factoring.

## Standards

*Oklahoma Academic Standards Mathematics (Algebra 2)*

**A2.A.1.4:** Solve polynomial equations with real roots using various methods (e.g., polynomial division, synthetic division, using graphing calculators or other appropriate technology).
**A2.A.2.1:** Factor polynomial expressions including, but not limited to, trinomials, differences of squares, sum and difference of cubes, and factoring by grouping, using a variety of tools and strategies.

## Attachments

- Diamond Puzzle—Finding Factors, Part 2 - Spanish.docx
- Diamond Puzzle—Finding Factors, Part 2 - Spanish.pdf
- Diamond Puzzle—Finding Factors, Part 2.docx
- Diamond Puzzle—Finding Factors, Part 2.pdf
- Factor Finder—Finding Factors, Part 2 - Spanish.docx
- Factor Finder—Finding Factors, Part 2 - Spanish.pdf
- Factor Finder—Finding Factors, Part 2.docx
- Factor Finder—Finding Factors, Part 2.pdf
- Get Your Factors Straight—Finding Factors, Part 2 - Spanish.docx
- Get Your Factors Straight—Finding Factors, Part 2 - Spanish.pdf
- Get Your Factors Straight—Finding Factors, Part 2.docx
- Get Your Factors Straight—Finding Factors, Part 2.pdf
- Guided Notes—Finding Factors, Part 2 - Spanish.docx
- Guided Notes—Finding Factors, Part 2 - Spanish.pdf
- Guided Notes—Finding Factors, Part 2.docx
- Guided Notes—Finding Factors, Part 2.pdf
- Lesson Slides—Finding Factors, Part 2.pptx
- Perfect Pairings—Finding Factors, Part 2 - Spanish.docx
- Perfect Pairings—Finding Factors, Part 2 - Spanish.pdf
- Perfect Pairings—Finding Factors, Part 2.docx
- Perfect Pairings—Finding Factors, Part 2.pdf

## Materials

- Lesson Slides (attached)
- Perfect Pairings handout (attached; one per student; printed front only)
- Diamond Puzzle handout (attached; one per pair; printed front only)
- Guided Notes handout (attached; one per student; printed front/back)
- Factor Finder handout (attached; one per student; printed front only)
- Get Your Factors Straight handout (attached; one per student; printed front only)
- Pencils
- Paper
- Scissors
- Coloring utensils (optional)
- Poster paper (optional)
- "Finding Factors, Part 1" (optional; lesson series)

10 minutes

# Engage

**Teacher's Note: Lesson Order**

The order of this lesson is as follows: Engage, Explore, Explain 1, Extend 1, Explain 2, Extend 2, Evaluate.

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Display **slide 5** and give each student a copy of the attached **Perfect Pairings** handout. Instruct students to work individually to match each given graph with each given equation.

Move to **slide 6**. Using the Inverted Pyramid strategy, have students find a partner to discuss how they matched the graphs and equations. Guide students to work with their partner to then write the equation for the last graph on their handout.

After a few minutes, show **slide 7** and have each pair of students find another pair of students (creating groups of four) to compare their results and reasoning.

After a few minutes, bring the class together for a whole-group discussion. Have one student from each group share their equation and reasoning with the class.

Use student responses to see if the class needs a quick review of the relationship between the intercept form of a quadratic and the $x$-intercepts of a parabola. If needed, guide the conversation so that students draw the logical conclusion that a polynomial could cross the $x$-axis more than twice and that there is a relationship between the factored form of a polynomial and its $x$-intercepts.

