ARTIFACT 1 (SAMPLE RESPONSES)

On a piece of notebook paper, prove the following identities.

Part A

Find the identity for $\tan(\alpha+\beta)$.

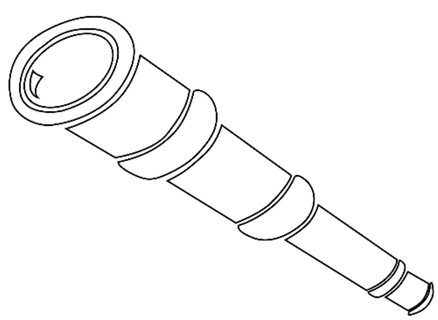
$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$=\frac{\sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)}{\cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta)}\cdot\frac{\left(\frac{1}{\cos(\alpha)\cdot\cos(\beta)}\right)}{\left(\frac{1}{\cos(\alpha)\cdot\cos(\beta)}\right)}$$

$$=\frac{\frac{\sin(\alpha)\cdot\cos(\beta)}{\cos(\alpha)\cdot\cos(\beta)}+\frac{\cos(\alpha)\cdot\sin(\beta)}{\cos(\alpha)\cdot\cos(\beta)}}{\frac{\cos(\alpha)\cdot\cos(\beta)}{\cos(\alpha)\cdot\cos(\beta)}-\frac{\sin(\alpha)\cdot\sin(\beta)}{\cos(\alpha)\cdot\cos(\beta)}}$$

$$=\frac{\frac{\sin(\alpha)}{\cos(\alpha)}+\frac{\sin(\beta)}{\cos(\beta)}}{1-\frac{\sin(\alpha)}{\cos(\alpha)}\cdot\frac{\sin(\beta)}{\cos(\beta)}}$$

$$=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}$$



ARTIFACT 2 (SAMPLE RESPONSES)

On a piece of notebook paper, prove the following identities.

Part B

Find the identity for $\sin(\alpha - \beta)$.

$$\begin{aligned}\sin(\alpha-\beta) &= \sin(\alpha+(-\beta))\\ &= \sin(\alpha)\cos(-\beta)+\cos(\alpha)\sin(-\beta)\\ &= \sin(\alpha)[\cos(\beta)]+\cos(\alpha)[-\sin(\beta)]\\ &= \sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)\end{aligned}$$

Part C

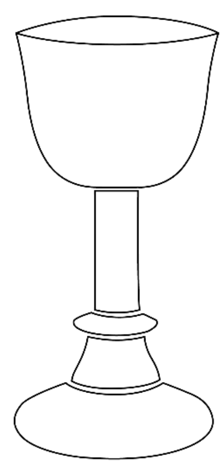
Find the identity for $\cos(\alpha - \beta)$.

$$\begin{aligned}\cos(\alpha-\beta) &= \cos(\alpha+(-\beta))\\ &= \cos(\alpha)\cos(-\beta)-\sin(\alpha)\sin(-\beta)\\ &= \cos(\alpha)[\cos(\beta)]-\sin(\alpha)[-\sin(\beta)]\\ &= \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)\end{aligned}$$

Part D

Find the identity for $\tan(\alpha - \beta)$.

$$\begin{aligned}\tan(\alpha-\beta) &= \frac{\tan(\alpha)+\tan(-\beta)}{1-\tan(\alpha)\cdot\tan(-\beta)}\\ &= \frac{\tan(\alpha)+[-\tan(\beta)]}{1-\tan(\alpha)\cdot[-\tan(\beta)]}\\ &= \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\cdot\tan(\beta)}\end{aligned}$$



ARTIFACT 3 (SAMPLE RESPONSES)

On a piece of notebook paper, prove the following identities.

Part E

Find the identity for $\sin(2\theta)$.

$$\begin{aligned}\sin(2\theta) &= \sin(\theta+\theta)\\ &= \sin(\theta)\cos(\theta)+\cos(\theta)\sin(\theta)\\ &= \sin(\theta)\cos(\theta)+\sin(\theta)\cos(\theta)\\ &= 2\sin(\theta)\cos(\theta)\end{aligned}$$

Part F

Find the identity for $\cos(2\theta)$.

$$\begin{aligned}\cos(2\theta) &= \cos(\theta+\theta)\\ &= \cos(\theta)\cos(\theta)-\sin(\theta)\sin(\theta)\\ &= \cos^2(\theta)-\sin^2(\theta)\end{aligned}$$

Part F: Challenge

Write two other identities for $\cos(2\theta)$ using what you know.

$$\begin{aligned}\cos^2(\theta)-\sin^2(\theta) &= \left(1-\sin^2(\theta)\right)-\sin^2(\theta)\\ &= 1-2\sin^2(\theta)\end{aligned}$$

$$\begin{aligned}\cos^2(\theta)-\sin^2(\theta) &= \cos^2(\theta)-\left(1-\cos^2(\theta)\right)\\ &= \cos^2(\theta)-1+\cos^2(\theta)\\ &= 2\cos^2(\theta)-1\end{aligned}$$