K20 LEARN

Transformers, Part 2

Function Transformations

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Grade Level	10th – 11th Grade	**Time Frame**	5 class periods
Subject	Mathematics	**Duration**	205 minutes
Course	Algebra 2		

Essential Question

How does changing the equation of a function change its graph? How can we generalize these changes?

Summary

Students will use a graphing calculator to explore function transformations on polynomial (quadratic and cubic), radical (square root and cube root), and transcendental (exponential and logarithmic) functions. Students then generalize these transformations using function notation.

Snapshot

Engage

Students are introduced to function transformations through a class discussion about functions with which they are already familiar.

Explore

Students explore function transformations through a guided investigation with a graphing utility on several parent graphs.

Explain

Students generalize transformations using function notation.

Extend

Students further demonstrate their understanding of transformations in a class presentation.

Evaluate

Students demonstrate their understanding by writing about how changing the equations changes the graph.

Standards

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.F.1.2: Identify the parent forms of exponential, radical (square root and cube root only), quadratic, and logarithmic functions. Predict the effects of transformations [f(x + c), f(x) + c, f(cx), and cf(x)] algebraically and graphically.

Attachments

- Graphing A—Transformers, Part 2 - Spanish.docx
- Graphing A—Transformers, Part 2 - Spanish.pdf
- Graphing A—Transformers, Part 2.docx
- Graphing A—Transformers, Part 2.pdf
- Graphing B—Transformers, Part 2 - Spanish.docx
- Graphing B—Transformers, Part 2 - Spanish.pdf
- Graphing B—Transformers, Part 2.docx
- Graphing B—Transformers, Part 2.pdf
- Graphing C—Transformers, Part 2 - Spanish.docx
- Graphing C—Transformers, Part 2 - Spanish.pdf
- Graphing C—Transformers, Part 2.docx
- Graphing C—Transformers, Part 2.pdf
- Lesson Slides—Transformers, Part 2.pptx
- Put It All Together—Transformers, Part 2 - Spanish.docx
- Put It All Together—Transformers, Part 2 - Spanish.pdf
- Put It All Together—Transformers, Part 2.docx
- Put It All Together—Transformers, Part 2.pdf

Materials

- Lesson Slides (attached)
- Graphing A handout (attached; one per pair; printed front/back)
- Graphing B handout (attached; one per pair; printed front/back)
- Graphing C handout (attached; one per pair; printed front/back)
- Put It All Together handout (attached; one per pair; printed front/back)
- Chart or poster paper
- Sticky Notes (two colors; one of each color per student)
- Pencils
- Paper
- Graphing calculator (or other graphing utility)

10 minutes

Engage

Teacher's Note: Lesson Pacing

During the Explore portion of this lesson students investigate three sets of functions: polynomials, radicals, and transcendental functions. This lesson is designed for students to complete the Engage and the first third of Explore: investigating polynomials for Day 1. Then for Days 2-3, students continue exploring the radical and transcendental functions. Students complete the Explain during Day 4 while Day 5 is for Extend and Evaluate.

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential questions: *How does changing the equation of a function change its graph? How can we generalize these changes?* **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Display **slide 5** and present for the students the prompt: *What do the graphs of* $y = x + 3$ *and* $y = |x| + 3$ *have in common?* Allow students time to answer for themselves and then, using the Elbow Partner strategy, have students share with an elbow partner. Take volunteers for responses.

Show **slide 6** and provide the example: $y = 4x$ and $y = |4x|$. Then ask, *How are these equations similar?* Allow for discussion before asking a few students to share.

Summarize their observations.

Teacher's Note: Purpose

Use these discussions as a formative assessment to determine if students are ready to continue the lesson or need a refresher. If students need additional review, complete the "Transformers, Part 1" lesson then resume this lesson. For more direct feedback from students, consider using a strategy like Fist to Five to determine whether students are in need of additional support.

105 minutes

Explore

Students need a graphing calculator or access to the Desmos Studio graphing calculator to complete the activity. Display **slide 7** and give each pair a copy of the attached **Graphing A** handout and a graphing utility. Here students are asked to explore the graphs of the parent graph and a few transformations for polynomials: both quadratic and cubic functions.

In Part I, students should make predictions about graphing and compare $y = x^2$, $y = (x - 3)^2$, $y = x^2 + 3$, and $y = 2x^2$. Students do this again in Part II, but with cubic functions: $y = x^3$, $y = (x - 3)^3$, $y = x^3 + 3$, and $y = 2x^3$.

In Part III, students are asked to compare their quadratic and cubic graphs to list observations and patterns.

In Part IV, each group then joins another group to compare what they observed.

Teacher's Note: Graphing Tools

If this is the students' first chance to use the graphing utility, a short introduction may be required.

It is best practice to have students regularly use the same graphing utility so that they become familiar with the tool. If a class set of graphing calculators is not available, consider using Desmos Studio by having students go to www.desmos.com/calculator or have them use Graphing Calculator X84 on their iPad.

Allow students 30-35 minutes to work through the activity. Walk around and guide students who are lost but try not to lead them outright. Some good questions to ask are:

- What does your partner think?
- How did you come to that conclusion?
- Can you show me that it works in [this] case? (try a negative number or a fraction)
- Why do you think that?
- What have you tried already?

