Do Three Sides Make a Right?

Converse of the Pythagorean Theorem and Inequality Theorems

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Grade Level	9th – 12th Grade	**Time Frame**	60-75 minutes
Subject	Mathematics	**Duration**	1-2 class period(s)
Course	Geometry		

Essential Question

How do I use side lengths to determine if a triangle is acute, obtuse, or right?

Summary

In this lesson, students will discover how to determine if any three side lengths make a triangle through a GeoGebra activity. Then, students will formalize their knowledge of the Converse of the Pythagorean Theorem before applying their knowledge through a Card Matching activity and a real-world scenario.

Snapshot

Engage

Students look at diagrams of attic rafters and use their prior knowledge to try to determine the third length of a triangle.

Explore

Students work with partners to complete a GeoGebra activity and discover a pattern to help determine when three sides make a triangle.

Explain

Students participate in a whole-class discussion to formalize their findings and introduce mathematical inequalities and equations to classify triangles as acute, right, or obtuse.

Extend

Students work in groups to complete a Card Matching activity to apply their new knowledge.

Evaluate

Students are posed with a construction task to create triangles out of lengths of lumber to demonstrate their understanding.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G602: Use the Pythagorean theorem

Oklahoma Academic Standards Mathematics (Geometry)

G.RT.1.1: Apply the distance formula, the Pythagorean theorem, and the Pythagorean theorem converse (approximate and exact values, including Pythagorean triples) to solve problems, using algebraic and logical reasoning and mathematical models.

Attachments

- Is it a Triangle (Sample Response)—Do Three Sides Make a Right.docx
- Is it a Triangle (Sample Response)—Do Three Sides Make a Right.pdf
- Is it a Triangle—Do Three Sides Make a Right - Spanish.docx
- Is it a Triangle—Do Three Sides Make a Right - Spanish.pdf
- Is it a Triangle—Do Three Sides Make a Right.docx
- Is it a Triangle—Do Three Sides Make a Right.pdf
- Lesson Slides—Do Three Sides Make a Right.pptx
- Three Sides Card Sort (Teacher Resource)—Do Three Sides Make a Right.docx
- Three Sides Card Sort (Teacher Resource)—Do Three Sides Make a Right.pdf
- Three Sides Card Sort Mat—Do Three Sides Make a Right - Spanish.docx
- Three Sides Card Sort Mat—Do Three Sides Make a Right - Spanish.pdf
- Three Sides Card Sort Mat—Do Three Sides Make a Right.docx
- Three Sides Card Sort Mat—Do Three Sides Make a Right.pdf
- Three Sides Sorting Cards—Do Three Sides Make a Right - Spanish.docx
- Three Sides Sorting Cards—Do Three Sides Make a Right - Spanish.pdf
- Three Sides Sorting Cards—Do Three Sides Make a Right.docx
- Three Sides Sorting Cards—Do Three Sides Make a Right.pdf

Materials

- Lesson Slides (attached)
- Is It a Triangle? handout (attached; 1 per student; printed front only)
- Is It a Triangle? (Sample Response) document (attached; for teacher use)
- Three Sides Card Sort Mat (attached; 1 per group; printed front only)
- Three Sides Sorting Cards (attached; half page per group; printed front only)
- Three Sides Card Sort Teacher Resource document (attached; for teacher use)
- Student devices with internet access (1 per student)
- Scratch paper (3 per student) or personal whiteboard
- Sandwich bag (1 per card set)
- Scientific calculators
- AngLegs (optional)

10 minutes

Engage

Teacher's Note: Lesson Preparation

The Extend section of this lesson includes a card sort for students to practice classifying triangles using their knowledge of the converse of the Pythagorean Theorem. Before the lesson, print the attached **Three Sides Card Sort Mat** handout and the attached **Three Sides Sorting Cards.** There are two sets of cards per page. Additionally, consider printing the cards on card stock paper, especially if you plan to reuse these cards.

Once printed, cut out the cards. All of these cards are the same size for easy cutting. Use a sandwich bag or paper clip to keep the mat and cards together.

Teacher's Note: Pacing the Lesson

If you are completing this lesson over two class periods, complete the Engage, Explore, and Explain sections on Day 1 and the Extend and Evaluate sections on Day 2.

