Trig Identities, Part 2

Reciprocal, Quotient, and Pythagorean Identities

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Grade Level	11th – 12th Grade	**Time Frame**	75-90 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Precalculus		

Essential Question

How do we use trigonometric identities?

Summary

Students will use the reciprocal, quotient, and Pythagorean trigonometric identities to solve equations. They will listen to a petroleum engineer describe his work and how trigonometric identities are used to solve problems when drilling. This lesson is the second lesson of four in a “Trig Identities” lesson series.

Snapshot

Engage

Students use the graphs of the sine and cosine functions to solve a trigonometric equation.

Explore

Students use their algebraic knowledge to solve trigonometric equations that are of the same format.

Explain

Students complete guided notes with the class and learn how to solve trigonometric equations using reciprocal, quotient, and Pythagorean identities.

Extend

Students listen to a petroleum engineer then apply what they have learned to solve a real-world problem.

Evaluate

Students demonstrate their understanding by independently solving a trigonometric equation.

Standards

Oklahoma Academic Standards Mathematics (Precalculus)

PC.T.3.1: Algebraically manipulate the structure of a trigonometric expression to identify ways to rewrite it.

PC.T.3.3: Graphically and algebraically verify solutions to trigonometric equations.

Attachments

- Appointment Clocks—Trig Identities, Part 2 - Spanish.docx
- Appointment Clocks—Trig Identities, Part 2 - Spanish.pdf
- Appointment Clocks—Trig Identities, Part 2.docx
- Appointment Clocks—Trig Identities, Part 2.pdf
- Bell Ringer—Trig Identities, Part 2 - Spanish.docx
- Bell Ringer—Trig Identities, Part 2 - Spanish.pdf
- Bell Ringer—Trig Identities, Part 2.docx
- Bell Ringer—Trig Identities, Part 2.pdf
- Drilling for Filling (Play-Doh)—Trig Identities, Part 2 - Spanish.docx
- Drilling for Filling (Play-Doh)—Trig Identities, Part 2 - Spanish.pdf
- Drilling for Filling (Play-Doh)—Trig Identities, Part 2.docx
- Drilling for Filling (Play-Doh)—Trig Identities, Part 2.pdf
- Drilling for Filling—Trig Identities, Part 2 - Spanish.docx
- Drilling for Filling—Trig Identities, Part 2 - Spanish.pdf
- Drilling for Filling—Trig Identities, Part 2.docx
- Drilling for Filling—Trig Identities, Part 2.pdf
- Exit Ticket—Trig Identities, Part 2 - Spanish.docx
- Exit Ticket—Trig Identities, Part 2 - Spanish.pdf
- Exit Ticket—Trig Identities, Part 2.docx
- Exit Ticket—Trig Identities, Part 2.pdf
- Guided Notes (Model Notes)—Trig Identities, Part 2.docx
- Guided Notes (Model Notes)—Trig Identities, Part 2.pdf
- Guided Notes—Trig Identities, Part 2 - Spanish.docx
- Guided Notes—Trig Identities, Part 2 - Spanish.pdf
- Guided Notes—Trig Identities, Part 2.docx
- Guided Notes—Trig Identities, Part 2.pdf
- Lesson Slides—Trig Identities, Part 2.pptx
- Oil Drilling (Sample Responses)—Trig Identities, Part 2.docx
- Oil Drilling (Sample Responses)—Trig Identities, Part 2.pdf
- Oil Drilling—Trig Identities, Part 2 - Spanish.docx
- Oil Drilling—Trig Identities, Part 2 - Spanish.pdf
- Oil Drilling—Trig Identities, Part 2.docx
- Oil Drilling—Trig Identities, Part 2.pdf