15 minutes

# Explore

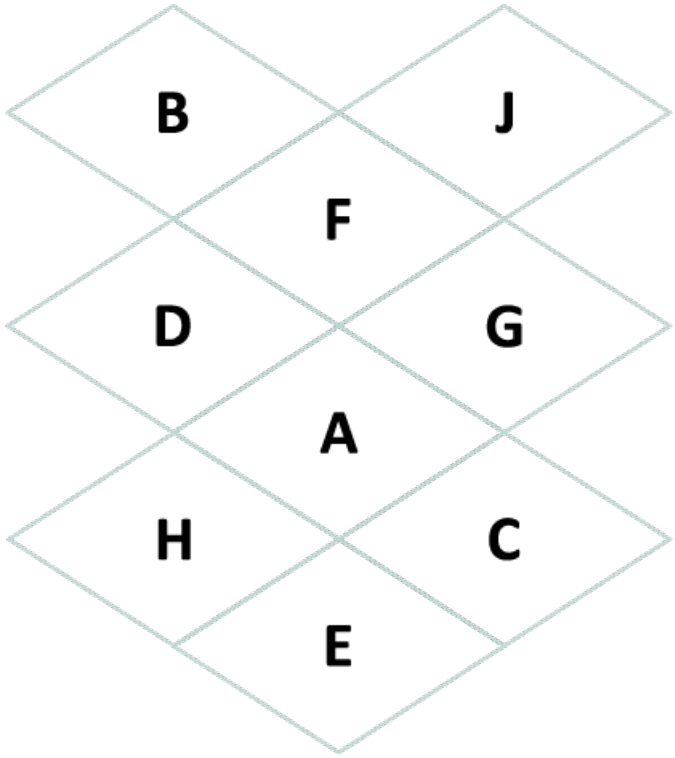
**Teacher's Note: Lesson Differentiation**

Consider removing the template image from slide 8 to increase the rigor of the puzzle for students who are apt for the challenge.

Instruct students to find their original partner or to find a new partner. Display **slide 8** and inform students that they will be practicing expanding polynomials. Pass out scissors and a copy of the attached **Diamond Puzzle** handout to each pair of students.

Direct students to cut out the nine diamond tiles. Have students rearrange the tiles into the pattern shown on the slide so that the sides touching are equivalent factored forms and expanded forms of the same expressions.

As students work together to solve the puzzle, use the image below as a quick way to check students' work. This image is also available on the hidden **slide 9**.



**Teacher's Note: Guiding the Activity**

Even though it is tempting, do not give students the answer to this puzzle. Give students time and allow them to have a healthy struggle. It is also important that students do not get frustrated and give up. To help with this balance, consider giving the following hints, depending on students' needs:

- Give feedback for correct matches.
- Help them place their correct matches in the correct place of the bigger puzzle. For example, "H, E, and C are correctly matched but are in the wrong places."
- If students are struggling to start, tell them where one tile goes. For example, "Tile B should be placed in the top-left corner of the puzzle."

15 minutes

# Explain 1

Display **slide 10**. Give each student a copy of the attached **Guided Notes** handout and complete the front side as a class.

After completing example 1 from the handout, which is attempting to factor $x^2+16$, show how to factor $x^2-16$ and then compare and contrast the two expressions. Help students understand why a sum of two squares is unfactorable.

### Teacher's Note: Reference List

If students are struggling to identify perfect squares or perfect cubes, consider having students create a quick-reference list like the example below. Make the reference list as long as you see fit to meet the needs of your students.

- Perfect Squares.....Perfect Cubes
  - $1^2=1$.....$1^3=1$
  - $2^2=4$.....$2^3=8$
  - $3^2=9$.....$3^3=27$
  - ...

### Teacher's Note: ACT Prep

Regarding the sum or difference of two cubes, students just need to recall the first factor. On the ACT, students are occasionally asked to factor a sum or difference of two cubes. Often the available options require students to recall which operation goes into the first factor: $(a+b)$ or $(a-b)$. Sometimes they will also need to recall that the second factor's middle term has the opposite operation but not need to recall any additional details of that second factor. Pattern recognition is what is really being assessed.

After completing only the front side of the handout, direct students to set it aside. Students will complete the back side later in the lesson.

15 minutes

# Extend 1

Now it is time for students to apply what they have learned; display **slide 11**. Give each student a copy of the attached **Factor Finder** handout and direct their attention to the first part of the handout: Factoring Polynomials. Have students work with a partner of their choice or assign partners to complete this handout. Instruct students that they are to factor each expression for questions 1–2 with their partner.

As you see students finishing questions 1–2, transition through **slides 12–13** so students can check their work.

Move to **slide 14** and bring the class together to discuss the following questions:

- What was similar about questions 1–2?
- What was different about questions 1–2?
- How did this impact your approach to factoring these expressions?

### Sample Student Responses

- Both expressions had two terms: $x$ to a power minus 64.
- The exponent was different for the two questions.
- The exponent told me which pattern to follow: difference of two squares or difference of two cubes.

