GUIDED NOTES (MODEL NOTES)

Examples

Solve each of the following equations for all radian values of θ.

Look Familiar?

Hint: Use your knowledge of factoring or other algebraic methods like the quadratic formula.

1) $2\sin\theta\cos\theta = \sqrt{3}\cdot\cos\theta$

$2\sin\theta\cos\theta - \sqrt{3}\cdot\cos\theta = 0$

$\cos\theta\left(2\sin\theta - \sqrt{3}\right) = 0$

$\cos\theta = 0$ and $2\sin\theta - \sqrt{3} = 0$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$\boxed{\begin{array}{l}\theta = \dfrac{\pi}{2}\pm 2\pi \\[2mm] \theta = \dfrac{3\pi}{2}\pm 2\pi\end{array}}$ and $\boxed{\begin{array}{l}\theta = \dfrac{\pi}{3}\pm 2\pi \\[2mm] \theta = \dfrac{2\pi}{3}\pm 2\pi\end{array}}$ $\boxed{\theta = \dfrac{\pi}{2}\pm\pi}$

or

Try Identities?

Hint: When you see more than one type of trig expression, try using a Pythagorean identity.

2) $\sec\theta = 1 - \tan^2\theta$

$\sec\theta = 1 - \left(\sec^2\theta - 1\right)$

$\sec\theta = 2 - \sec^2\theta$

$\sec^2\theta + \sec\theta - 2 = 0$

$\left(\sec\theta - 1\right)\left(\sec\theta + 2\right) = 0$

$\sec\theta - 1 = 0$ and $\sec\theta + 2 = 0$

$\sec\theta = 1$ and $\sec\theta = -2$

$\boxed{\theta = 0\pm 2\pi}$ *and* $\boxed{\begin{array}{l}\theta = \dfrac{2\pi}{3}\pm 2\pi \\[2mm] \theta = \dfrac{4\pi}{3}\pm 2\pi\end{array}}$

What if…?

Hint: If you squared both sides, could you then use a Pythagorean identity? Watch out for extraneous solutions.

3) $\csc\theta + \cot\theta = 1$

$\csc\theta = 1 - \cot\theta$

$\csc^2\theta = 1 - 2\cot\theta + \cot^2\theta$

$1 + \cot^2\theta = 1 - 2\cot\theta + \cot^2\theta$

$0 = -2\cot\theta$

$\cot\theta = 0$

$\theta = \dfrac{\pi}{2} \pm 2\pi, \ \dfrac{3\pi}{2} \pm 2\pi$

Check :

$\csc\left(\dfrac{\pi}{2}\right) + \cot\left(\dfrac{\pi}{2}\right) = 1?$

$1 + 0 = 1? \ Yes$

$\csc\left(\dfrac{3\pi}{2}\right) + \cot\left(\dfrac{3\pi}{2}\right) = 1?$

$-1 + 0 = 1? \ No$

$$\boxed{\theta = \dfrac{\pi}{2} \pm 2\pi}$$