Use the attached **Lesson Slides** to guide the lesson. Review the essential question and learning objectives on **slides 3 and 4**.

Place students into groups of 3–4, transition to **slide 5**, ask students to to get out a piece of paper, and introduce the Think-Pair-Share strategy. Preview the activity with the students by explaining that they will be given a problem on the next two slides and they must use prior knowledge to find the side marked with a question mark. They will be given time to independently find an answer or write questions they have on their paper before discussing the picture with their group. Lastly, have groups share with the class what they decided during their conversation.

Teacher's Note: Guiding the Activity

During this activity, encourage students to do their best and let the class know this is not an assessment. Ask students to recall prior knowledge to solve the problem on **slide 6** and have students predict the solution on **slide 7** or consider what information is needed to solve that problem. Consider revisiting these two slides at the end of the Explain section and ask groups to use their new knowledge to find the third side.

Show **slide 6** and give individual students about 2 minutes to think about their answer. Next, ask groups to turn and talk with each other about the displayed image and come to a decision as a group on the answer. After a couple of minutes, ask groups to volunteer to share their thoughts with the class.

After two or three groups have shared, move to **slide 7** and repeat the process again with the second image.

Ask students to set the work they did for slides 6 and 7 aside to revisit later in the lesson.

Sample Student Responses

Slide 6: Because this is a right triangle you can use Pythagorean Theorem to solve.

$a^2 + b^2 = c^2$

$(9.75)^2 + (9.75)^2 = c^2$

$190.13 = c^2$

$c = 13.79$

Slide 7: Students may attempt to use Pythagorean Theorem to solve this problem as well. This is not possible because it is not a right triangle. Students may ask if this triangle has a right angle; however, since it has a 100-degree angle, this would not be possible because the angle sum of a triangle is 180 degrees.

10 minutes

Explore

Teacher's Note: Activating Prior Knowledge

If students need to review triangle properties unhide **slides 8-10**. Show **slide 8** to the class and introduce the Chain Notes strategy.

Arrange students into groups of 3–4, then have each group get one piece of paper and appoint a person to write first. Instruct groups to take turns writing everything they know about triangles on the paper, one piece of information at a time.

After 3 to 5 minutes of writing, transition to **slide 9** and have groups share what is on their paper while the teacher writes responses directly on the slide. Use **slide 10** to review all triangle properties if any are missed by the class.

Have students find a new partner and give each student a copy of the **Is It a Triangle?** Handout. Show **slide 11** and provide students with the link to the GeoGebra activity: geogebra.org/m/tgwg6tnj.

Explain to students that centuries ago, Egyptians used knotted cords (ropes with knots indicating certain lengths) to measure distances and construct triangles. These ancient surveyors used stretched rope to ensure that measurements were consistent. This activity lets you imagine that you are a new surveyor in ancient Egypt and have been given knotted ropes to test. They will get to find out if each rope is useful for creating triangles.

This interactive GeoGebra activity gives students 3 line segments to move to attempt to create triangles. Guide students to move points B, C, and/or D, as point A is fixed. When points A and D align, the enclosed region will turn blue, indicating that those three segments create a triangle.

While students are working with partners to complete the handout, walk around the classroom to keep students on task. This is a time in the lesson for students to discover patterns and relationships. Allow students time for productive struggle. You will clear up any misconceptions later in the Explain phase. Use the attached **Is it a Triangle? (Sample Response)** document for possible student answers. Encourage students to use a pencil to complete this handout in case they need to correct answers in the Explain phase.

Allow students to work together for at least 10 minutes. The information students record on this handout will be used in the Explain phase.

Alternative Activity

If you have students who need something tangible, consider using AngLegs as an alternative to the GeoGebra activity. Adjust the values on the **Is It a Triangle?** handout to match AngLeg colors rather than side length.

15 minutes

Explain

Move to **slide 12** and begin a whole class conversation about what they found in the Explore phase. The purpose of this class discussion is for students to interpret their findings and lead the class in learning while the teacher steps in when necessary to formalize the knowledge. Have groups volunteer to explain which sets were and were not triangles and what influenced their decision. Probe the rest of the class by asking who agrees with the explanations, and if the class has any questions for the answering group.