Display **slide 15** and direct students' attention to the second part of their handout: Solving Polynomials. As a reminder, ask the class what needs to be true about a polynomial equation before they begin factoring? If needed, use this time to review the importance of the polynomial equaling zero. Consider reminding students that quadratics are a type of polynomial, so the process of using factoring to solve will be the same as it was for quadratics.

As students progress through questions 3–6, transition through **slides 16–20** so students can check their work.

### Optional Addition

If time allows, consider showing the graphs of these equations and emphasizing the relationship between the factored forms and the $x$-intercepts. Tie it back to the Engage portion of the lesson.

Display **slide 21** and bring students back together. Then, ask them the questions on the slide:

- Is there a relationship between the number of solutions and the type of polynomial?
- If so, what do you think it is?

Give all students a chance to consider this question. Encourage them to review their work from their handout to answer the questions. Then, ask for volunteers to share their responses.

15 minutes

# Extend 1

Now it is time for students to apply what they have learned; display **slide 11**. Give each student a copy of the **Factor Finder** handout and direct their attention to the first part of the handout: Factoring Polynomials. Have students work with a partner of their choice or assign partners to complete this handout. Instruct students that they are to factor each expression for questions 1–2 with their partner.

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Show **slide 14** and bring the class together to discuss the following questions:

- What was similar about questions 1–2?
- What was different about questions 1–2?
- How did this impact your approach to factoring these expressions?

### Sample Student Responses

- Both expressions had two terms: $x$ to a power minus 64.
- The exponent was different for the two questions.
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Display **slide 15** and direct students' attention to the second part of their handout: Solving Polynomials. As a reminder, ask the class what needs to be true about a polynomial equation before they begin factoring? If needed, use this time to review the importance of the polynomial equaling zero. Consider reminding students that quadratics are a type of polynomial, so the process of using factoring to solve will be the same here as it was for quadratics.

As students progress through questions 3–6, transition through **slides 16–20** so students can check their work.

### Optional Addition

If time allows, consider showing the graphs of these equations and emphasizing the relationship between the factored forms and the $x$-intercepts. Tie it back to the Engage portion of the lesson.

Bring the class together and display **slide 21**. Ask the class the questions on the slide: *Is there a relationship between the number of solutions and the type of polynomial? If so, what do you think it is?*

Use wait time to give all students a chance to consider this question. Encourage students to review their work from their handout to answer the questions. Then ask for volunteers to share their responses.

15 minutes

# Extend 1

On the Dashboard, press the orange plus sign three times to allow students to progress through **screens 10–12**. Have students work with their partner to factor each expression on screens 10–11. Students will receive immediate feedback based on their entered responses.

As students complete screen 12, bring the class together to discuss the following questions:

- What was similar about questions on screens 10–11?
- What was different about questions on screens 10–11?
- How did this impact your approach to factoring these expressions?

### Sample Student Responses

- Both expressions had two terms: $x$ to a power minus 64.
- The exponent was different for the two questions.
- The exponent told me which pattern to follow: difference of two squares or difference of two cubes.

Press the orange plus sign on the Dashboard four times to allow students to progress through **screens 13–16**. As a reminder, ask the class what needs to be true about a polynomial equation before they begin factoring. If needed, use this time to review the importance of the polynomial equaling zero. Consider reminding students that quadratics are a type of polynomial, so the process of using factoring to solve will be the same as it was for quadratics. Students will receive immediate feedback based on their entered responses in the Desmos Classroom activity.

As students finish screen 16, press the orange plus sign on the Dashboard to allow students to progress to **screen 17**. Ask the class the questions on the screen:

- Is there a relationship between the number of solutions and the type of polynomial?
- If so, what do you think it is?

Give all students a chance to consider this question and then direct them to enter their responses on screen 17. Remind students that it is okay if they are unsure but that you would like to understand their thinking. Encourage students to review their work from screens 13–16 to answer the questions. Then, ask for volunteers to share their responses.

20 minutes

# Explain 2

Display **slide 22** and focus students' attention on question 3 of their Factor Finder handout. Use this slide to help summarize or correct how students answered the question about the relationship between the number of solutions and the type of polynomial from the Extend 1 section of this lesson.

Move to **slide 23** and use the same problem to explain the definition of "multiplicity."

