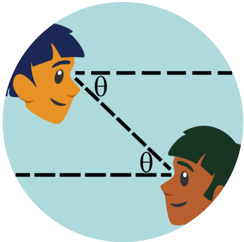

Elevating Angles, Part 1

Inverse Trigonometry: Introduction

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Published by *K20 Center*

Grade Level	9th – 10th Grade	**Time Frame**	75-85 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Geometry		

Essential Question

How is trigonometry used to find the measures of unknown angles?

Summary

Students will explore inputs and outputs of trigonometric functions and use this relationship to identify the need for inverse trigonometric functions to find an unknown angle measure. Students will then watch a video of a civil engineer share how he uses right triangle trigonometry and apply their new knowledge to a real-world scenario. Students need to know how to find the sine, cosine, and tangent of an acute angle of a right triangle and know the basic trigonometric values of special angles (30°, 45°, and 60°) – or be able to sketch the triangle to find those trigonometric values – before beginning this lesson. This is the first lesson in the “Elevating Angles” lesson duo.

Snapshot

Engage

Students recall inputs and outputs of trigonometric functions.

Explore

Students compare the inputs and outputs of trigonometric functions with the inputs and outputs of inverse sine with a focus on special angles.

Explain

Students complete guided notes with the class and learn notation for inverse trigonometric functions and how to find the value of an unknown angle.

Extend

Students learn how a civil engineer uses trigonometry and then solve a real-world problem.

Evaluate

Students reflect on their learning using the What Did I Learn? strategy.

Standards

Oklahoma Academic Standards Mathematics (Geometry)

G.RT.1.3: Use the definition of the trigonometric functions to determine the sine, cosine, and tangent ratio of an acute angle in a right triangle. Apply the inverse trigonometric functions to find the measure of an acute angle in right triangles.

Attachments

- Go With the Flow—Elevating Angles, Part 1.docx
- Go With the Flow—Elevating Angles, Part 1.pdf
- Guided Notes—Elevating Angles, Part 1.docx
- Guided Notes—Elevating Angles, Part 1.pdf
- Inputs and Outputs—Elevating Angles, Part 1.docx
- Inputs and Outputs—Elevating Angles, Part 1.pdf
- Lesson Slides—Elevating Angles, Part 1.pptx

Materials

- Lesson Slides (attached)
- Inputs and Outputs handout (attached; one per group; printed front only)
- Guided Notes handout (attached; one per student; printed front/back)
- Go With the Flow handout (attached; one per pair; printed front only)
- Scientific calculator (one per student)
- Pencil
- Paper

5 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objective. Review each of these with your class to the extent you feel necessary.

Direct students into small groups of 2-3. Display **slide 5** and introduce to the class the following questions: *What are the inputs to a trigonometric function? What are the outputs?* On the slide, students see an illustration of a function machine, where the input is on the top-left of the illustration and the output is in the bottom-right.

Transition to **slide 6** and assign each group one trigonometric function to focus their discussion: sine, cosine, or tangent. This can be done a number of ways, for example, divide the room into thirds. Groups in the left third focus on the sine function, groups sitting in the center third focus on cosine, and groups in the right third focus on tangent.

Ask for a few volunteers to share what the input would be and what the output would be to a trigonometric function. Next, transition to **slide 7**. Use this slide to ensure that everyone understands that the input of the function is the angle and the output is the ratio of the lengths of two sides of a right triangle. Use student responses as a formative assessment to check if students are ready to proceed with the lesson or if they need to review sine, cosine, and tangent. They will be reviewing special angles during the next portion of the lesson, so a solid understanding is necessary.

Teacher's Note: Guiding the Activity

Use this activity to also help students see that it did not matter which trigonometric function they were discussing, the input is always the angle, and the output is always a ratio. Students need to understand this relationship of inputs and outputs for trigonometric functions now so that later in the lesson it is easier to understand why the input to an inverse trigonometric function is the ratio and the output is the angle measure.

15 minutes

Explore

Show **slide 8** and give each group a copy of the attached **Inputs and Outputs** handout. Remind students that special angles are commonly used in questions because they have simple inputs and outputs. Have groups complete their handout.

Teacher's Note

In this handout, students are asked to find the sine, cosine, and tangent of 30°, 45°, and 60°. Encourage students to work together. This might mean that one student in the group draws a right triangle with a 30° angle and finds all three trigonometric values, while another group member works on the triangle with a 45° angle. Consider telling students that you expect to see more than one person's handwriting on the handout.

Once students complete their handouts, transition to **slide 9**. Remind students that they have been given an angle measure and asked to evaluate trigonometric functions. Ask them what they think they would do if they did not know the angle measure and wanted to find it. Have students write down any ideas on a piece of scratch paper at this time and do not yet share with the whole class. Give all students a chance to think on this.

After giving students time to think, click to display the next question on the slide. Ask the class a much simpler question: *How do we find the value of x if x + 3 = 5?* Here, allow volunteers to share out or call on students to respond. For students who respond with "Do the opposite of plus 3," respond by asking them to use more academic language. Push students to use the word "inverse" operation.

Now, display **slide 10** and introduce students to the I Notice, I Wonder strategy. Ask students to compare what they see on the slide with what is on their handout. Here, students see the inverse sine function evaluated for three ratios yielding three special angles. Do not yet use the vocabulary of "inverse sine." In fact, do not even read the slide to the class.

Ask the class what they notice. Next, ask the class what they think $\sin^{-1}(\quad)$ means by gesturing to the notation and referring to it as "that." (*What do you think "that" means?*)

Teacher's Note: Guiding the Activity

Let students know that this is not yet the time to focus on how to read the notation—they will learn this later in the lesson—but you are only asking them to think about what it means or represents.