Materials

- Lesson Slides (attached)
- Bell Ringer handout (attached; one half per student; printed front only)
- Appointment Clocks handout (attached; one per student; printed front only)
- Guided Notes handout (attached; one per student; printed front/back)
- Guided Notes (Model Notes) document (attached; for teacher use)
- Drilling for Filling handout (attached; one per pair; printed front only)
- Drilling for Filling (Play-Doh) handout (optional; one per pair; printed front/back)

- Oil Drilling handout (attached; one per pair; printed front/back)
- Oil Drilling (Sample Responses) document (attached; for teacher use)
- Exit Ticket handout (attached; one half per student; printed front only)
- Pencil
- Paper
- Twinkies (one per student)
- Plastic, bendy straws (2 per student)
- Paper plates (1 per student)
- Play-Doh (optional; one 3 oz. container per student)
- Wax paper (optional)

5 minutes

Engage 1

Teacher's Note: Lesson Order

The order of this lesson is as follows: Engage 1, Explore, Explain, Engage 2, Extend, Evaluate.

Teacher's Note: Lesson Prep

The second day of this lesson begins with a Twinkie activity during Engage 2. There are a few things to consider before gathering the materials for this activity:

- Gluten Allergies: Some students cannot touch materials that contain gluten. There are usually gluten-free crème cakes sold in most major chain grocery stores.
- Food: If you would prefer to not have food in your classroom, there is a Play-Doh version of the activity available. Keep in mind that Play-Doh does contain gluten, and gluten-free modeling dough is often available from an online retail store.

Before the lesson, decide if you would like to use the Twinkie or Play-Doh version of the activity and gather materials accordingly. Both versions use bendy straws and paper plates. The Twinkie version also uses Twinkies and the attached **Drilling for Filling** handout, while the Play-Doh version uses Play-Doh, wax paper, and the attached **Drilling for Filling (Play-Doh)** handout.

Use the Bell Ringer strategy to begin the lesson. As students enter the classroom, display **slide 3** from the attached **Lesson Slides** and give each student a half-sheet of the attached **Bell Ringer** handout. Ask students to work independently to create the graphs and find the angle measure(s).

After giving students time to answer the question, transition to **slide 4** so that students can check their work. Use this time to address any misconceptions. If time allows, facilitate a brief discussion on alternative, non-graphical approaches to solving the equation.

Teacher's Note: Purpose

The purpose of this question is to remind students that they can use not only the unit circle or right-triangle trigonometry to solve an equation, but they can also use their knowledge of the graphs of trigonometric functions to solve.

Go to **slide 5** to share the lesson's essential question with students. Go to **slide 6** to identify the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

15 minutes

Explore

Display **slide 7** and give each student a copy of the attached **Appointment Clocks** handout. Use the slide to preview the activity. Explain that they will be using the Appointment Clocks strategy and will be working with 4 different partners to complete their handout. Use one of the handouts to help indicate to students to write the name of their partner for each appointment time in the corresponding box. Having students record their partner will help you know who worked with whom if you choose to collect this handout.

Transition to **slide 8** and start the 3-minute timer on the slide. Direct students to find a partner and work together to solve questions 1-2 on their handout.

Teacher's Note: Guiding the Activity

Here students are recalling their algebraic knowledge and applying it to solving trigonometric equations.

There is a 3-minute timer for questions 1–2 and 5-minute timers for questions 3–4 and 5–6. Students will likely not need the entire time given for each set of questions, so have them give you a thumbs-up to indicate when they are done. This will allow you to progress the lesson as needed. Also, as you observe them finding partners, remind them to find a partner quickly, so there is plenty of time to complete the math problems before the timer expires.

If students do seem to need the entire time, use this as a formative assessment and ask the class what they feel is the most challenging aspect of these questions. Is it recalling their algebraic knowledge? Is it recalling which angle yields that value for that specific trigonometric function? Determining where students are struggling here can help inform where students might need more practice.

As students complete questions 1–2, transition to **slide 9** so students can check their work. Give students time to correct their work and ask questions.