### Teacher's Note: Guiding the Lesson

Express to your students your expectation of how you would prefer they write their solutions. Do you want them to use the vocabulary of multiplicity or not? Do you want them to only write unique solutions or all solutions?

Be sure that students at least understand the idea of multiplicity so that they can use it to ensure that they have found all of the solutions for a given polynomial.

Have students get out their **Guided Notes** handout from earlier in the lesson and move to **slide 24**. Complete the back of the handout as a class.

After completing the handout, direct students to add this to their math notebook or otherwise save it according to classroom norms.

20 minutes

# Explain 2

On the Dashboard, press the orange plus sign to allow students to progress to **screen 18**. Tell students that this screen is referring to question 3 from screen 13. Use this screen to help summarize or correct how students answered the question about the relationship between the number of solutions and the type of polynomial from the Extend 1 portion of this lesson.

Press the orange plus sign on the Dashboard to allow students to progress to **screen 19**. Use the same problem to explain the definition of "multiplicity."

**Teacher's Note: Guiding the Lesson**

Express to your students your expectation of how you would prefer they write their solutions. Do you want them to use the vocabulary of multiplicity or not? Do you want them to only write unique solutions or all solutions?

The remainder of this lesson does not include any additional problems with zeros having a higher than one multiplicity.

Be sure that students at least understand the idea of multiplicity so that they can use it to ensure that they have found all of the solutions for a given polynomial.

Have students get out their **Guided Notes** handout from earlier and press the orange plus sign on the Dashboard three times to allow students to progress through **screens 20–22**. Direct students to use the examples and explanations in the activity to complete the back of the Guided Notes handout.

After completing the handout, direct students to add this to their math notebook or otherwise save it according to classroom norms.

20 minutes

# Explain 2

On the Dashboard, press the orange plus sign to allow students to progress to **screen 18**. Tell students that this screen is referring to question 3 from screen 13. Use this screen to help summarize or correct how students answered the question about the relationship between the number of solutions and the type of polynomial from the Extend 1 portion of this lesson.

Press the orange plus sign on the Dashboard to allow students to progress to **screen 19**. Use the same problem to explain the definition of *multiplicity*.

**Teacher's Note: Guiding the Lesson**

Express to your students your expectation of how you would prefer they write their solutions. Do you want them to use the vocabulary of multiplicity or not? Do you want them to only write unique solutions or all solutions?

The remainder of this lesson does not include any additional problems with zeros having a higher than one multiplicity.

Be sure that students at least understand the idea of multiplicity so that they can use it to ensure that they have found all of the solutions for a given polynomial.

Have students get out their **Guided Notes** handout from earlier and press the orange plus sign on the Dashboard three times to allow students to progress to **screens 20–22**. Direct students to use the examples and explanations in the activity to complete the back of the Guided Notes handout.

After completing the handout, direct students to add this to their math notebook if that is a classroom norm.

10 minutes

# Extend 2

Display **slide 25** and introduce the Choice Board strategy. Pass out a copy of the attached **Get Your Factors Straight** handout to each student. Explain the directions for the activity to the class.

The handout contains nine tasks, divided into three columns (A, B, and C) and three rows (1, 2, and 3). Students are to select one task per column and one task per row, completing a total of three tasks.

In columns A and B, students are to factor the given polynomial. In column C, students are to solve the given polynomial by factoring.

### Teacher's Note: Guiding the Activity

Before beginning this activity, ensure that all students understand the directions. Consider giving students an example. If they choose the first question in column A—let's call it question 1A—they may not choose another question from row 1 or column A for credit. In other words, if they pick question 1A, they may not do question 1B, 1C, 2A, or 3A for credit. Then, let's say they choose the third question in column B. In this case, they must complete the second question in column C. In other words, they would complete questions 1A, 2C, and 3B.

Possible combinations are as follows:

- 1A-2B-3C
- 1A-2C-3B
- 1B-2A-3C
- 1B-2C-3A
- 1C-2A-3B
- 1C-2B-3A

Have pairs select which tasks they both want to answer.

Once students have selected their three tasks, direct students to work independently. After completing the three tasks, have students check their work with their partner.

10 minutes

# Extend 2

Display **screen 23** and introduce the Choice Board strategy.

The Choice Board contains nine tasks, divided into three columns (A, B, and C) and three rows (1, 2, and 3). In columns A and B, students are to factor the given polynomial. In column C, students are to solve the given polynomial by factoring.

