

TRIG SPIES RECORD SHEET (SAMPLE REPONSES)

Keep in mind that there is more than one correct way to simplify or verify a trigonometric identity. The next page contains the work for the most common approaches. Below are examples of two identities that were likely used. Students are asked to write only one. If students' responses deviate, check students' scratch work.

Card Number	Identity	Points
A	$1 - \cos^2 \theta = \sin^2 \theta; \csc \theta = \frac{1}{\sin \theta}$	4
B	$\cos^2 \theta = 1 - \sin^2 \theta; \frac{1}{\sec \theta} = \cos \theta$	3
C	$1 - \cos^2 \theta = \sin^2 \theta; \cot \theta = \frac{\cos \theta}{\sin \theta}$	4
D	$\sec^2 \theta = 1 + \tan^2 \theta$	4
E	$1 - \cos^2 \theta = \sin^2 \theta; \csc \theta = \frac{1}{\sin \theta}$	4
F	$\sin^2 \theta = 1 - \cos^2 \theta; \tan \theta = \frac{\sin \theta}{\cos \theta}$	3
U	$1 - \cos^2 \theta = \sin^2 \theta; \frac{\sin \theta}{\cos \theta} = \tan \theta$	6
V	$1 - \sin^2 \theta = \cos^2 \theta; \sec \theta = \frac{1}{\cos \theta}$	6
W	$1 - \sin^2 \theta = \cos^2 \theta; \tan \theta = \frac{\sin \theta}{\cos \theta}$	5
X	$\cot \theta = \frac{\cos \theta}{\sin \theta}; \tan \theta = \frac{\sin \theta}{\cos \theta}$	6
Y	$1 - \cos^2 \theta = \sin^2 \theta; \csc \theta = \frac{1}{\sin \theta}$	5
Z	$\tan \theta = \frac{\sin \theta}{\cos \theta}; \cot \theta = \frac{\cos \theta}{\sin \theta}$	6
Total		56

Card A

$$\text{Simplify: } \csc \theta - \cos \theta \cot \theta = \frac{1}{\sin \theta} - \cos \theta \frac{\cos \theta}{\sin \theta} = \frac{1 - \cos^2 \theta}{\sin \theta} = \frac{\sin^2 \theta}{\sin \theta} = \sin \theta$$

Card B

$$\text{Verify: } \frac{1}{\sec^2 \theta} = \cos^2 \theta = 1 - \sin^2 \theta = (1 + \sin \theta)(1 - \sin \theta)$$

Card C

$$\text{Simplify: } \cot \theta - \cos^3 \theta \csc \theta = \frac{\cos \theta}{\sin \theta} - \cos^3 \theta \frac{1}{\sin \theta} = \frac{\cos \theta (1 - \cos^2 \theta)}{\sin \theta} = \frac{\cos \theta \sin^2 \theta}{\sin \theta} = \sin \theta \cos \theta$$

Card D

$$\begin{aligned} \text{Verify: } 2 \sec^2 \theta - \sec^4 \theta &= 2(1 + \tan^2 \theta) - (1 + \tan^2 \theta)^2 = 2 + 2 \tan^2 \theta - (1 + 2 \tan^2 \theta + \tan^4 \theta) \\ &= 2 + 2 \tan^2 \theta - 1 - 2 \tan^2 \theta - \tan^4 \theta = 1 - \tan^4 \theta \end{aligned}$$

Card E

$$\text{Simplify: } (\csc \theta - \cot \theta)(\csc \theta + \cot \theta) = \csc^2 \theta - \cot^2 \theta = \frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1 - \cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta}{\sin^2 \theta} = 1$$

Card F

$$\text{Verify: } \tan^2 \theta \cos^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta = \sin^2 \theta = 1 - \cos^2 \theta$$