Display **slide 10** and start the 5-minute timer on the slide. Direct students to find a new partner, and work together to solve questions 3–4 on their handout. Remind them to write their partner's name on their handout for their 6 o'clock appointment.

As students complete questions 3–4, transition to **slide 11** so students can check their work. Give students time to correct their work and ask questions.

Move to **slide 12** and start the 5-minute timer on the slide. Direct students to find a new partner, and work together to solve questions 5–6 on their handout. Remind them to write their partner's name on their handout for their 9 o'clock appointment.

As students complete questions 5–6, transition to **slide 13** so students can check their work. Give students time to correct their work and ask questions.

Show **slide 14** and direct students to find a new partner.

Transition to **slide 15** and ask the class to discuss with their partner what was similar and what was different about the problems they solved. Start the 3-minute timer on the slide. As time allows, ask for a few volunteers to share with the whole class.

Sample Responses:

- We noticed that the first steps of each set of problems were the same, but the trigonometric equations all required one more step.
- We noticed that the even-numbered problems (the trigonometric problems) all had more answers than the odd-numbered problems (the algebraic problems).

20 minutes

Explain

Transition to **slide 16**. Give each student a copy of the **Guided Notes** handout.

Explain to the class that if they can recognize the style, or format, of a trigonometric equation, then determining the first step of solving is much easier. Ask students to reflect on the Appointment Clock activity and point out that they were able to solve all of these new trigonometric equations by relating them to a style of problem that they already knew how to solve.

Help students see that questions 1-2 from the Appointment Clock activity both use their knowledge of how to solve multi-step linear equations. Questions 3-4 both use their knowledge of solving a quadratic equation by taking a square root. Questions 5-6 both use their knowledge of factoring a quadratic expression. So, if they can recognize the format, "how" to solve the problem becomes much easier.

Teacher's Note: Guiding the Lesson

The example problems in the Guided Notes are designed to help students understand the sequence of thinking that is involved when attempting to solve some of these more complicated trigonometric equations. As you go through the examples, help students understand that they should ask themselves (1) Can I solve this using an algebraic approach that I am familiar with? If not, (2) Can I use a trigonometric identity? If not, (3) Could I square both sides so that I could then use a trigonometric identity?

Helping students understand the thinking and reasoning that goes on, that no one sees, is vital for students to be able to apply their understanding to new situations.

Direct students' attention to example 1 on their Guided Notes handout. Complete example 1 as a class. Use the attached **Guided Notes (Model Notes)** document as needed for additional support and recommendations for the Guided Notes. Explain that factoring or using their knowledge of algebraic approaches—even the approaches they used during the Explore portion of this lesson—is often the first place to start.

Help students understand that example 1 has two terms that both have a common factor, so that is why factoring is the approach for solving this equation. Remind students that in the same way one should not divide both sides of an equation by x (potentially dividing by zero), one also should not divide both sides by a trigonometric expression.

Direct students' attention to example 2. Ask if they think factoring or another algebraic approach would work to begin this problem. Explain that when using an algebraic method is not helpful for the first step, we ask ourselves: "Could I use a trigonometric identity?"

Challenge students to start example 2 on their own. After about 1 minute, giving everyone a chance to try the first step, ask for volunteers to share what they think the first step should be.

Once the class agrees on what the first step should be, have students complete example 2 on their own. While students work, monitor progress by circulating the room. Depending on time, write the steps on the board slowly so students can check their work as they go or have a volunteer go to the board to share their work.

Direct students' attention to the back of their handout and emphasize the thought process of solving these problems by asking them the following guiding questions:

- *What question should we ask ourselves first?* Can we start algebraically?
- *Is there an algebraic method that would be a helpful first step?* No.
- *What do we ask ourselves next?* Can we use a trigonometric identity?
- *Is there a trigonometric identity we can use?* No.

Explain that what we do when this happens is that we make using a trigonometric identity an option by squaring both sides.

