Function Rationally

Investigating Graphs of Rational Functions

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Published by *K20 Center*

Grade Level	10th – 11th Grade	**Time Frame**	120-135 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	Algebra 2		

Essential Question

What is a rational function? What might we use a rational function to model?

Summary

In this lesson, students will use an investigation to explore rational functions. Students will formalize their understanding of a rational function and continue to investigate the relationship between the equation of a simple rational function and its graph.

Snapshot

Engage

Students are provided prompts that encourage them to imagine the graphs of data relationships.

Explore

Students investigate a relationship that can be modeled as a rational function.

Explain

Students formalize their understanding of rational functions and are introduced to the vocabulary of hyperbolas, branches, and asymptotes.

Extend

Students further investigate rational functions through a Desmos Classroom Marbleslides activity.

Evaluate

Students draw conclusions and analyze the standard form of a rational function with the It Says, I Say, and So strategy.

Standards

Oklahoma Academic Standards Mathematics (Algebra 2)

A2.F.1.6: Graph a rational function and identify the domain (including holes), range, x- and y-intercepts, vertical and horizontal asymptotes, using various methods and tools that may include a graphing calculator or other appropriate technology (excluding slant or oblique asymptotes).

Attachments

- Lesson Slides—Function Rationally.pptx
- Note Catcher—Function Rationally - Spanish.docx
- Note Catcher—Function Rationally - Spanish.pdf
- Note Catcher—Function Rationally.docx
- Note Catcher—Function Rationally.pdf
- Pasta Branches—Function Rationally - Spanish.docx
- Pasta Branches—Function Rationally - Spanish.pdf
- Pasta Branches—Function Rationally.docx
- Pasta Branches—Function Rationally.pdf

Materials

- Lesson Slides (attached)
- Pasta Branches handout (attached; one per group; printed front only)
- Note Catcher handout (attached; one per student; printed front only)
- 1 bag or box of dry spaghetti noodles
- 1 bag of dried beans
- Plastic cups (2–3 oz., one per group)
- String (one 10–12-in. piece per group)
- Permanent markers (one per group)
- Rulers (one per group)
- Tape (painter's tape is recommended)
- Pencils

10 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to share the lesson's essential question with students. Go to **slide 4** to share the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

Show **slide 5** and ask students if they agree or disagree with the given prompt: *A house cat can crawl farther out onto a tree branch than a firefighter.* Help the class come to the consensus that this is true because the further away from the tree trunk, the less weight a branch can support.

Display **slide 6** and introduce the idea of wanting to know the relationship between the length (distance from the tree trunk to the breaking point of the branch) and the mass it would take to break the branch.

Show **slide 7** and then ask the class to consider what the graph of this relationship would look like, where *x* is the length and *y* is the mass. Allow students time to think individually and use the options on the slide to predict what shape the graph would most likely be.

As time allows, have students share with an Elbow Partner why they picked the curve they picked. Then ask for volunteers to share with the class.

Show **slide 8** and ask, *Why might this be good information to know?* Engineers need to know how much weight a beam could safely support. Allow students to share their ideas. Tell them that we are going to explore this kind of relationship today.

45 minutes

Explore

Teacher's Note: Activity Preparation

Plan ahead to organize your classroom for the following activity. Students need space to tape a spaghetti noodle to a desk or table allowing 8 inches to hang over the edge, and there needs to be enough room for groups of three students to gather around as they add a container and weight to the pasta noodle and watch it break.

Having materials prepared before class saves time. Having the materials easy to gather also streamlines the process. Consider the following preparations:

- Cut string into 10–12-inch pieces such that there is one piece per group of three students.
- Gather enough tape dispensers such that there is one available for each group of three. Alternatively, tear tape into 1.5–2-foot pieces and adhere to the edge of a table for students to take to their workspace and tear into smaller pieces as needed.
- Make available one permanent marker and one ruler per group of three.
- Place dry spaghetti noodles into a container or open one end of the bag or box for students to gather 5 noodles per group of three.
- Fill plastic cups approximately 1.75 oz. full of dried beans such that there is one available for each group of three. Alternatively, you could have a container of dried beans and have students use the plastic cup to scoop out what they need.

Teacher's Note: Using the Desmos Studio Graphing Calculator

Try to become familiar ahead of time with the Desmos Studio graphing calculator to better help students navigate through the following activity. It is not necessary for students to have Desmos accounts to use the Desmos Studio graphing calculator. For more information, go to https://k20center.ou.edu/externalapps/graphing-calculator.

Assign or have students choose groups of three to work. Display **slide 9** and preview the activity with the class. Pass out the attached **Pasta Branches** handout to each group of students.

Show **slide 10** and review the following roles: counter, recorder, and catcher with the students. Direct students to decide within their groups who should take on which role.

Display **slide 11** and direct students to where they can gather their supplies for this activity.