Card U

$$\begin{aligned} \text{Simplify: } \frac{\sin \theta - 1}{\cos \theta} - \frac{\cos \theta}{\sin \theta - 1} &= \frac{(\sin \theta - 1)^2}{\cos \theta (\sin \theta - 1)} - \frac{\cos^2 \theta}{\cos \theta (\sin \theta - 1)} \\ &= \frac{\sin^2 \theta - 2 \sin \theta + 1 - \cos^2 \theta}{\cos \theta (\sin \theta - 1)} = \frac{\sin^2 \theta - 2 \sin \theta + (\sin^2 \theta)}{\cos \theta (\sin \theta - 1)} \\ &= \frac{2 \sin^2 \theta - 2 \sin \theta}{\cos \theta (\sin \theta - 1)} = \frac{2 \sin \theta (\sin \theta - 1)}{\cos \theta (\sin \theta - 1)} = \frac{2 \sin \theta (\sin \theta - 1)}{\cos \theta (\sin \theta - 1)} \\ &= \frac{2 \sin \theta}{\cos \theta} = 2 \cdot \frac{\sin \theta}{\cos \theta} = 2 \tan \theta \end{aligned}$$

Card V

$$\begin{aligned} \text{Verify: } \frac{\sec \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} &= \frac{\left(\frac{1}{\cos \theta}\right)}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \frac{1}{\sin \theta \cos \theta} - \frac{\sin^2 \theta}{\sin \theta \cos \theta} = \frac{1 - \sin^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{\cos^2 \theta}{\sin \theta \cos \theta} = \frac{\cos \theta}{\sin \theta} = \cot \theta \end{aligned}$$

Card W

$$\begin{aligned} \text{Simplify: } \frac{\sec \theta \csc \theta - \tan \theta}{\sec \theta \csc \theta} &= \frac{\sec \theta \csc \theta}{\sec \theta \csc \theta} - \frac{\tan \theta}{\sec \theta \csc \theta} = 1 - \frac{\left(\frac{\sin \theta}{\cos \theta}\right)}{\left(\frac{1}{\cos \theta}\right)\left(\frac{1}{\sin \theta}\right)} \\ &= 1 - \left(\frac{\sin \theta}{\cos \theta}\right) \frac{\sin \theta \cos \theta}{1} = 1 - \sin^2 \theta = \cos^2 \theta \end{aligned}$$

Card X

$$\begin{aligned} \text{Verify: } \frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} &= \frac{\sin \theta}{1 - \left(\frac{\cos \theta}{\sin \theta}\right)} + \frac{\cos \theta}{1 - \left(\frac{\sin \theta}{\cos \theta}\right)} = \frac{\sin^2 \theta}{\sin \theta - (\cos \theta)} + \frac{\cos^2 \theta}{\cos \theta - (\sin \theta)} \\ &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} - \frac{\cos^2 \theta}{\sin \theta - \cos \theta} = \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} \\ &= \frac{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}{\sin \theta - \cos \theta} = \sin \theta + \cos \theta \end{aligned}$$

Card Y

$$\begin{aligned} \text{Simplify: } (\csc \theta + \cot \theta)(1 - \cos \theta) &= \csc \theta - \cos \theta \csc \theta + \cot \theta - \cos \theta \cot \theta \\ &= \frac{1}{\sin \theta} - \cos \theta \cdot \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} - \cos \theta \cdot \frac{\cos \theta}{\sin \theta} \\ &= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} - \frac{\cos^2 \theta}{\sin \theta} = \frac{1 - \cos^2 \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta}{\sin \theta} = \sin \theta \end{aligned}$$

Card Z

$$\begin{aligned}\text{Verify: } \frac{1 - \tan^2 \theta}{1 - \cot^2 \theta} &= \frac{1 - \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right)}{1 - \left(\frac{\cos^2 \theta}{\sin^2 \theta}\right)} = \frac{\left(\frac{\cos^2 \theta}{\cos^2 \theta}\right) - \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right)}{\left(\frac{\sin^2 \theta}{\sin^2 \theta}\right) - \left(\frac{\cos^2 \theta}{\sin^2 \theta}\right)} = \frac{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta}} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} = \frac{-\sin^2 \theta (\sin^2 \theta - \cos^2 \theta)}{\cos^2 \theta (\sin^2 \theta - \cos^2 \theta)} = -\frac{\sin^2 \theta}{\cos^2 \theta} = -\tan^2 \theta\end{aligned}$$