

Trig Identities, Part 1

Reciprocal, Quotient, and Pythagorean Identities

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Grade Level	11th – 12th Grade	**Time Frame**	90-120 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	Precalculus		

Essential Question

How do we use trigonometric identities?

Summary

Students will prove the reciprocal, quotient, and Pythagorean trigonometric identities. They will then use those identities to practice simplifying and verifying trigonometric identities during this spy-themed lesson. This lesson is an introduction to identities and is the first lesson of four in a “Trig Identities” lesson series.

Snapshot

Engage

Students recall special angles and find patterns between trigonometric expressions.

Explore

Students discover the three Pythagorean trigonometric identities.

Explain

Students complete guided notes with the class and learn how to simplify and verify trigonometric identities using reciprocal, quotient, and Pythagorean identities.

Extend

Students practice simplifying and verifying trigonometric identities through a card game.

Evaluate

Students reflect and summarize their learning while sharing advice for peers.

Standards

Oklahoma Academic Standards Mathematics (Precalculus)

PC.T.3.1: Algebraically manipulate the structure of a trigonometric expression to identify ways to rewrite it.

PC.T.3.2: Choose and produce an equivalent form of an expression to explain the properties of the quantity represented by the expression.

Attachments

- Gathering Intel—Trig Identities, Part 1 - Spanish.docx
- Gathering Intel—Trig Identities, Part 1 - Spanish.pdf
- Gathering Intel—Trig Identities, Part 1.docx
- Gathering Intel—Trig Identities, Part 1.pdf
- Intel Notes (Model Notes)—Trig Identities, Part 1.docx
- Intel Notes (Model Notes)—Trig Identities, Part 1.pdf
- Intel Notes—Trig Identities, Part 1 - Spanish.docx
- Intel Notes—Trig Identities, Part 1 - Spanish.pdf
- Intel Notes—Trig Identities, Part 1.docx
- Intel Notes—Trig Identities, Part 1.pdf
- Lesson Slides—Trig Identities, Part 1.pptx
- Secret Agent Pythagoras—Trig Identities, Part 1 - Spanish.docx
- Secret Agent Pythagoras—Trig Identities, Part 1 - Spanish.pdf
- Secret Agent Pythagoras—Trig Identities, Part 1.docx
- Secret Agent Pythagoras—Trig Identities, Part 1.pdf
- Trig Spies Cards—Trig Identities, Part 1 - Spanish.docx
- Trig Spies Cards—Trig Identities, Part 1 - Spanish.pdf
- Trig Spies Cards—Trig Identities, Part 1.docx
- Trig Spies Cards—Trig Identities, Part 1.pdf
- Trig Spies Record Sheet (Sample Responses)—Trig Identities, Part 1.docx
- Trig Spies Record Sheet (Sample Responses)—Trig Identities, Part 1.pdf
- Trig Spies Record Sheet—Trig Identities, Part 1 - Spanish.docx
- Trig Spies Record Sheet—Trig Identities, Part 1 - Spanish.pdf
- Trig Spies Record Sheet—Trig Identities, Part 1.docx
- Trig Spies Record Sheet—Trig Identities, Part 1.pdf

Materials

- Lesson Slides (attached)
- Gathering Intel handout (attached; one per pair; printed front only)
- Secret Agent Pythagoras handout (attached; one per student; printed front only)
- Intel Notes handout (attached; one per student; printed front/back)
- Intel Notes (Model Notes) document (attached; for teacher use)
- Trig Spies Cards (attached; one set per group; printed front/back)
- Trig Spies Record Sheet handout (attached; one per group; printed front only)
- Trig Spies Record Sheet (Sample Responses) document (attached; for teacher use)
- Pencil
- Paper
- Green plastic cups (1 per pair)
- Yellow plastic cups (1 per pair)
- Red plastic cups (1 per pair)

20 minutes

Engage

Teacher's Note: Lesson Prep

Before the lesson, print the attached Trig Spies Cards (one set per group of 2-3 of students in your class). These cards have a front and back to indicate the two types of cards, so be sure to print pages 1-2 as a front/back pair and pages 3-4 as a front/back pair. Consider printing on cardstock paper, especially if you plan to reuse these cards; see the Extend portion of the lesson for more details.

