

Risk and Reward

Discrete Random Variables and Probability Distribution

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Grade Level	11th – 12th Grade	**Time Frame**	105 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	AP Statistics, Statistics		

Essential Question

How do I determine the likelihood of the outcomes of a scenario?

Summary

In this lesson, students will calculate the probability of a discrete random variable and represent the probability distribution as a table and histogram. Then, they will determine the mean and standard deviation using the probability distribution.

Snapshot

Engage 1

Students play the dice game “Greedy Pig” to investigate the best chance of winning the game.

Explore

Students imagine flipping a coin three times. Then, they record the possible outcomes and likelihood of the discrete random variable representing the number of heads.

Explain

Students complete guided notes as a group to learn more about discrete random variables, probability distribution, mean, and standard deviation.

Engage 2

Students challenge their knowledge of probability through the “Monty Hall Problem.”

Extend

Students watch an ICAP video and answer reflection questions about the quality control underwriter’s career.

Evaluate

Students complete a Choice Board to demonstrate their understanding.

Standards

AP Statistics Course and Exam Description (Course at a Glance)

4.7: Introduction to Random Variables and Probability Distributions

Attachments

- Benefits (Model Notes)—Risk and Reward.pdf
- Benefits—Risk and Reward - Spanish.pdf
- Benefits—Risk and Reward.pdf
- Choice Board (Sample Responses)—Risk and Reward.docx
- Choice Board (Sample Responses)—Risk and Reward.pdf
- Choice Board—Risk and Reward - Spanish.docx
- Choice Board—Risk and Reward - Spanish.pdf
- Choice Board—Risk and Reward.docx
- Choice Board—Risk and Reward.pdf
- Greedy Pig—Risk and Reward - Spanish.docx
- Greedy Pig—Risk and Reward - Spanish.pdf
- Greedy Pig—Risk and Reward.docx
- Greedy Pig—Risk and Reward.pdf
- Heads or Tails—Risk and Reward - Spanish.docx
- Heads or Tails—Risk and Reward - Spanish.pdf
- Heads or Tails—Risk and Reward.docx
- Heads or Tails—Risk and Reward.pdf
- Lesson Slides—Risk and Reward.pptx
- Video Reflection Questions—Risk and Reward - Spanish.docx
- Video Reflection Questions—Risk and Reward - Spanish.pdf
- Video Reflection Questions—Risk and Reward.docx
- Video Reflection Questions—Risk and Reward.pdf

Materials

- Lesson Slides (attached)
- Greedy Pig handout (attached; one per student; printed front only)
- Heads or Tails handout (attached; one per pair; printed front only)
- Benefits handout (attached; one per student; printed front/back)
- Benefits (Model Notes) document (attached; one per student; printed front/back)
- Choice Board handout (attached, one per student; printed front only)
- Choice Board (Sample Responses) document (attached; for teacher use)
- Video Reflection Questions handout (optional; attached; one per student; printed front only)
- Sticky notes (optional)
- Dice (one per student or one per pair)
- Coins (optional; one per pair)
- Pencils
- Paper
- Graphing calculators
- Student devices with internet access (optional)

15 minutes

Engage 1

Teacher's Note: Lesson Order

The order of this lesson is as follows: Engage 1, Explore, Explain, Engage 2, Extend, Evaluate.

Alternative Digital Dice

Students need dice to play the game Greedy Pig. If you prefer students to use a digital version, you can use the CPM Probability Generators and select the dice option.

Introduce the lesson using the attached **Lesson Slides**. **Slides 3–4** display the lesson's essential question and learning objectives. Review each of these with your class to the extent you feel necessary.

Give each student a copy of the attached **Greedy Pig** handout. Using **slide 5**, introduce the Greedy Pig game and explain the game's rules and procedures. Explain to students that they will each roll a die. If they roll a 1, 3, 4, 5, or 6, they get to write down that value as points they earned. However, if they roll a 2, they lose all points for that round (they need to roll again if their first round rolls a 2).

After the first round, they get to decide if they want to stop and keep their points for the round, or roll again with the risk of losing their points. If they choose to roll again and get any number but 2, they get to add more points, but if they roll a 2, they lose all points for that round.

Pairs of students begin each round together but end each round independently. In other words, Students A and B both begin each round at the same time, but each decides for themselves when to end their round.

Teacher's Note: Guiding the Activity

For more information about how to play the game, watch the "Greedy Pig" video.

Embedded video
https://youtube.com/watch?v=4KgRfUwpxPU

Display **slide 6** to show students two round outcome examples. Once students understand the rules, go back and display slide 5 for their reference as they play. Have students work with their Elbow Partners to play the first five rounds and record their results on their handouts. After a few minutes, move to **slide 7** and have the class answer:

- How did you decide when to end your round?
- What are some possible strategies to maximize your score?