At the end of the class/work period, discuss the students' results.

Teacher's Note: Guiding the Activity

As tempting as it is, leave their observations as they are for now. They will be investigating several more functions and these observations will have a chance to be refined.

Repeat the same process over the next couple of class/work periods with **slide 8** and the attached **Graphing B** handout for radical functions and again with **slide 9** and the attached **Graphing C** handout for exponential and logarithmic functions.

Teacher's Note: Grouping Students

It is your discretion whether groups need to be mixed up for each of the subsequent explorations, or not.

Sample Student Responses:

Graphing A (Polynomials)

- All of the graphs in Part I had the same shape: a parabola. The graphs in Part II all looked similar.
- The – 3 in (x – 3) made the graphs move 3 units right of the parent graph.
- The + 3 after the exponent made the graphs move up 3.
- The times 2 in the front made the graphs skinnier than the parent graph.
- So, when there was a number multiplied by the x to a power and another number added to it, it was skinnier and moved up from the parent graph.

Graphing B (Radicals)

- The graphs in Part I all look like half of a parabola fallen over. The graphs in Part II all look like a stretched-out S.
- The number in front of the x, inside the radical made the graph taller, as if it were stretched up and down.
- When there was a number added to x (inside the radical), it moved left that many units.
- The – 2 made the graphs 2 units down from their parent graph.
- Inside the radical, when you subtract a number from x, then the graph moves right. And when you add a number (outside the radical), the graph moves up.

Graphing C (Transcendentals)

- The graphs in Part I all have the same shape. The graphs in Part II all have the same shape.
- When you multiply a function by a number, the graph feels taller/skinnier/stretched upward. If you only multiply x by a number, that seems to be similar stretching, but not quite the same.
- If you add or subtract a number from x, then the graph shifts left or right.
- If you add or subtract a number from the type of function (not inside with x), then the graph moves up or down.

45 minutes

Explain

Show **slide 10** and split students into new groups of 2-3. Provide each group with a copy of the attached **Put It All Together** handout. Students will need access to the graphing utility again. Let students know that they are going to be generalizing their observations. Use **slide 11** for a short explanation of the usage of function notation, if needed, before students start working.

Display **slide 12** and allow students time to work through the handout, and then come back together as a class. Help them recall their previous observations and compare/contrast them with the observations they have just made.

Summarize the generalized transformations together as a class and record them in a visible place.

Once students are confident in how to represent their observations with function notation and how different constants and coefficients affect the graph of a function, move to **slide 13**. Direct students' attention to the *Prediction* portion of their handout and ask the class to make a prediction about how $y = 3 \cdot \sin(x + 4) - 2$ compares to $y = \sin(x)$.

After a few minutes, transition to **slide 14** and have students graph the two functions in their graphing calculator and compare their results with their predictions. Ask for volunteers to share their thinking to share how they were able to accurately predict how the graphs would differ.

40 minutes

Extend

Have students get into groups of 3-4 or assign groups then display **slide 15**. Assign each group one type of transformation: $f(x) + c$, $f(x + c)$, $f(c \cdot x)$, and $c \cdot f(x)$. Have each group create an Anchor Chart with everything they know about their assigned transformation.

If you have more than four groups, consider either giving groups the same transformations (i.e. two groups might have $f(x) + c$) or giving some groups the challenge of multiple transformations (i.e. groups might have $a \cdot f(x + c)$ or $f(b \cdot x) + d$). If you have fewer than four groups, consider making one of the Anchor Charts yourself. Use the hidden **slide 16** for sample student responses. Give students approximately 15 minutes to create their poster.

Show **slide 17** and introduce students to the Gallery Walk strategy. Give each student two different colored sticky notes. Indicate to students which color indicates “This transformation makes the ***most*** sense,” and which color indicates “This transformation makes the ***least*** sense.” Emphasize to students that they are not evaluating the quality of the Anchor Chart but are expressing their level of understanding. Explain to students that they are to read each poster then put their sticky notes on the corresponding posters.

5 minutes

Evaluate

Use the Exit Ticket strategy to individually assess what students have learned from the lesson. Display **slide 18** and provide the prompt on the slide asking students to explain how a, b, c, and d in the equation $y = a \cdot f(b \cdot x + c) + d$ affect the parent graph. Allow students a couple of minutes to respond using an index card, sticky note, piece of paper, etc. Use these responses as a formative assessment to see students' understanding of this concept.

Resources

- IncptMobis. (2024). *Graphing Calculator X84* (Version 3.2) [Mobile app]. App Store. https://iphonecalculator.com/

- K20 Center. (n.d.). Anchor Chart. Strategies. https://learn.k20center.ou.edu/strategy/58
- K20 Center. (n.d.). Bell Ringers and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Desmos Studio. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2356
- K20 Center. (n.d.). Elbow Partners. Strategies. https://learn.k20center.ou.edu/strategy/116
- K20 Center. (n.d.). Fist to Five. Strategies. https://learn.k20center.ou.edu/strategy/68
- K20 Center. (n.d.). Gallery Walk / Carousel. Strategies. https://learn.k20center.ou.edu/strategy/118
- Pixabay. (December 9, 2018). Figurine [Photograph]. Pixabay. https://pixabay.com/images/id-3871893/