Once printed, cut out the cards. All of these cards are the same size for easy cutting. Use envelopes, zip-top bags, or paper clips to organize the cards. The cards will be used during the Extend portion of the lesson.

This lesson is heavily spy-themed. If you would like additional information about some of the spy lingo, check out the Language of Espionage from the Spy Museum (in Washington DC) or the Glossary of Spy Terms from SPYSCAPE museum (in New York City).

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with the class to the extent you feel necessary.

Ask students to find a partner or assign pairs. Display **slide 5** and pass out the **Gathering Intel** handout. Introduce the students to the I Notice, I Wonder instructional strategy and help set the scene for their "spy mission" lesson by telling them that they are a "covert operative" assigned with the task of gathering intel.

Assign each pair one of the following angle measures to investigate: 30°, 45°, 60°, $\pi/6$, $\pi/4$, or $\pi/3$. This can be done a number of ways: the easiest is to start in one corner of the room and assign the first pair 30°, the next pair 45°, ..., this pair $\pi/3$, then the next pair 30°, and continue to repeat the 6 angle measures. The importance is that students are flexible with degrees and radians.

Instruct students to write their assigned angle measure at the top of their handout. Explain to them that they are to use that angle measure to complete the table.

Once they have completed the table, have pairs review their table and look for patterns. Have students write what they notice and what they wonder on their handout.

Teacher's Note: Guiding the Activity

The purpose here is for students to move from recognizing patterns to proving. Use the I Notice, I Wonder strategy to guide the discussion from "We noticed a pattern with our assigned angle." to "I wonder if this pattern holds true for everyone's angle?" to "I wonder if this is true for all angle measures?"

Use this activity to also help students see that it did not matter which angle they were discussing, the relationships between the trigonometric functions are always the same. Students will learn during the Explain portion of the lesson that this is true because these relationships are "identities." So, wait to introduce students to this vocabulary until later in the lesson.

Slides 7, 9, 11, 13, and 14 are hidden and have been designed to be used for reference. However, you may want to consider unhiding the slides if you want to use them to provide extra support as students transition from recognizing patterns to proving during this portion of the lesson.

Display **slide 6** and pose the following question to the students: *Sine and Cosecant seem to be conspiring. What is their relationship?*

Ask for volunteers to share anything they noticed. Use the hidden **slide 7** to help move students from noticing the relationship between sine and cosecant for their angle to proving that the sine and cosecant are reciprocals.

Show **slide 8** and pose the following question to the students: *Cosine and Secant are plotting together. What is their connection?*

Ask for volunteers to share anything they noticed. Use the hidden **slide 9** to help move students from noticing the relationship between cosine and secant for their angle to proving that the cosine and secant are reciprocals.

Move to **slide 10** and pose the following question to the students: *Tangent and Cotangent seem to be scheming. What is their alliance?*

Ask for volunteers to share anything they noticed. Use the hidden **slide 11** to help move students from noticing the relationship between tangent and cotangent for their angle to proving that the tangent and cotangent are reciprocals.

Display **slide 12** and pose the following question to the students: *Sine and Cosecant seem to be in cahoots with Tangent and Cotangent but how?*

Ask for volunteers to share anything they noticed. Use the hidden **slides 13-14** to help move students from noticing the relationships between sine and cosine with tangent and sine and cosine with cotangent for their angle to proving those relationships.

25 minutes

Explore

Alternative Materials

If finding colorful cups is a challenge, consider making red, yellow, and green cones out of construction paper.

Transition to **slide 15** and give each student a copy of the attached **Secret Agent Pythagoras** handout. Explain to the class that they are now "intelligence analysts" and need to look closely at the Pythagorean Theorem and see what they can uncover.