Transition to **slide 13** and ask students to look at the 4th column on the first table to find a pattern that could explain how set 1, 2, 3 is not a triangle and set 3, 4, 5 is a triangle. Show **slide 14-15** to confirm the Notation on their table and introduce the inequality that must be true for three lengths to make a triangle.

Ask students to write this inequality in the box under "What algebra can help us calculate this?"

Move to **slide 16** and provide students a chance to share which sets were right triangles and how they knew.

Teacher's Note: Guiding the Lesson

Example solutions for the examples on **slides 19, 23, and 27** are on the second page of the **Is it a Triangle? sample response** document.

Move through **slides 17-19** to reveal which set was a right triangle, how to determine this algebraically, and complete an example to formalize their knowledge.

Ask students to write the equation/inequality that proves three lengths make a right triangle in the table at the bottom of the page under "What Algebra can help us calculate this?"

Repeat this same process for **slides 20-23** for acute triangles.

Repeat this same process for **slides 24-27** for obtuse triangles.

Teacher's Note

The purpose of the *Triangle Inequality Theorem* is to determine if three line segments create a triangle.

The purpose of the *Converse of the Pythagorean Theorem* is to determine if three sides lengths create a right triangle. $a^2 + b^2 = c^2$ must be true in order for a triangle to be right.

If $a^2 + b^2$ *does not equal* c^2, then you must use the *Pythagorean Inequality Theorem* decide what type of triangle the three sides create. That is where the two inequalities, $a^2 + b^2 > c^2$ for acute and $a^2 + b^2 < c^2$ for obtuse, help to aid students in this decision-making process.

Teacher's Note: Pacing the Lesson

If you are completing this lesson over two class periods, consider ending Day 1 of the lesson at this time.

15 minutes

Extend

Teacher's Note: Pacing the Lesson

If you are completing this lesson over two class periods, consider beginning Day 2 by revisiting **slide 6 and 7** to apply new learning to the first task students were given in the lesson.

Review **slide 6** and give students a couple of minutes to solve the problem independently. Next, give groups the opportunity to talk about the image and decide on a solution. Hold a whole class discussion on how students decided the answer or what questions they have.

Repeat the same Think-Pair-Share activity on **slide 7**.

Have students find a partner, then transition to **slide 28** and introduce the Card Matching strategy. Give each pair a copy of the attached **Three Sides Card Sort Mat** and **Three Sides Sorting Cards.**

Instruct students to use their new knowledge to sort each set of side lengths into the box that represents the type of triangle it is: acute, right, obtuse, or not a triangle. Remind students that they must determine if the three lengths make a triangle before they classify the triangle.

As pairs work together, rotate around the room to keep students on task and clear up misconceptions as necessary. Utilize the attached **Three Sides Card Sort Teacher Resource** document to review the correct placement of the cards.

As groups complete the card sort, review their answers and point out any cards that are in the wrong category and ask students to look back at their Is It a Triangle? Handout and make corrections before putting away the activity.

10 minutes

Evaluate

Show **slide 29** and remind students of the building task from the beginning of the lesson. Explain that in construction this sort of planning for attic supports is important to providing clients with the kinds of roofs they prefer. Construction workers have to plan carefully because one wrong measurement can ruin their lumber supply and lead to waste and lost money.

Give the students the following scenario: You are a building and have been given 4 pieces of lumber to cut into 3 segments. The lumber is 8 feet long and the client would like to see options for an acute, right, and obtuse triangle. Ask the class if it is possible to have leftover lumber in construction, and if that idea can be extended to this task?

After giving students about 10 minutes to decide on their lengths and write the answers with all necessary work shown on a piece of paper, use already established classroom procedures to turn in the work.

Sample Student Response

Right: 3, 4, 5: $a^2 + b^2 = c^2$

Acute: 4, 4, 3: $a^2 + b^2 > c^2$

Obtuse: 3, 4, 6: $a^2 + b^2 < c^2$

There are several correct answers to this task. Ensure that the students' side lengths work with the notation listed above and the sum of the three lengths does not exceed 32 feet.

Resources

- K20 Center. (n.d.). Chain notes. Strategies. https://learn.k20center.ou.edu/strategy/52
- K20 Center. (n.d.). Card matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- K20 Center. (n.d.). GeoGebra. Tech tools. https://learn.k20center.ou.edu/tech-tool/2352
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/139