

Making Connections

Connecting Functions and Their Derivatives

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Grade Level	12th Grade	**Time Frame**	95-135 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	AP Calculus		

Essential Question

How are the graphs of a function and its derivatives related?

Summary

The goal of this lesson is to help students understand the relationships between a function and its first and second derivatives. Students will analyze graphs recalling their knowledge of sketching graphs by hand using the first and second derivatives and apply their graphical knowledge verbally through a Leap Frog game.

Snapshot

Engage

Students make observations about four functions and their corresponding derivative as the derivative of the function is graphed in real time.

Explore

Students use one graph, viewing it once as the first derivative and once as the second derivative, to draw conclusions about the function.

Explain

Students look at three unlabeled curves and label them as f, f ', and f '', and as a class, formalize their understanding of how the function and its derivatives are related.

Extend

Students apply their understanding of the relationships between the function and its first and second derivatives through a Leap Frog game.

Evaluate

Students complete an Exit Ticket where they answer an AP exam-style question to demonstrate their understanding of derivative relationships.

Standards

AP Calculus AB and BC Course and Exam Description (AP Calculus AB & BC (2020))

FUN-4: A function's derivative can be used to understand some behaviors of the function.
FUN-4.A: Justify conclusions about the behavior of a function based on the behavior of its derivatives.
FUN-4.A.11: Key features of the graphs of f, f', and f'' are related to one another.

Attachments

- Exit Ticket—Making Connections - Spanish.docx
- Exit Ticket—Making Connections - Spanish.pdf
- Exit Ticket—Making Connections.docx
- Exit Ticket—Making Connections.pdf
- Exploring Relationships—Making Connections - Spanish.docx
- Exploring Relationships—Making Connections - Spanish.pdf
- Exploring Relationships—Making Connections.docx
- Exploring Relationships—Making Connections.pdf
- Leap Frog Student Cards—Making Connections - Spanish.docx
- Leap Frog Student Cards—Making Connections - Spanish.pdf
- Leap Frog Student Cards—Making Connections.docx
- Leap Frog Student Cards—Making Connections.pdf
- Leap Frog Teacher Cards—Making Connections.docx
- Leap Frog Teacher Cards—Making Connections.pdf
- Lesson Slides—Making Connections.pptx
- Which Graph is Which—Making Connections - Spanish.docx
- Which Graph is Which—Making Connections - Spanish.pdf
- Which Graph is Which—Making Connections.docx
- Which Graph is Which—Making Connections.pdf

Materials

- Lesson Slides (attached)
- Exploring Relationships handout (attached; one per pair; printed front and back)
- Which Graph is Which? handout (attached; one per student; printed front only)
- Leap Frog Student Cards handout (attached; one set per student; printed front only)
- Leap Frog Teacher Cards document (attached; for teacher use; one set; printed front only)
- Exit Ticket handout (attached; one half per student; printed front only)
- Pencils
- Coloring utensils (3 colors per student; highlighters, colored pencils/pens, etc.)
- Graphing calculators
- Student devices with internet access
- Card stock (optional)

15 minutes

Engage

Teacher's Note: Spanish Handouts

The learner handouts are available in English and Spanish to meet your students' needs. Keep in mind that the AP exam is only administered in English.

Teacher's Note: Lesson Preparation

Before you begin, print the attached Leap Frog Student Cards handout (one copy per student) and Leap Frog Teacher Cards document (one copy for you). Consider printing the cards on card stock paper, especially if you plan to reuse these cards. Consider using different colors of paper, printing each set in a different color to avoid mixing the card sets. This can be accomplished by using at least four different colors and passing out different colored sets to students sitting next to each other.

Once printed, cut out the cards. All of the cards are the same size for easy cutting. These cards will be used during the Extend portion of the lesson.

Teacher's Note: Using the Desmos Studio Graphing Calculator

Try to become familiar ahead of time with the Desmos Studio graphing calculator to better help students navigate through the following activity. It is not necessary for students to have Desmos accounts to use the Desmos Studio graphing calculator. For more information, go to https://k20center.ou.edu/externalapps/graphing-calculator.

