

Angles and Sides, Sides and Angles

Triangle Congruence

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Grade Level	9th – 10th Grade	**Time Frame**	3-4 class period(s)
Subject	Mathematics	**Duration**	150 minutes
Course	Geometry		

Essential Question

Are there congruence shortcuts for triangles?

Summary

Students investigate triangle congruence and complete proofs using the theorems they verified.

Snapshot

Engage

Students respond to a prompt asking the essential question.

Explore

In groups, students test possible congruence theorems.

Explain

Students present the possible shortcuts, explaining why they did or did not work.

Extend

Students create a construction project using congruence theorems.

Evaluate

Students complete proof puzzles to solidify their knowledge of the congruence theorems.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G603: Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles

Oklahoma Academic Standards Mathematics (Geometry)

G.2D.1.9: Construct logical arguments to prove triangle congruence (SSS, SAS, ASA, AAS and HL).

Attachments

- Proof Puzzles—Angles and Sides, Sides and Angles.pdf
- Triangle Congruence Possibilities Investigation 1—Angles and Sides, Sides and Angles - Spanish.docx
- Triangle Congruence Possibilities Investigation 1—Angles and Sides, Sides and Angles - Spanish.pdf
- Triangle Congruence Possibilities Investigation 1—Angles and Sides, Sides and Angles.docx
- Triangle Congruence Possibilities Investigation 1—Angles and Sides, Sides and Angles.pdf
- Triangle Congruence Possibilities Investigation 2—Angles and Sides, Sides and Angles - Spanish.docx
- Triangle Congruence Possibilities Investigation 2—Angles and Sides, Sides and Angles - Spanish.pdf
- Triangle Congruence Possibilities Investigation 2—Angles and Sides, Sides and Angles.docx
- Triangle Congruence Possibilities Investigation 2—Angles and Sides, Sides and Angles.pdf
- Triangle Congruence Possibilities—Angles and Sides, Sides and Angles.docx

Materials

- Triangle Congruence Possibilities (attached)
- Triangle Congruence Investigations 1 and 2 (attached)
- Proof Puzzles (attached)
- Access to dynamic geometry software, like Geogebra for each group (A physical manipulative, like AngLegs, or ability to construct segments and angles would work too.)

Engage

Provide the following prompt for students: "A contractor has just assembled two massive triangular trusses to support the roof of a recreation hall. Before they are hoisted into place, the contractor needs to verify that the two triangles are identical.How should he do this?" Allow students time to think and take five student responses.

Teacher's Note

We would expect students at this point to say that, in order to be certain of congruence, the contractor would need to measure and compare all six parts of the triangles: each of the three sides and each of the three angles.

Pose the question: "Might there be a shortcut?" Students will probably readily agree that there should be. Ask, "What do you think is the minimum number of parts we can test to reach a conclusion?"

Teacher's Note

If students suggest two parts, have them test it. This could lead to a great discussion, as they realize you need at least three. You can focus the class on using three parts since then they'll know at least one combination of three works. Perhaps one group of students will discover one of the congruency theorems.

Ask students to provide all combinations of three adjacent parts they can think of (you may suggest S for side and A for angle). Allow them to work in pairs to create a list. Choose two groups to write their lists on the board, then other groups can add to what they have. Discuss the possibilities together.

Teacher's Note

The list that students might create is: SSS, AAA, SAA, SSA, SAS, ASA, AAS, ASS. Note that AAS is equivalent to SAA (two angles and a non-included side), and ASS is equivalent to SSA (two sides and a non-included angle).

Teacher's Note

Students may already know the third angle conjecture and argue that AAA is true because of that. Clarify that, from the conjecture, all we know for certain is that the *angles* are congruent and ask, "Does that guarantee the *triangles* are congruent, too?"

Come to a consensus: "Are these are the only six ways that the same three parts of two triangles may be compared?" (Yes, as any additional ones would be equivalent.) Then we will investigate to see which of these work.

Teacher's Note

You may display the "Triangle Congruence Possibilities" handout to remind students during the investigations which methods are being explored.

Explore

Day 1: Split students into groups of two or three. Provide each group with one investigation (either 1A, 1B, or 1C from the "Triangle Congruence Possibilities Investigation 1" handout) and a device on which to access Geogebra. Allow students time to work through their investigations and reach a conclusion. Walk around as students are working, helping students who struggle, but be careful not to direct them outright.

Some good questions to ask might be:

- Are you sure?
- Does your group agree?
- What have you tried already?
- What do you think you should try next?
- Does that work in any case?
- Have you tried another case? This could be for students who draw conclusions very quickly; encourage them to create another case and try it too.

Teacher's Note

See Geogebra for more information about this free Dynamic Geometry tool. It can be downloaded onto a machine or accessed from a browser, like Chrome.

Because it is likely that more than one group will have completed each A, B, and C, have the A groups get together to discuss their findings and do the same for the B groups and the C groups. Have each new group reach a consensus about the results of their investigation.

Day 2: Similar to Day 1, split students into groups of two or three (it's not necessary to keep the same groups from Day 1, so keep or change them as you wish). Provide each group with either investigation 2A, 2B, or 2C from the "Triangle Congruence Possibilities Investigation 2" handout and a device on which to access Geogebra. Repeat the process from Day 1 with this second investigation.

Teacher's Note

If the third angle conjecture has been investigated rigorously in your class, there may be no need to investigate AAA. If that is the case, then only investigations 2A and 2B need to be completed.

Explain

Day 1: Have a representative (or two) from group 1A present their findings and then do the same with the groups that went over 1B and 1C. Visual aids are suggested during these presentations, since the other groups have not had a chance to think about the conjecture being presented each time.

After the presentations, discuss the results as a class and come to a conclusion. Record the SSS, SAS, and SSA conjectures for future reference.

Day 2: Similarly to Day 1, have a representative (or two) from group 2A present their findings and do the same with 2B and 2C. Discuss as a class and reach consensus. Record the ASA, SAA, and AAA conjectures for future reference.

Extend

Student groups of two or three should create a presentation in response to the following prompt:

- The PTA from our school is looking to sponsor a student project for the beautification of our grounds. Create a proposal for this project that is made of congruent triangles. The proposal should include a detailed sketch of the item, with all congruencies clearly labeled.

Evaluate

Split students into groups of two or three and supply them with the "Proof Puzzles" handout (only proofs one through four). Give them time to piece together the puzzles in a logical order, encouraging them to think about how they constructed their triangles in their previous investigation. After they have completed the puzzles, they should write them up with a justification of their sequence.

Teacher's Note

The "Proof Puzzles" document includes a fifth puzzle that utilizes the hypotenuse-leg theorem. This could be a great challenge for students if they are ready for it.

Student groups should compare their solutions and come to consensus.

Resources

- Serra, M. (2003). Discovering geometry: An investigative approach (Vol. 4). Emeryville, CA: Key Curriculum Press.
- K20 Center. (n.d.). Geogebra. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2352