Teacher's Note: Guiding the Lesson

This approach is often uncomfortable for students. They will often ask questions like, "What do you mean that we just square both sides?" or "Why would anyone square both sides?" And they ask these questions because they want to understand the reasoning behind this approach. So please help them understand. Remind them that they took what they knew during the Appointment Clock activity and applied it to trigonometric equations. They took their knowledge of factoring and applied it to example 1 of the Guided Notes, and used their knowledge of substitution with trigonometric identities to then, again, factor – something they already knew how to do. This approach is how we all tackle new problems. We think, *I know how to solve an equation that looks like "this," I wonder if I can make the given equation look like the "this" that I am familiar with.* So, we take something that we do not know how to solve and make it look like something we do know how to solve and solve it.

Remind students that just like solving radical equations, it is easiest to get one trigonometric equation isolated before squaring both sides. As a class, work through the first few steps together. After squaring both sides, point out that now the equation looks like example 2. Then direct students to complete example 3 on their own.

While students work, monitor progress by circulating the room. Depending on time, write the steps on the board slowly so students can check their work as they go or have a volunteer go to the board to share their work. Be sure to remind students that the process of squaring both sides of an equation increases the likelihood of extraneous solutions. So, it is important that they check their results when using this approach.

Once finished, have students add the handout to their math notebook if that is a classroom norm.

10 minutes

Engage 2

Teacher's Note: Activity Prep

Again be sure to be mindful of those with gluten allergies and make accommodations as needed.

Decide how you want students to get their materials for this activity. Consider having a space, or spaces, in your classroom where the materials will all be located for students to gather. Consider having smaller quantities of students distributing each supply. For example, one student could pass out 4 straws to each group, while another student passes out 2 Twinkies to each group, etc. Decide what would work best for your classroom setup and have a procedure to ensure this activity goes smoothly.

Cost-Saving Option

One way to save some money on this activity is to only purchase enough Twinkies for each pair to share one Twinkie instead of each having their own.

The Play-Doh version of this activity has similar up-front costs, but can be reused from year to year, overall saving money. See the "Alternative Activity: Play-Doh" note at the end of Engage 2 for more details on this option.

Have students find a partner or assign students partners. Show **slide 17** and read the following scenario:

Petroleum engineers are professionals who specialize in the extraction of oil and gas from the Earth's crust. They use advanced drilling techniques to access underground reservoirs and retrieve these valuable resources. By analyzing geological data and employing innovative technology, petroleum engineers play a crucial role in locating and drilling wells to extract oil, ensuring a steady supply of energy for various industries and daily life.

Transition to **slide 18** and ask students to discuss with their partner the question on the screen: *Which method of drilling do you think will be more effective: vertical or horizontal? Why?*

While students are discussing, pass out a copy of the attached **Drilling for Filling** handout to each pair.

Show **slide 19** and direct their attention to their handout. Preview the activity with the class and explain that they will be using straws to drill for filling to see if vertical or horizontal drilling yields more filling. Direct students to gather the materials listed on their handout.

Teacher's Note: Pacing the Activity

The steps for the activity are on the slides so that the class progresses together. To ensure that students do not jump to conclusions and not follow directions, consider setting the scene for the science experiment that they are about to conduct. Remind them that they only have one place to test (their one Twinkie), so it is important that the whole class follows the directions and completes the same experiment for consistent data. How else can they compare results if their methods are significantly different?

Display **slide 20** and give the overview of the activity: each student should have two straws, one for vertical and one for horizontal drilling. When they extract their drilling devices (straws), they will place it on their data table, under the Drilling Records portion of their handout, and shade in the number of squares indicating the length of the filling extracted.

Transition through **slides 21-24**, reading the steps on each slide. Move through the slides at a pace that allows students time to complete the steps on the screen before moving to the next steps.

Alternative Activity: Play-Doh

If you are using the **Drilling for Filling (Play-Doh)** handout, the directions are on the handout. Skip slides 20-24 and have students follow the printed directions.