Sample Student Responses

Ending the Round:

- I rolled until it felt like a good time to stop.
- After earning 10 points, I stopped.
- I only rolled 3 times at most.

Maximizing My Score:

- Last time, I rolled until I couldn't and got a lot of 2's. I rolled almost 4 times each round—maybe I should always stop after 4 rolls.

After discussing strategies, have students return to their Elbow Partners and start a new five-round game.

After a few minutes, gather the whole class again and show **slide 8**. Determine two students with the highest scores during the second game. Then, have the two highest scorers share their scores and their strategies with the class. If both students have the same strategy, discuss with the class why one strategy can produce different scores.

Teacher's Note: Guiding the Lesson

There are many strategies that students can choose to maximize their scores. Examples include deciding to roll *n* times or deciding to roll until they get *T* points. Ultimately, for a student to determine if their strategy will maximize their score, it will take thousands of trials to test their theory. The purpose of this activity is to help students familiarize themselves with making predictions and collecting data.

For more information on what the data might look like after playing the game a thousand times, watch "The Math of Being a Greedy Pig" by Numberphile starting at 19:46.

Embedded video
https://youtube.com/watch?v=ULhRLGzoXQ0

15 minutes

Explore

Display **slide 9** and pass the attached **Heads or Tails** handout to each student pair. Then, give each pair a coin to flip or have students use the CPM Probability Generators (alternatively, let them imagine flipping a coin).

Tell students to write all possible outcomes when flipping a coin 3 times on their handout, as well as the probability of 0, 1, 2, or 3 heads occurring in the situation. To stretch their knowledge, have students create a histogram to display their data.

Circulate around the room to help students and answer questions. Be sure not to give students the possible outcomes, probabilities, or formal language yet. This is the time for students to demonstrate what they do and do not know. Make notes of questions and misunderstandings to address during the Explain phase.

After about 10 minutes, have some groups share the possible outcomes of flipping a coin three times. As students share their answers, fill the table on slide 9 with the answers for students to see as they complete their own tables.

Transition to **slide 10** and complete the probability table. Then, ask students to reflect on the following questions:

- Do you see a pattern? If so, describe it.
- Are TTH and HTT the same outcome? How do you know if order matters in an outcome?

Have some groups share their answers.

Sample Student Responses

- You are equally likely to get 0 heads as 3 heads. You are equally likely to get 1 head as 2 heads.
- They are not the same outcome because these two outcomes represent the 1st, 2nd, and 3rd coin flip, which means that order matters.

Use the hidden **slide 11** for sample responses to the Heads or Tails handout.

Once students complete their work, have them set aside their Heads or Tails handout. Students will check their answers during the Explain phase.

25 minutes

Explain

Teacher's Note: Activity Preparation

Before this portion of the lesson, decide how you would like your students to calculate expected value and standard deviation: by hand or using a calculator. If using a calculator, ensure that students are practicing with a calculator permitted on the AP exam.

Teacher's Note: Pacing the Lesson

If you have a traditional 45-minute class period, this lesson can be paused at the end of the first page of notes (**slide 24**) or during the middle of example 2 after the probability distribution table and before calculating the mean and standard deviation (**slide 29**). Consider sending students home to try creating the probability distribution table on their own.

Then begin day 2 with the Engage 2 portion of the lesson before completing the notes.

Show **slide 12** and pass out the attached **Benefits** handout to each student. Move through **slides 12–14** and go over the definitions: random variables, discrete random variables, continuous random variables, and probability distribution. After each definition, ask students to come up with and share an example for that term.

Teacher's Note: Guiding the Lesson

In this phase of the lesson, use the attached **Benefits (Model Notes)** document as needed.

Move to **slide 15** and direct students' attention to *Example 1: Flipping a Fair Coin*. Have students look back at their Heads or Tails handout and match their answers with the ones on the slide. Clarify any misconception if necessary and have them correct any wrong answer on their handout.

Emphasize to students that they already know what a random variable is, but they now have the formal academic language to use.

Transition through **slides 16–19** to review with students how to create a probability distribution graph. This is the time to clear up any misconceptions and strengthen academic vocabulary.

Display **slide 20** and ask students to predict what the notation is asking them to do. Once students share their answers, tell them that the notation is asking them to decide the probability of flipping a coin three times in which 1, 2, or 3 heads come up. Move through **slides 21–23** to model how to calculate that probability.

On **slide 24**, review the possible values of $P(X)$ and the sum of all probabilities in any given scenario.

Transition to **slide 25** and direct students' attention to the back of their handout. Introduce students to the formulas for mean (expected value) and standard deviation of a discrete random variable. Then, introduce students to the notation in each equation. Formally name the Greek letters—μ (mu) and σ (sigma)—so that students feel more comfortable reading and discussing the formulas.

