Measuring Borders

Convergent and Divergent Series

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Grade Level	10th – 12th Grade	**Time Frame**	1-2 class period(s)
Subject	Mathematics	**Duration**	60 minutes
Course	AP Calculus, Algebra 2, Precalculus		

Essential Question

How long is a border and how should you measure it?

Summary

Students will explore what happens when one measures a "rough" object using different units of measure. This lesson is adapted from Benoit Mandelbrot's famous problem, "Measuring the Coastline of Britain." Students will learn about the concepts of convergent and divergent series and the paradoxical coastline phenomenon. This lesson can also be used as an introduction to converging and diverging limits in calculus.

Snapshot

Engage

Students quickly explore the question, "How long is Oklahoma's border, and what is the best way of measuring it?"

Explore

Students explore Benoit Mandelbrot's technique of measuring a coastline using different sized "rulers."

Explain

Students share the results of their investigation.

Extend

Students research areas in the world where coastlines or borders are having an impact in the world.

Evaluate

Students present their findings and reflect on the process.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

F703: Exhibit knowledge of geometric sequences

AP Calculus AB and BC Course and Exam Description (AP Calculus AB & BC (2020))

LIM-7.A: Determine whether a series converges or diverges. bc only

Attachments

- How I Know It - Spanish.docx
- How I Know It.docx
- How Long is a Border - Spanish.docx
- How Long is a Border - Spanish.pdf
- How Long is a Border.docx
- How Long is a Border.pdf
- What, So What, Now What - Spanish.docx
- What, So What, Now What.docx

Materials

- Rulers (included in the handout)
- Map of Oklahoma
- Handout
- Pencil

Engage

Teacher's Note: Spanish Handouts

The learner handouts are available in English and Spanish to meet your students' needs. Keep in mind that the AP exam is only administered in English.

Display the first question: "Without googling the answer, how long is Oklahoma's border?"

To answer the question, pass out the "How I Know It" handout, which is attached to this lesson. Have students write in the circle what they believe is the answer to the question. After listing their estimate, ask the students to think about how they know this information and how they came to their conclusions. In the rectangle surrounding the circle, have students write out how they know it.

Teacher's Note

Teachers may want to prompt students in this part of the activity with questions like, "Where did your estimate come from?" and, "What is the source?" This may require some deep thinking on the part of students as it asks them to think of their knowledge as constructed and influenced by multiple sources

After filling out the graphic organizer for the first question, display the second question: "What would be the best way of measuring the border?" Repeat the process of the How I Know It strategy

After going through both questions, have a few volunteers share out their answers to the first question and then also to the second.

Explore

Pair students together. To do this, you can use the strategy called Fold the Line. Have students line up according to their estimates of how long the Oklahoma border is, from the students with the longest estimates to those with the shortest. To "fold the line," have the student at one end of the line walk to the student at the other end, the rest of the line following suit. The student who is first in line should stop in front of the student who is last in line, and the rest of the students do something similar (the second student pairs with the second to last and so on).

Teacher's Note

Sometimes it is helpful for students to hear viewpoints that differ from their own. This form of heterogeneous pairing can potentially produce interesting dialogue between students.

Variation

Split and Slide: The line is split in the middle. Half of the line takes three steps forward. The line then slides down so every student faces another student. Students who were in the middle are now faced with students from the ends.

After pairing students, provide each pair with the "How Long is a Border?" handout, which can be found under Attachments. Give students time to work through the questions, providing assistance for students who may be struggling but not by supplying answers to them.

You may consider prompting students with questions such as:

- Does your partner agree with your measurement?
- How did you come to that conclusion?
- What do you think the goal of this question is?

Explain

Students should be able to articulate what is happening to the perimeter (border) of the state and the area of the state as the measuring sticks become shorter. The perimeter is diverging while the area is converging. Encourage students to share their findings and explain what is happening in their own words. Encourage students to justify their answers to clear up misconceptions. If a student gets the answers wrong, coach them through their thinking and allow them to hear other justifications to help change their thinking.

Sample Student Responses

Some sample student responses are: "I see that as I measure with smaller rulers, the perimeter seems to be getting longer," or, "I am noticing that the area of the border seems to become more defined as I measure with smaller rulers."

Teacher's Note

Coaching a student through their thinking may include prompts like, "What led you to your result?" or, "Walk me through your process so I can understand your thinking."

Extend

Have pairs form groups of four. Ask students to brainstorm different areas in the world where borders are an issue of importance. In rounds, ask students to articulate why they chose those particular areas.

Ask students to answer these questions (optional):

- What is a border and how would you define it?
- Why do we have borders?
- Is there a right way to measure borders and/or coastlines?
- Knowing how long some natural borders can be, what would be the best way to secure a border?
- Are political borders or natural boundaries more effective?

Teacher's Note

Knowing that some of these topics can be sensitive, please use your discretion when asking students to discuss them.

Evaluate

The student reflection here uses the What? So What? Now What? strategy. Please use the graphic organizer attached titled "What? So What? Now What?"

Have students write in each column as you prompt them:

"In the WHAT section, describe what it was that we did today." Give students 2 - 3 minutes to write

"In the SO WHAT section, explain the implications of what we did today. In other words, why does this activity matter, and what impact did it have on you?"

"In the NOW WHAT section, talk about how this activity applies to your life and what you will take away from it."

Have students turn in their work and their reflections.

Resources

- K20 Center. (n.d.). Fold the Line. Strategies. https://learn.k20center.ou.edu/strategy/171
- K20 Center. (n.d.). How I Know It. Strategies. https://learn.k20center.ou.edu/strategy/144
- K20 Center. (n.d.). What? So What? Now What?. Strategies. https://learn.k20center.ou.edu/strategy/95
- Thompson-Grove, G. (2012). What? So What? Now What? http://schoolreforminitiative.org/doc/what_so_what.pdf
- TED on Benoit Mandelbrot, Mathematician: http://www.ted.com/speakers/benoit_mandelbrot