20 minutes

Extend

Teacher's Note: Preparation

Decide whether you want the Oil Drilling handout to be guided practice or independent practice. Part A should be guided practice, but Parts B-E could be either. The sample responses to the handout are on hidden slides, so if you would like the class to check their work as they go, unhide **slides 34-35, 37, 39, and 41**. You can also use these slides or the attached **Oil Drilling (Sample Responses)** document for an idea of what work should look like.

If you choose independent practice, plan to have students submit their handout and use it as a formative assessment.

Show **slide 25** and introduce the "Petroleum Engineering and Trigonometry" video, which features an interview with Derek Draper, a petroleum engineer, talking about his career and how trigonometry is used in his job.

Embedded video
https://youtube.com/watch?v=vNBCiu7Tgns

After the video, transition to **slide 26** and ask: *What is an advantage of horizontal drilling?* If needed, ask guiding questions or remind the class to think about the video and the Engage 2, Twinkie, activity to help them come to the conclusion that extracting more resources is a definite advantage of horizontal drilling over vertical drilling.

Give each student a copy of the attached **Oil Drilling** handout.

Transition through **slides 27-29** and read the following scenario:

A well planner is working with a petroleum engineer to design a well. There will be three sections of drilling: the vertical portion, the curved portion, and the horizontal portion. The point where the drilling transitions from the vertical to curved portion is known as the kickoff point.

The drilling will need to transition from the vertical portion to the curved portion to bypass a salt dome section. Salt domes cause expensive challenges when drilling, so it is best if they are avoided.

The desired horizontal width, x, for the curved portion is 1,500 feet. The true vertical depth includes the vertical height of the curved portion, y, and vertical length of the vertical portion; this total needs to be 10,000 feet.

Display **slide 30** and direct students' attention to Part A of their handout. Direct students to discuss with their partner how they might start the problem.

Teacher's Note: Guiding the Activity

Part A of this activity is the most challenging since it is not a typical style of problem that they see at this grade level. For this reason, you will likely need to work through Part A as a class, then let students finish Parts B-E on their own.

This activity was adapted from a directional drilling calculation example by DrillingFormulas.com.

Move through **slides 31-32** and work through Part A as a class.

Show **slide 33** and direct students' attention to the back of their handout: Part B. Have students use their work from Part A to find θ.

To help with pacing, use **slide 36** to direct their attention to Part C, finding the value of *r*. Then use **slide 38** to direct their attention to Part D, calculating the measure depth. Lastly, use **slide 40** to direct their attention to Part E, finding the buildup rate.

Optional Slides

If you would like, you may unhide and transition through slides 34-35, 37, 39, and 41 so students can check their work. As time allows, ask for volunteers to explain their work for each question.

5 minutes

Evaluate

Display **slide 42** and use the Exit Ticket strategy to individually assess what students have learned from the lesson. Give each student a half-sheet of the attached **Exit Ticket** handout.

Alternative Pacing

After collecting the Exit Ticket handout, and if there is enough time, consider unhiding and reviewing the solutions on **slide 43**. You may choose to assign the Exit Ticket as homework and review the solutions on slide 43 as bellwork during the next class period.

Collect student responses and use them to see which misconceptions persist before moving on to the next lesson: "Trig Identities, Part 3."

Teacher's Note: ACT Prep

Using basic identities to solve is a skill assessed on the ACT exam, but those questions are likely less rigorous than the question from the Evaluate portion of this lesson.

Resources

- DrillingFormulas.Com. (2011, January 12). *Directional drilling calculation example?*. Drilling Formulas and Drilling Calculations | Learn about drilling formulas frequently used in drilling and workover operation. https://www.drillingformulas.com/directional-drilling-calculation-example/
- K20 Center. (2021, September 21). *K20 Center 3 minute timer*. [Video]. YouTube. https://youtu.be/iISP02KPau0
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