BENEFITS OF RANDOM DISCRETE VARIABLES (MODEL NOTES)

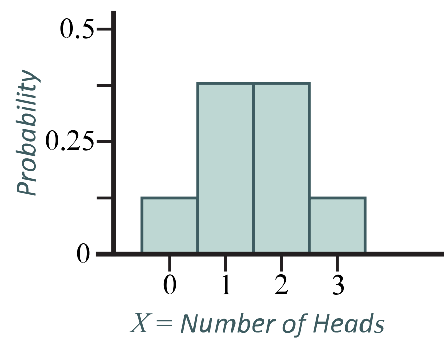
Definitions

- **Random variables:** a numerical representation of an outcome from a random experiment
    - **Notation:** Use the capital letter X.
    - **Examples:** X = number of tails from flipping a coin 4 times
    - **Discrete random variables:** values can only be countable numbers (positive integers); typically result from counting something.
        - **Examples**: number of students in a grade; number of red marbles in a bag
    - **Continuous random variables:** values can be any real number; typically result from measuring something.
        - **Examples**: heights of students in a grade; distance between home and a grocery store
- **Probability distribution:** a table or graph that lists the probability of each outcome

Example 1: *Heads or Tails*

Let X be the number of heads showing. Create a probability distribution table and graph. Then determine $P(1 \le X \le 3)$ and explain its meaning.

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



$$P(1 \le X \le 3) = P(X = 1 \text{ or } X = 2 \text{ or } X = 3)$$

$$= P(X = 1) + P(X = 2) + P(X = 3)$$

$$= \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8} \Rightarrow$$ 87.5% of the time, we should see at least 1 head with 3 coin flips.

Take Note

- Each probability, $P(X)$, must be between 0 and 1, inclusive: $0 \le P(X) \le 1$.
- The sum of all the possible probabilities is 1: $\sum P(x_i) = 1$.

Definitions

- **Mean (expected value):** $\mu_X = E(X) = \sum x_i \cdot P(x_i)$; is not an ordinary average; it is a weighted average.
- **Standard deviation:** $\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 p_i}$

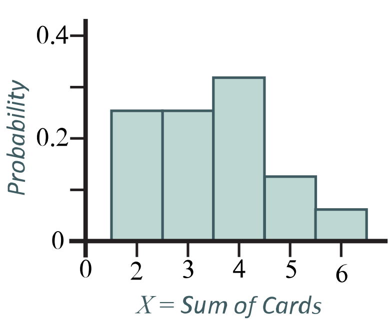
***Example 2:** Drawing Cards*

There is a deck of four cards: an ace, 2, and 3 of hearts, and an ace of spades. One card is randomly drawn, replaced, and a second card is drawn. Let X be the sum of the two drawn cards, where the ace has a value of 1. Create a probability distribution table and graph. Then calculate the expected value and standard deviation.

Sample Space	X
A♡, A♡	2
A♡, 2♡	3
A♡, 3♡	4
A♡, A♤	2
2♡, A♡	3
2♡, 2♡	4
2♡, 3♡	5
2♡, A♤	3

Sample Space	X
3♡, A♡	4
3♡, 2♡	5
3♡, 3♡	6
3♡, A♤	4
A♤, A♡	2
A♤, 2♡	3
A♤, 3♡	4
A♤, A♤	2

X	$P(X)$
2	$\frac{1}{4}$
3	$\frac{1}{4}$
4	$\frac{5}{16}$
5	$\frac{1}{8}$
6	$\frac{1}{16}$



$\mu_X = 3.5$

$\sigma_X = 1.173$

The sum of two randomly selected cards will typically vary from the mean (3.5) by 1.173 units.