Show **slide 12** and go through the steps of the activity on the Pasta Branches handout with the students. Use the picture on this slide to point out the markings and how to prevent the container string from sliding off of the pasta noodle.

As students complete their investigation and return their materials, show **slide 13**. Direct students to go to desmos.com and click "Graphing Calculator." Have students add a table by clicking the plus sign in the top-left corner of their screen. Guide students to enter their data into the table.

Alternative Pacing

If time is a concern, this activity can be shortened by having two groups of three work together to complete the table of data. For example, one group could collect data for the 1" and 2" markers, while the other group could collect data for the 4" and 5" markers; then the group who finishes first would collect the data for the 3" marker.

15 minutes

Explain

After students have completed entering their data into the Desmos Studio graphing calculator, show **slide 14**. Ask students to talk with their group about which graph most closely matches their data points and to reflect if this graph is the same one as they selected earlier in the lesson. Facilitate a class discussion on their data and ask students to use the data trend to complete the following sentence: *As the distance from the tree trunk (length) increases, the mass it takes to break the branch ___.*

Show **slide 15** and use this slide to direct students on how to generate a curve that models their data.

Display **slide 16** and explain that the graph on their screen is a hyperbola with two branches and two asymptotes. Explain these vocabulary terms to your students. Clarify to students that the asymptotes on the slide are not visible on the graphing calculator because they represent where the function is approaching but not what the function equals. Explain that we often draw them by hand when sketching hyperbolas. Then share the general equation for a simple rational function: $y = a/(x–h) + k$. Review this equation, the equation of the parent graph $y = 1/x$, and the definition of a rational function to the extent you see necessary but wait to explain the direct relationship between the equation and the graph, as students discover this later in the lesson.

45 minutes

Extend

Teacher's Note: Desmos Classroom Activity Preparation

To use this Desmos Classroom activity, select the following link: "Marbleslides: Rationals." Create an account or sign in under the "Activity Sessions" heading. After you log in, the green "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

Provide students with your session code. Then, have students go to student.amplify.com/join/ and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Display **slide 17** and give each pair of students a copy of the **Note Catcher** handout to use as they progress through the Desmos Classroom activity.

Screen 1 gives a preview to Desmos Classroom Marbleslides, which is a creative way for students to explore the relationship between equations and their graphs by trying to get a marble to follow the path of the curve to roll or go through a series of stars on the screen.

Screens 2 and 4–7 ask students to make one change to the given rational function to complete the Marbleslides challenge. Screen 3 gives directions on how to reset their graphing calculator screen within the Desmos Classroom activity.

Screens 10–15 ask students to make predictions about how changing a specific value will affect the graph.

The activity continues with less scaffolding in place, continuing to challenge students.

Optional Differentiation

Depending on your class, expect to have students complete the activity through screen 15, then challenge them to see how much further they can progress through the Desmos Classroom activity.

Teacher's Note: Guiding the Desmos Classroom Activity

Starting with screen 17, students are given the challenge of creating their own rational functions to get the marble from its starting point and through the stars. If this lesson is taught as an introduction to rational functions, where students are mostly familiar with simple rational functions of the form $y = 1/(x–h) + k$, then students may struggle with these Marbleslides challenges. However, this can be overcome by encouraging students to use more than one rational function, to use domain restrictions, and to think outside the box. If this lesson is not an introduction to rational functions, consider challenging students to write equations for more advanced rational functions to complete the given challenges.

As time allows, consider asking students who only used two functions (or the least number of functions of the class) to complete a screen to share their strategy and thinking with the class.

5 minutes

Evaluate

Display **slide 18** and introduce students to the It Says, I Say, and So strategy. Direct students' attention to the bottom of the Note Catcher handout and ask students to use their notes from the Desmos Classroom Marbleslides activity, where they circled what they changed (It Says) and described the change in the graph (I Say), to explain how *a*, *h*, and *k* of y = a/(x–h) + k each affect the graph (and So).

Sample Responses

- The *h*-value shifts the graph left and right.
- The *k*-value shifts the graph up and down.
- The *a*-value stretches the graph.

Resources

- Desmos Classroom. (n.d.). Marbleslides: Rationals [Interactive activity]. Desmos. https://teacher.desmos.com/activitybuilder/custom/566b31794e38e1e21a10aae8?collections=featured-collections%2C5e73b3a1bb8b0c7628d2809c
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081
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- Murdock, J., Kamischke, E., & Kamischke, E. (2010). Introduction to Rational Functions. In *Discovering advanced algebra: An investigative approach* (Second section, pp. 490–491). Key Curriculum Press.
- Sheppard, V. (n.d.). Homemade Tomato Sauce [Photograph]. The Vegetarian Ginger. https://thevegetarianginger.com/2012/09/06/homemade-tomato-sauce/