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question with students. Move to **slide 4** to share the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Ask students to find a partner or assign them. Display **slide 5** and give each pair of students a copy of the attached **Exploring Relationships** handout. Introduce students to the I Notice, I Wonder strategy and direct them to draw a table on the back of their handout.

Show **slide 6** and have students go to the Making Connections collection of graphs. These graphs have an animated tangent line and first derivative graph. Direct students to click on any two of the graphs to create and complete an I Notice, I Wonder table. Give students some time to reflect and talk with their partner about their discoveries.

Ask for volunteers to share which graphs they selected and what they noticed and wondered about.

Teacher's Note: Guiding the Activity

Since these graphs are animated, they can be quite captivating. It will be tempting to spend a long time looking at these graphs. If needed, remind students after a few minutes that they need to move on to the second graph.

These graphs are intended to give students exposure to a variety of types of functions. They should be making general observations, as they will be expected to be more detailed later in the lesson. If students are struggling, though, consider asking the following questions:

- What do you notice about the function's behavior and the graph of the derivative?
- What correlations do you see in the slope of the line tangent to the graph and the derivative?

30 minutes

Explore

Display **slide 7** and introduce students to the Inverted Pyramid strategy. Direct students' attention to the front of the Exploring Relationships handout. Have students graph the given function on their graphing calculator. This could be done by hand but the calculator will save time.

For Questions 1 and 2, have students imagine that the graph on their calculator is the first derivative of a function. Question 1 asks students to describe the function, using the graph of the derivative (the graph on their calculator). Question 2 asks students to describe the second derivative.

Have students work in pairs to answer Questions 1 and 2. After a few minutes, show **slide 8** and have pairs of students find another pair of students (creating a group of four) to compare their results and reasoning.

Bring the class together for a whole group discussion. Have one student from each group share their responses and write the responses on the board for all to see. Before moving on, make sure students have represented everything for Questions 1 and 2.

Give students time to ask questions, and encourage students to justify their answers.

Sample Student Responses

Use the hidden **slides 9–10** as a resource for sample responses for Questions 1 and 2.

Show **slide 11** and have students split back into their original pairs to answer the remaining questions. For Questions 3 and 4, students are looking at the same graph on their calculator, but now imagining that it is the second derivative of some function. Question 3 asks students to describe the function, and Question 4 asks them to describe the first derivative.

After a few minutes, display **slide 12** and have pairs of students create new groups of four to share and compare their results and reasoning. Working with different peers fosters the development of academic vocabulary and encourages students to consider different approaches to a problem.

Bring the class together for a whole group discussion. Have one student from each group share their responses and write the responses on the board for all to see. Before moving on, make sure students have represented everything for Questions 3 and 4.

Give students time to ask questions, and encourage students to justify their answers.

Sample Student Responses

Use the hidden **slides 13–14** as a resource for sample responses for Questions 3 and 4.

20 minutes

Explain

Show **slide 15** and give each student a copy of the attached **Which Graph is Which?** handout and three coloring utensils.

Students are to use the coloring utensils to color coordinate their functions. In other words, students should use one color for the first derivative on the first and second graph, a second color on the second derivative of both graphs, and a third color for $f(x)$ on both graphs. Therefore, students could trace each curve on the first graph now or after the class decides which graph is which.

After students have had some time to analyze the graphs and make notes, ask if anyone thinks they know which graph is the original function and why. If there are not any volunteers, ask what types of notes they have made to help the class see how using the properties of the graphs and using process of elimination they are able to make a decision.

Use **slide 16** to share and review one approach to the question. Give students time to process and ask questions.

Show **slide 17** and ask students to work with their partner to try the second example on their handout. As students work, circulate the room and listen to student discussions. Once a few pairs of students finish, display **slide 18** so that students can check their work.

Once the class has finished, ask for volunteers to explain why the graph on the slide is labeled correctly.

25 minutes

Extend

Teacher's Note: Leap Frog Game Preparation

For the card game, students who answer questions correctly "leap over" a classmate who did not get the correct answer. The student who answers the question correctly moves to the next empty seat. The goal for the students is to get back to their original seats.