Students should be noticing that the inputs and outputs are opposite on the slide to what they have in the table on their handout. Use prompting questions as needed to help students see the relationship. You should try to ask more questions than you give answers during this time. Consider the following questions if the class do not make any immediate observations:

- What are the outputs on the slide? What were those values (inputs or outputs) on your handout?
- What are the inputs on the slide? What were those values (inputs or outputs) on your handout?

Try your best to not yet say “inverse sine” as students are noticing and wondering. Try to let students figure out the language of “inverse sine” themselves. Students may think that $\sin^{-1}(\quad)$ means “the opposite of sine,” which is fantastic. If that is the conclusion that the class comes to, remind them that just like “the opposite of plus 3” (from the previous slide), we formally say “inverse operation.” Instead of “opposite of sine,” we formally say “inverse sine.”

20 minutes

Explain

Display **slide 11**. Give each student a copy of the attached **Guided Notes** handout and a scientific calculator.

Explain to the class how to read the notation. Note that negative one looks like an exponent, but is not an exponent. Explain to students that the inverse notation is not preferred by all, and that arcsine, arccosine, and arctangent are sometimes used instead. During Algebra 2, though, students will learn more about inverse functions and use that "negative 1 exponent-looking" notation, $f^{-1}(x)$, so this is the preferred notation to practice with for the time.

Take a moment to ask the class what the inputs and outputs of an inverse trigonometric function are and help tie it back to the function machine activity from the Engage portion of the lesson.

Complete example 1 as a class. Let students know that β (beta) is just another Greek letter, like θ, used to represent an angle. Students will also see α (alpha) later in the lesson.

Teacher's Note: Example 1

Here, help students understand how to approach this problem. In relation to the given β students are given the opposite and adjacent side lengths. Consider having students label their picture with the words "opposite" and "adjacent" to reinforce those relationships. Then, ask the class which trigonometric function relates the opposite and adjacent sides (tangent). Therefore, the tangent of β is 8 over 6. Taking the inverse tangent of both sides gives us the fact that β is the inverse tangent of the quantity 8 over 6. Using a calculator, β is approximately 53.130°. Let students know your expectations when it comes to rounding or truncating decimal values.

Then, ask the class what they would need to know if they wanted to use the sine or cosine function instead. Since they would need to know the length of the hypotenuse, ask how they would determine that value. This question is a Pythagorean triple, but, that said, using the Pythagorean Theorem works as well and it is not a lot of work. Have students try to finish this example by using the inverse sine or inverse cosine. Consider having half of the class use inverse sine while the other uses inverse cosine. Help students see that regardless of the inverse trigonometric function used, the angle measure is the same. Also emphasize to students that using more than one inverse trigonometric function is a great way to check one's work!

Direct students' attention to the back of their handout and introduce the vocabulary of *angle of elevation*, also known as *angle of incline* or *angle of inclination*. (Students will see *angle of depression* during the next lesson of this series.) Use **slide 12** to either show students how to make notes on their handout or use it as a guide to demonstrate to students how to label their image on their handout.

Have students work independently or in pairs to find the angle of elevation in example 2. As students work, circulate the room. Ask for a volunteer to share what they found to be the angle and how they found it (the angle being approximately 34.992°). Consider having pairs of students share work by having one student verbally explain and the other write their algebraic work on the board.

30 minutes

Extend

Teacher's Note: Preparation

Decide whether you want the Go With the Flow handout to be guided practice or independent practice. The sample responses to the handout are on hidden slides, so if you would like the class to check their work as they go, unhide slides 15-18.

Display **slide 13** and introduce the video on the slide: "K20 ICAP - Civil Engineering and Water Drainage." The video introduces civil engineer Bobby Williams, who shares insights and advice about his profession. At the end of the video, Williams' interview is linked to the real-world issue of preventing erosion. By using the right triangle trigonometry to determine value of the angle of elevation, it is possible to determine whether a ditch needs to be lined with concrete or grass.

Embedded video
https://youtube.com/watch?v=JVUnXjCW2-o

Show **slide 14** and give each student a copy of the attached **Go With the Flow** handout. Here, students are given four drainage ditch segments and are asked to find the angles of elevation. Next, they must determine if that segment needs a concrete liner or if it can remain grassed. If the angle of elevation is less than 0.3° or greater than 6°, then a concrete liner should be used.

Teacher's Note: Guiding the Activity

If students wonder why they do not just put concrete on the entire ditch which "would be easier" than doing the math, remind students that government agencies try to use taxpayer money wisely and that it is cheaper to do the calculations to see where the concrete is needed than spending money on concrete if grass will work.

Optional Slides

Unhide and display **slides 15-18** so students can check their work and review the process of determining which segments do or do not need concrete.

5 minutes

Evaluate

Move to **slide 19**. Use the What Did I Learn Today? strategy to have students reflect on their learning. On the back of their Go With the Flow handout, have students write 2-3 sentences and consider the following questions:

- What did you learn that was new?
- How was what you learned related to something you already knew?
- What question(s) do you still have?

Collect students' Go With the Flow handout to assess student learning and determine if students are ready for to the next lesson, "Elevating Angles, Part 2."

Resources

- K20 Center. (n.d.). What Did I Learn Today?. Strategies. https://learn.k20center.ou.edu/strategy/169
- K20 Center. (n.d.). I Notice, I Wonder. Strategies. https://learn.k20center.ou.edu/strategy/180
- K20 Center. (May 2023). K20 ICAP - Civil Engineering and Water Drainage. YouTube. https://youtu.be/JVUnXjCW2-o