INTEL NOTES (MODEL NOTES)

Reciprocal and Quotient Identities

- $\frac{1}{\sin\theta} = \csc\theta$
- $\frac{1}{\cos\theta} = \sec\theta$
- $\frac{1}{\tan\theta} = \cot\theta$
- $\frac{1}{\csc\theta} = \sin\theta$
- $\frac{1}{\sec\theta} = \cos\theta$
- $\frac{1}{\cot\theta} = \tan\theta$
- $\frac{\sin\theta}{\cos\theta} = \tan\theta$
- $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Pythagorean Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Notice the shorthand notation for exponents of trigonometric expressions: $\sin^2\theta = (\sin\theta)^2$.

Simplifying Expressions

Simplify each expression. <u>Hint</u>: *Rewrite the expression using sine and/or cosine.*

1) $$\frac{3 - \frac{3}{\csc^2\theta} - \frac{2}{\sec^2\theta}}{\left(\frac{1+\cos\theta}{1+\sec\theta}\right)^2} = \frac{3 - \frac{3}{\left(\frac{1}{\sin\theta}\right)^2} - \frac{2}{\left(\frac{1}{\cos\theta}\right)^2}}{\left(\frac{1+\cos\theta}{\left(\frac{\cos\theta}{\cos\theta}\right)+\left(\frac{1}{\cos\theta}\right)}\right)^2} = \frac{3 - 3\sin^2\theta - 2\cos^2\theta}{\left(\frac{1+\cos\theta}{\left(\frac{\cos\theta+1}{\cos\theta}\right)}\right)^2}$$

$$= \frac{3 - 3\left(1-\cos^2\theta\right) - 2\cos^2\theta}{\left(\frac{(1+\cos\theta)}{1}\cdot\left(\frac{\cos\theta}{\cos\theta+1}\right)\right)^2} = \frac{3 - 3 + 3\cos^2\theta - 2\cos^2\theta}{(\cos\theta)^2} = \frac{\cos^2\theta}{\cos^2\theta} = 1$$

2) $$\frac{\sin^3 t}{\cos t} + \sin t\cos t = \frac{\sin^3 t}{\cos t} + \sin t\cos t\cdot\frac{\cos t}{\cos t} = \frac{\sin^3 t}{\cos t} + \frac{\sin t\cos^2 t}{\cos t}$$

$$= \sin t\left(\frac{\sin^2 t + \cos^2 t}{\cos t}\right) = \sin t\left(\frac{1}{\cos t}\right) = \frac{\sin t}{\cos t} = \tan t$$

Verifying Trigonometric Identities

Verifying an identity is proving that one side actually equals the other side. This is done by algebraically manipulating one side of the equation, often using identities, until it equals the other side. *Hint: Start with the scariest-looking side.*

Verify the following identity.

3) $\dfrac{1-\csc^2 x}{\csc^2 x} = \dfrac{-1}{\sec^2 x}$

$$\frac{1-\csc^2 x}{\csc^2 x} = \frac{1-\left(1+\cot^2 x\right)}{\left(\dfrac{1}{\sin^2 x}\right)} = \frac{\left(1-1-\cot^2 x\right)}{1}\cdot\left(\frac{\sin^2 x}{1}\right) = \left(-\cot^2 x\right)\left(\sin^2 x\right)$$

$$= \left(-\frac{\cos^2 x}{\sin^2 x}\right)\left(\sin^2 x\right) = -\cos^2 x = \frac{-1}{\left(\dfrac{1}{\cos^2 x}\right)} = \frac{-1}{\sec^2 x}$$