Display **slide 16** and introduce the Try It, Talk It, Color It, Check It instructional strategy and provide each pair with a stack of three cups: one green, one yellow, and one red. Preview the activity by explaining that they are to begin by trying step 1 on their own, then compare their work with their partner, then use the stack of cups to indicate their confidence level of their final answer, then check their work. They will repeat this process for each of the four steps and work through this handout step-by-step as a class.

Teacher's Note: Pacing the Activity

This should be a relatively quickly paced activity; however, make modifications to meet the needs of the class. For each step, give students 1-2 minutes to try the step themselves, 1-2 minutes to talk with their partner, less than a minute to select a color, and 1-2 minutes to check their work. Be sure to clearly communicate with students what phase of the instructional they are on for each of the steps on the handout.

Let students know that the green cup means "We got it and can teach others!" The yellow cup indicates "We are a bit uncertain." And the red cup means "We need help. We may be wrong." Have each group start with the yellow cup on top, so that it is easy to see when there is a change.

Display **slide 17** and instruct students to complete step 1 on their own. Then have them compare their answers, discuss their work, and agree upon a final result. Direct pairs to indicate their confidence level with their stack of cups: if they feel confident in their work to put the green cup on top, if they are unsure to put the yellow on top, and if they are completely lost to put the red cup on top.

Transition to **slide 18** and have students check their work for step 1. Give students time to correct their work and ask questions as needed. As students are checking their work, assist pairs who have a red or yellow cup on top to ensure everyone is understanding before moving to the next step.

Repeat this process for step 2. Use **slides 19-20**. For step 3, use **slides 21-22**. For step 4, use **slides 23-24**. On completion of the handout, display **slide 25** and share with the class the truth: We derived three trigonometric equations from the Pythagorean Theorem.

20 minutes

Explain

Transition to **slide 26**. Give each student a copy of the **Intel Notes** handout.

Explain to the class that the equations they have seen during this lesson are known as "trigonometric identities." Give students the definition of "identity," which is on the slide.

Show **slide 27** and ask the class to identify which of the given equations is an identity.

Teacher's Note: Guiding the Lesson

Help students see that no matter which angle value they choose, the first equation is always true and is, therefore, an identity, while the second equation is only true when $\theta = \pi/4 + 2\pi$ and $\theta = \pi/4 - 2\pi$, so it is not an identity.

Direct students' attention to their handout and explain to students that they already proved the reciprocal, quotient, and Pythagorean identities earlier in the lesson.

Display **slide 28** and explain how to read and write the shorthand notation for trigonometric expressions raised to the second power. Help students see this shorthand notation in the Pythagorean identities on their handout.

Complete example 1 as a class. Use the hint of rewriting expressions using *sine* and *cosine* to simplify the given expression.

Challenge students to try example 2 on their own. While students work, monitor progress by circulating the room. Depending on time, write the steps on the board slowly so that students can check their work as they go or have a volunteer go to the board to share their work.

Direct students' attention to the back of their handout and explain how similar, yet different, verifying trigonometric identities is from simplifying trigonometric identities. Let students know that they can start with either side of the equation, but it is recommended—as it is often simpler—to make the complicated side look like the less complicated side.

Based on this advice, direct students to start example 3 on their own. After a few minutes, facilitate a brief discussion having students share why they started on the side they started and what their first couple of steps look like. Use student responses to determine whether the class is ready to finish example 3 on their own or whether they need to work through it together as a class.

Teacher's Note: Guiding the Lesson

For more support and recommendations for the Intel Notes, use the attached **Intel Notes (Model Notes)** document.

Once finished, have students add the handout to their math notebook if that is a classroom norm.

20 minutes

Extend

Teacher's Note: Pacing the Activity

The amount of time you give students to play the following Trig Spies card game can vary based on your classroom needs. It is recommended that you give students at least 15 minutes to play, and it is unlikely that students would need more than 30 minutes to play through the entire deck of cards.

Display **slide 29** and direct students to get into small groups of 2-3 or assign groups. Students could continue to work in the same pairs as earlier in the lesson or use this time to have students work with someone they have not yet worked with during this lesson. Working with different peers fosters the development of academic vocabulary and encourages students to consider different approaches to a problem.