Before students begin, decide how students will move throughout the room. Depending on your classroom setup, it might be helpful to have students "snake" through the room, where the first row all moves left, the second row all moves right, etc., and the person in the last chair in the back row moves to the empty chair in the front row.

Also decide the rules of how to define a winner(s). The simplest way is that a student wins when they are back in their original seat or have just passed it. It can be fun to also play the game where the person must get back to their original seat (where making it past their seat would not count). Often the person near their original seat is sabotaged by the person in that seat getting the question incorrect intentionally, so that person would have to answer enough questions to make another lap around the room.

The game can also end by using a timer. For example, the class plays the Leap Frog game for 25 minutes, and whoever is closest to their original seat is the winner.

Display **slide 19**. Remind students to be kind and careful with the printed cards, then pass out a set of the **Leap Frog Student Cards** to each student and direct them to arrange the cards on their desk such that they could easily select one as the answer to a question. Give the warning that students should not spend too much time arranging their cards, as they will be at different desks throughout the game.

Display **slide 20**. Explain how the game is played and then use one of the attached **Leap Frog Teacher Cards** to model the procedure. This is an ideal time to select a card that everyone should get correct.

Explain to students that only one of their cards is correct even if multiple feel like they could be. The correct card is the one that creates a biconditional statement (if and only if). In other words, if the teacher card is true, then the student card is true AND if the student card is true, then the teacher card is true. So students need to think about that before selecting a card. For example, if the function has a relative maximum (teacher card), then the first derivative changes from positive to negative (student card). The first derivative would also be zero (student card), but it being zero does not determine if the function has a relative maximum (teacher card). Therefore, the correct card is the student card that states that the first derivative changes from positive to negative and not the card that says that the first derivative is zero.

Read the teacher card aloud, repeating it at least once, then have students select a student card from their desk and place it on their forehead, facing you. Scan the room to get an idea of who answered the question correctly but do not try to keep track of this information. Read the correct response aloud and direct students to place the card back on their desk. Have students who selected the correct card stand up and move along the path you explained earlier to the next empty seat. Students usually do a good job of honestly self-managing.

Repeat this until a student makes it back to his/her seat (or time is called).

Teacher's Note: Guiding the Activity

As you read the cards, help your students with academic vocabulary by reading the same card multiple ways. For example, select the card that says, "If f(x) is increasing, f'(x) _____." Then read it aloud three times:

- If f(x) is increasing, f'(x) *blank*.
- If the function is increasing, the first derivative *blank*.
- If f(x) is increasing, the first derivative *blank*.

It is possible, but unlikely, that only one student gets a question correct and "makes the full lap" in one turn. If this happens, pause the game and review the card and answer with the class. Then set the card aside to make sure that it is asked again in the near future.

If time allows, play another round by having students go back to their original seats and begin the game again.

5 minutes

Evaluate

Display **slide 21** and use the Exit Ticket strategy to assess what students have learned individually. Give each student a copy of the attached **Exit Ticket** handout. Students are asked to determine which graph would have a relative minimum given graphs of the derivative.

Teacher's Note: Exit Ticket Question

This question is similar to an AP exam multiple-choice question. Remind students that the real AP exam only has four options (A, B, C, or D) and does not require them to show their work, which is slightly different than what they are being asked to do on this handout.

Resources

- K20 Center. (n.d.). Bell Ringers and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Desmos Studio. Tech tools. https://learn.k20center.ou.edu/tech-tool/2356
- K20 Center. (n.d.). I Notice, I Wonder. Strategies. https://learn.k20center.ou.edu/strategy/180
- K20 Center. (n.d.). Inverted Pyramid. Strategies. https://learn.k20center.ou.edu/strategy/173
- K20 Center. (n.d.). *Making connections*. HIVE. https://hive.k20center.ou.edu/boards/32
- Smalley, C. (2008, July) *Leap Frog*. Game presented at the AP Calculus AB session meeting of the AP Summer Institute, Norman, OK.