Display **slide 26** and read the following scenario to the class:

There is a deck of four cards: an ace of hearts, 2 of hearts, 3 of hearts, and an ace of spades. One card is randomly drawn, replaced, and a second card is drawn. Let X be the sum of the two drawn cards, where the ace has a value of 1.

Ask the class to create a probability distribution table and graph. Then give students a couple of minutes to discuss with their Elbow Partners what the first problem-solving steps are and then ask a few groups to share their plans with the class. Instruct student pairs to complete the sample space tables and probability distribution table.

Display **slide 27** and have students check their work with what is on the slide. Encourage them to correct information as necessary.

Show **slide 28** and ask pairs to now create a graph for their probability distribution table.

Once students complete their graphs, show **slide 29** so that students can check their work.

Transition through **slides 30–31** and demonstrate to students how to find the expected value and standard deviation using their probability distribution table.

Teacher's Note: Guiding the Lesson

This is the time to correct any misconceptions. Use guiding questions to ensure students are making accurate statements.

Once students finish filling in their guided notes, have them keep their handouts in their notebooks for future reference.

10 minutes

Engage 2

At the beginning of day 2, display **slide 32** and introduce the class to the Monty Hall problem. Ask students to pick a door. Designate three walls in the classroom to represent each of the three doors (one wall per door) and ask students to stand against the wall that corresponds to the door they chose.

Teacher's Note: Sticky Notes Alternative

If having students move around the room poses the class with a logistical challenge, you can give students sticky notes and have them write down their answers instead.

Once students have made their choice, display **slide 33**. Tell all students standing at door 2 that, if this were the real game "Let's Make a Deal", they would have lost. However, for this activity, they get to have an extra chance—have them choose a different door. Tell the other students that they can make a choice: either choose to switch doors or stay at the one they originally chose.

Teacher's Note: Guiding the Activity

If you choose, you can announce to the class that door 1 or 3 was the winning door before sending students back to their seats.

Send students back to their seats and transition through **slides 34–37**. Explain to students the statistics involved in answering this question and why it is statistically beneficial to switch doors instead of staying.

If time allows, play the "Monty Hall Problem" video by Numberphile on **slide 38**. If needed to save time, stop the video at 2:47 or 4:49.

Embedded video
https://youtube.com/watch?v=4Lb-6rxZxx0

10 minutes

Extend

Display **slide 39**. Have students take out a piece of paper or give each student a copy of the attached **Video Reflection Questions** handout.

Tell students that they will watch a video to learn how underwriters use discrete random variables in their careers and answer the provided questions.

Show **slide 40** and play the embedded ICAP video on the slide. Students will hear how a professional uses discrete random variables in their careers.

Embedded video
https://youtube.com/watch?v=CJ-SdJlLr3U

Show **slide 41** and, as long as time allows, facilitate a whole class discussion on:

- What discrete random variables does he mention?
- How does he use statistics and/or probability in his job?
- What two new things did you learn?

Ask students to share their answers.

Sample Student Responses

What discrete random variables does he mention?

- He mentions income, liabilities, and debts.
- The value of a mortgage could be a discrete random variable.
- He mentions seasonal/hourly/annual income/salary.

How does he use statistics and/or probability?

- He talks about the likelihood of someone paying back their loan. That helps the loan company decide if someone is a good risk or not.
- If there is a problem with a loan, he looks to see if it was probable that there would be an issue.
- He uses the values of those random variables to notice patterns and make decisions and predictions.

30 minutes

Evaluate

Display **slide 42** and share the Choice Board instructional strategy with the class. Give each student a copy of the attached **Choice Board** handout. Instruct students to work independently to complete enough tasks to earn 6 points.

Alternative Pacing

This activity can be completed in class or assigned as homework for students to turn in the next day. Consider going over some of the questions as bellwork the following day. Also, you may consider changing the number of points each column is worth or the total number of points that are needed for full credit to fit your needs.

Once students have completed the Choice Board, use established classroom procedures to collect their work.

Resources

- K20 Center. (n.d.). Choice Board. Strategies. https://learn.k20center.ou.edu/strategy/73
- K20 Center. (n.d.). Elbow Partners. Strategies. https://learn.k20center.ou.edu/strategy/116
- Numberphile. (2014, May 22). Monty hall problem [Video]. YouTube. https://youtu.be/4Lb-6rxZxx0
- Numberphile. (2021, April 28). The math of being a greedy pig. [Video]. YouTube. https://youtu.be/ULhRLGzoXQ0
- Minas, M. (2020, March 30). Greedy Pig. [Video]. YouTube. https://youtu.be/4KgRfUwpxPU
- Skitterphoto. (2017, December 23). Brain, mind, psychology [Photograph]. Pexels. https://www.pexels.com/photo/pair-of-white-dice-on-top-of-mirror-705171/