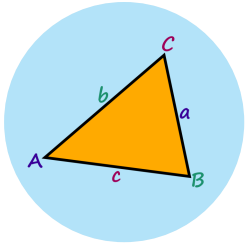

Law of Sines

AAS, ASA, SSA

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Grade Level	11th – 12th Grade	**Time Frame**	80-100 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Precalculus		

Essential Question

How can indirect measurements help calculate unknown distances and angle measurements?

Summary

In this lesson, students will discover how a given side, side, and angle could result in 0, 1, or 2 triangles. They will then learn the Law of Sines and use it to find missing side lengths and angle measures. This lesson is intended to be taught before the Law of Cosines lesson and after students learn about inverse trigonometric functions.

Snapshot

Engage

Students use the Not Like the Others strategy to recall triangle vocabulary.

Explore

Students explore the ambiguous case of side-side-angle using GeoGebra.

Explain

Students formalize their understanding of the ambiguous case and the Law of Sines.

Extend

Students apply what they have learned to solve the unknown side length within a puzzle.

Evaluate

Students reflect on their learning using the Stoplight Stickies strategy.

Attachments

- Guided Notes (Model Notes)—Law of Sines.docx
- Guided Notes (Model Notes)—Law of Sines.pdf
- Guided Notes—Law of Sines - Spanish.docx
- Guided Notes—Law of Sines - Spanish.pdf
- Guided Notes—Law of Sines.docx
- Guided Notes—Law of Sines.pdf
- How Many Triangles—Law of Sines - Spanish.docx
- How Many Triangles—Law of Sines - Spanish.pdf
- How Many Triangles—Law of Sines.docx
- How Many Triangles—Law of Sines.pdf
- Lesson Slides—Law of Sines.pptx
- Using the Law of SSSSines (Sample Response)—Law of Sines.docx
- Using the Law of SSSSines (Sample Response)—Law of Sines.pdf
- Using the Law of SSSSines—Law of Sines - Spanish.docx
- Using the Law of SSSSines—Law of Sines - Spanish.pdf
- Using the Law of SSSSines—Law of Sines.docx
- Using the Law of SSSSines—Law of Sines.pdf

Materials

- Lesson Slides (attached)
- How Many Triangles handout (attached; one per student; printed front/back)
- Guided Notes handout (attached; one per student; printed front/back)
- Guided Notes (Model Notes) document (attached; for teacher use)
- Using Law of SSSSines handout (attached; one per student; printed front only)
- Using Law of SSSSines (Sample Response) document (attached; for teacher use)
- Pencils
- Paper
- Sticky notes (preferably red, yellow, and green; 1 sticky note per student)
- Scientific calculator
- Student devices with internet access

10 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's Essential Question. **Slide 4** identifies the lesson's Learning Objectives. Review each of these with your class to the extent you feel necessary.

Show **slide 5** and introduce students to the Not Like the Others strategy. Instruct students to decide which given triangle is not like the others. Then ask for a few volunteers to share their reasoning. If time allows, ask for students who have different answers to share or ask the class if anyone can find an answer that someone has not yet heard.

Teacher's Note: Activity Purpose

The purpose of this activity is for there to be more than one correct response and for students to use academic vocabulary to help you determine students' prior knowledge.

Sample Student Responses:

Responses will likely vary but encourage students to use academic vocabulary with their justifications.

- The first triangle is not like the others because it is an acute triangle.
- The second triangle is not like the others because it is the only triangle with a right angle.
- The third triangle is not like the others because only one angle measure is given.
- The third triangle is not like the others because it has more known side lengths.

20 minutes

Explore

Display **slide 6** and introduce students to the Think-Pair-Share strategy. Explain that students are going to start the GeoGebra activity individually as the "Think" portion of the strategy. Provide students with the link to the GeoGebra activity: geogebra.org/m/rsw7dspt. This interactive GeoGebra activity gives students the opportunity to see why certain given side-side-angle combinations result in 0, 1, or 2 triangles. It includes two applets: The first has a given acute angle, while the second has a given obtuse angle.