Let students know that they are now "spymasters" and responsible for selecting agents for the next mission. Give each group a copy of the attached **Trig Spies Record Sheet** and a set of **Trig Spies Cards** (containing 6 Undercover Agents cards and 6 Deep Cover Agents cards). Direct students to place each stack of cards face down so that the names of the cards with their icons are facing up. Make sure each student has a piece of scratch paper and writing tool for this game. Remind students to not draw on the cards, especially if you plan to reuse the cards.

Display **slide 30** and ask each team to check that they have everything they need for the mission (card game):

- 2 Stacks of Cards
- Scratch Paper
- Writing Utensil
- Trig Spies Record Sheet

Display **slide 31** and go over the directions for the card game. Explain that teams will pick a stack and flip over the top card. On their scratch paper, each student works independently to complete the question, and then they compare their work with their teammate(s). Once everyone on the team agrees, they write the card letter (A, B, C, etc.), the identity they used, and the points they earned on the team's Trig Spies Record Sheet.

Show **slide 32** and explain that each stack is worth different points. The cards in the Undercover Agent stack are worth 3-4 points; the Deep Cover Agent cards are worth 5-6 points and are more challenging. Share that they will have 15 minutes to earn as many points as they can. So, they need to select their agents wisely.

Teacher's Note: Guiding the Activity

There are two stacks of cards with different difficulty levels to give students the opportunity to challenge themselves when they feel ready. Most teams will start with a card from the Undercover Agent stack, then decide whether they are ready for a more challenging question from the Deep Cover Agent stack or need a little more practice from the Undercover Agent stack.

If students draw from the Deep Cover Agent stack "too soon" and get stuck, ask them to put that card back in the stack randomly. Then they will need to successfully answer two questions from the Undercover Agent stack before trying the Deep Cover Agent stack again.

If you observe a team quickly going through the Undercover Agent stack, consider pushing them to try a card from the Deep Cover Agent stack before they run out of Undercover Agent cards.

Transition to **slide 33**, begin the 15-minute timer, and tell the teams to begin.

Once the timer expires, ask students to add up their points. Then instruct students to put their name on their scratch paper and staple everyone's scratch paper to their group's Trig Spies Record Sheet. Collect these papers to assess students' learning. Use the **Trig Spies Record Sheet (Sample Responses)** document for possible student responses.

Quickly skim the papers and announce the highest score, but not which team achieved that score. This will give students the opportunity to self-reflect and compare their score with the highest. If the class really wants to know which team earned the most points, remind them that you do not want to reveal any secret identities in the middle of such an important mission.

5 minutes

Evaluate

Move to **slide 34**. Inform students that their expertise is needed for a new mission, so they need to pass along their knowledge to the new spymaster taking over. Have students write 2-3 pieces of advice for the new spymaster. Students can write their advice on an index card, piece of scratch paper, etc. Have students consider the following questions:

- What strategies did you find effective?
- How did you decide how to start a problem?
- What would you not recommend?

Alternative Pacing

This can be completed in class or assigned for students to return to class the next day.

Use student responses to see which misconceptions persist before moving on to the next lesson: "Trig Identities, Part 2."

Teacher's Note: ACT Prep

Using basic identities is a skill assessed on the ACT exam. The questions from the stack of Undercover Agent cards are most similar to the ACT questions about identities since they are relatively short and can be completed in less than a minute.

Resources

K20 Center. (n.d.). I Notice, I Wonder. Strategies. https://learn.k20center.ou.edu/strategy/180

K20 Center. (n.d.). Try It, Talk It, Color It, Check It. Strategies. https://learn.k20center.ou.edu/strategy/2329

International Spy Museum. (2023). *Language of espionage*. https://www.spymuseum.org/education-programs/spy-resources/language-of-espionage/

Spyscape. (2023). *The Spyscape glossary of spy terms*. SPYSCAPE. https://spyscape.com/article/spy-glossary