Have students get with a partner and decide which tasks they will select. They should select one task per column and one task per row, completing a total of three tasks.

### Teacher's Note: Guiding the Activity

Before beginning this activity, ensure that all students understand the directions. Consider giving students an example. If they choose the first question in column A—let's call it question 1A—they may not choose another question from row 1 or column A for credit. In other words, if they pick question 1A, they may not do question 1B, 1C, 2A, or 3A for credit. Then, let's say they choose the third question in column B. In this case, they must complete the second question in column C. In other words, they would complete questions 1A, 2C, and 3B.

Possible combinations are as follows:

- 1A-2B-3C
- 1A-2C-3B
- 1B-2A-3C
- 1B-2C-3A
- 1C-2A-3B
- 1C-2B-3A

When partners have selected which questions they want to complete, have students complete the problems individually. As they come to their answers, have them transition through **screens 24-26** to input and verify their answers.

Once students have confirmed that they have the correct answers, have them get back with their partners to discuss the processes they used to reach a their final answers.

10 minutes

# Extend 2

Display **slide 25** and introduce the Choice Board strategy. Pass out the attached **Get Your Factors Straight** handout to each student.

Explain the directions for the activity with the class.

The handout contains nine tasks, which are divided into three columns (A, B, and C) and three rows (1, 2, and 3). Students are to select one task per column and one task per row, completing a total of three tasks.

In columns A and B, students are to factor the given polynomial. In column C, students are to solve the given polynomial by factoring.

### Teacher's Note: Guiding the Activity

Before beginning this activity, ensure that all students understand the directions. Consider giving students an example. If they choose the first question in column A - let's call it question 1A - they may not choose another question from row 1 or column A for credit. In other words, if they pick question 1A, they may not do question 1B, 1C, 2A, or 3A for credit. Then, let's say they choose the third question in column B - in this case, they must complete the second question in column C. In other words, they would complete questions 1A, 2C, and 3B.

Possible combinations are as follows:

1A-2B-3C 1B-2A-3C 1C-2A-3B

1A-2C-3B 1B-2C-3A 1C-2B-3A

Have pairs select which tasks they both want to answer.

Once students have selected their three tasks, direct students to work independently. After completing the three tasks, have students check their work with their partner.

20 minutes

# Evaluate

### Teacher's Note: Activity Preparation

During this portion of the lesson, students will create flowcharts that another student could use to learn how to factor polynomials. Consider what you would like students to do with their completed flowcharts. They could:

- Display them on the wall for students to reference.
- Use poster paper and coloring utensils to make their flowcharts into Anchor Charts.
- Trade flowcharts and use their peers' flowcharts to factor polynomials (if time allows or as later review or practice).
- Use notebook or copy paper—or even tape multiple sheets of paper together—to create their flowcharts.

Regardless of what you choose, it is likely that students will need a place to plan their flowcharts and a place to put their finished ideas.

### Optional Technology Integration

If you would like students to digitally create their flowcharts, have students use Google Drawings. Keep in mind that a digital creation will likely double the amount of time needed for this activity.

Give students the following advice for using this tool to create a digital flowchart:

- Use copy and paste for the flowchart shapes (and some text).
- Select "Insert," then hover over "Shape" to find the options for shapes to insert.
- Double-click on the shape to add text.
- Select "Format," then hover over "Text" to find the superscript option for exponents.

Display **slide 26** and introduce students to the idea of a flowchart. The shapes in a flowchart indicate meaning, as indicated on the slide.

Give students paper (and coloring utensils if you prefer) and direct them to work with their partner to create their own flowchart that another student could use to learn how to factor polynomials. Communicate your expectations of this project with students.

### Optional Scaffolding

Unhide and use **slide 27** if needed to help students get started.

## Resources


- K20 Center. (n.d.). Anchor Charts. Strategies. https://learn.k20center.ou.edu/strategy/58
- K20 Center. (n.d.). Card Matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- K20 Center. (n.d.). Choice Boards. Strategies. https://learn.k20center.ou.edu/strategy/73
- K20 Center. (n.d.). Google Drawings. Tech tools. https://learn.k20center.ou.edu/tech-tool/629
- K20 Center. (n.d.). Inverted Pyramid. Strategies. https://learn.k20center.ou.edu/strategy/173