After giving students a couple of minutes to explore the GeoGebra activity, pass out the attached **How Many Triangles** handouts to each student. Direct their attention to the front of the handout: *Acute Angle* and the first applet: *SSA Applet – Acute Angle*. Remind them that they are to be individually thinking and completing their table.

Teacher's Note: Providing Guidance

It is best practice to become familiar with the activity yourself before completing it with your students. This will ensure you can guide students and help them find success.

If students are struggling with the activity, consider giving more specific directions such as "Drag point C to the x-axis to create a triangle." Consider having students complete the table from bottom to top instead. This could help if students are unsure of how to create the possible triangles and feel stuck.

Instruct students to find a partner or assign partners. Have pairs compare their results and confirm or revise each other's noticed patterns.

Sample Student Responses:

Push students to write statements that work for more than one triangle or to rephrase what they have to be a more general statement – something that is always true, not just true for this triangle. There are many ways to write the observed patterns.

- If the first leg ≥ the second leg, then there is only one triangle.
- If the first leg < the second leg, then there are 0, 1, or 2 triangles.
- If the first leg is half of the second leg, then there is only 1 triangle.
  **If this statement arises, do not yet correct it. It will be revised for accuracy during the Explain portion of the lesson.*
- If the first leg < 4, then there are 0 triangles.
- If the 4 < first leg < second leg, then there are 2 triangles.

Display **slide 7** and ask for volunteers to share their generalized observations. Write these on the board so that the class can see the list. Continue until all patterns have been shared. Ask the class to compare what is on the board with their list and if anything is on the board that they did not record to check the statement. If students find counterexamples, remove or revise the claims accordingly.

Teacher's Note: Guiding the Activity

At this time, do not correct any students' statements. However, ask clarifying questions that promote academic language. For example, if a student said, "If the first one is less than the second one, then ..." then you would ask, "The first what?"

At this time, students do not need to write the patterns listed on the board on their handout. They can, but at this time, they just need to compare and analyze the list. The patterns will be formalized and recorded elsewhere during the Explain portion of the lesson.

Now direct students' attention to the back of the How Many Triangles handout: *Right or Obtuse Angle* and the second applet: *SSA Applet – Right or Obtuse Angle*. Ask students to work in pairs to complete the table and record any observed patterns. This applet only uses an obtuse angle, but the observed patterns apply to both right and obtuse angles.

Sample Student Responses:

- If the first leg ≤ second leg, then there are 0 triangles.
- If the first leg > second leg, then there is 1 triangle.

Display **slide 8** and ask for volunteers to share their generalized observations. Write these on the board so that the class can see the list. Continue until all patterns have been shared. Ask the class to compare what is on the board with their list and if anything is on the board that they did not record to check the statement. If students find counterexamples, remove or revise the claims accordingly.

Make note of any misconceptions or concerns from the student responses and be sure to address them during the Explain portion of the lesson.

25 minutes

Explain

Teacher's Note: Pacing the Lesson

If you have a traditional 45-minute class period, it is recommended that you finish the first day by completing the table for the ambiguous case on the Guided Notes. Consider sending your students home with the challenge to prove the given Law of Sines by giving them the hint of drawing the height of the triangle. Then, you can begin the next class period by inviting students to share their approaches.

Show **slide 9** and tell the class that they now need to work together to write three statements that when given a side, side, and acute angle yield 0, 1, or 2 triangles (one statement each). Have students discuss with their partner what those three statements should be. As students are discussing, give each student a copy of the attached **Guided Notes** handout.

After a few minutes, if the class observed a relationship to the first leg length and 4 (or half of eight), ask the class the following questions to get them to see that 4 is unique to this triangle and that the 4 is 8·sin (30°) = (hypotenuse)·sin (θ), also known as the height of the right triangle.

- What was significant about the triangle when the first leg was 4 units long?
- What is the relationship between 4 and the other side?
- Is there something that would relate 30° to one-half? Is there a trigonometric function that relates an angle, the side opposite of the angle, and the hypotenuse? We are going to be talking about the sine function today.
- Is there another name we could use for that opposite side of the triangle?

Teacher's Note

These are typical student responses to the guided questions above:

- It was a right triangle.
- Four is half of the length of the other side.
- The sin (30°) is one-half.
- The side with a length of 4 is also the height of the triangle.

If student responses vary significantly, this is the time for you to correct their thinking.

Now that students have the vocabulary of “height” for the side length of 4, give students a few more minutes to think of their three statements about what would cause 0, 1, or 2 triangles.

Bring the class together for a whole-class discussion and determine what should actually be written in the notes.

Repeat this with two statements for a given side, side, and obtuse angle. Let students know that the patterns for obtuse angles also apply to a right angle.

Teacher's Note: Guiding the Lesson

This is the time to correct student's misunderstandings and help them write accurate generalizations. Use the attached **Guided Notes (Model Notes)** document to guide what could be written in the table.

Display **slide 10** and complete the examples on the Guided Notes handout with the class.

Teacher's Note: Guiding the Lesson – Example 2

Help students understand why the two possible angle measures are θ° and (180–θ)° by using the unit circle or the graph of the sine function. Consider using 30° and 150° as a specific example since they both have the same reference angle measurement and evaluating the sine function for those values has a simple output.

Optional Scaffolding

If students are struggling to see why they need to check how many triangles might exist given a side, side, and angle or if they are struggling with how to compare the side length with the height of the triangle, consider sharing the following videos before moving to the next part of the lesson.

- Determining if Triangles Exist – Nerdstudy
- The Ambiguous Case for Sine Law – Nerdstudy

Have students add their completed Guided Notes to their math notebook if that is a classroom norm.

20 minutes

Extend

Teacher's Note

Since the following activity requires students to use one result to get the next result and so on, consider giving students the option of check-ins (opportunities for students to check their work). Otherwise, students have the potential of getting the wrong final answer due to only one minor mistake, since one incorrect result will cause each of the following results to be incorrect as well. Going back to fix the last 5 triangles could be frustrating, so consider encouraging students to come to your desk to check their work. Use the attached **Law of SSSSines (Sample Response)** document and consider the following approaches to check-ins:

- Students can check their third and sixth triangle's side lengths.
- Students are allowed to check any three, but only three, values.

Decide whether you want the Using Law of SSSSines handout to be guided practice or independent practice. Let your decision determine how many check-ins you would like students to have.

If you do not prefer using check-ins, be sure to use the first incorrect value and follow the student's work to see if that was the only error.

Instruct students to find a new partner, someone they have not yet worked with during this lesson or assign students new partners. Show **slide 11** and pass out the **Using Law of SSSSines** handout.

Now is the time for students to apply what they have learned to determine the unknown side length.

5 minutes

Evaluate

Teacher's Note: Activity Preparation

Students are going to use the Stoplight Stickies strategy and write a question or comment on a specific-colored sticky note regarding their confidence level of using the Law of Sines. Decide where you want students to put their sticky notes before starting this activity. Consider organizing your dry-erase board, or other space in the room, into a table with 3 columns. Label the columns: “Confident,” “Unsure,” and “Not Confident.” Students would place their green sticky notes in the “Confident” column, yellow in the “Unsure” column, and red in the “Not Confident” column.

Display **slide 12** and share the Stoplight Stickies strategy with the class and make available red, yellow, and green sticky notes to the students. Direct students to write a question or comment, on their sticky note, that represents their confidence level regarding their ability to use the Law of Sines. Guide students to where you want them to put their sticky notes.

If time allows, answer some of the questions on the red sticky notes to help resolve any confusion. Use students’ comments and questions to determine whether students need remediation or are ready for the next lesson: “Law of Cosines.”

Resources

- K20 Center. (n.d.). GeoGebra. Tech tools. https://learn.k20center.ou.edu/tech-tool/2352
- K20 Center. (n.d.). Not Like the Others. Strategies. https://learn.k20center.ou.edu/strategy/77
- K20 Center. (n.d.). Stoplight Stickies. Strategies. https://learn.k20center.ou.edu/strategy/92
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/139
- Nerdstudy. (2017, February 11). *Determining if Triangles Exist – Nerdstudy* [Video]. YouTube. https://youtu.be/iNCFx3XR-hw
- Nerdstudy. (2017, February 11). *The Ambiguous Case for Sine Law – Nerdstudy* [Video]. YouTube. https://youtu.be/RCyjglaJo5w