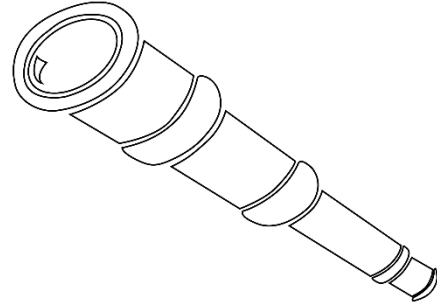


ARTIFACT 1

You are presented with what appears to be an ancient telescope. You gaze through the lens imagining that this could be the one used by Hipparchus. What's this? You notice that through the lens you read: $\tan(\alpha + \beta)$. Is this another identity we can find? On a piece of notebook paper, prove the following identity. *Do not write on the artifacts.*

Part A

Find the identity for $\tan(\alpha + \beta)$.

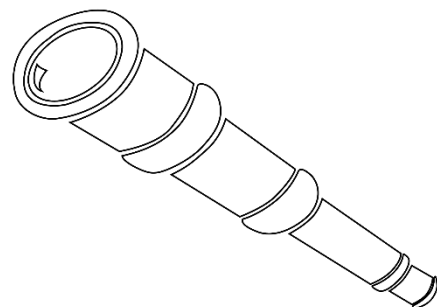


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ARTIFACT 2

What is this? A chalice? It's beautiful but covered in dirt. As you fill it with water, you notice 3 inscriptions. Are they more identities? On a piece of notebook paper, prove the following identities. *Do not write on the artifacts.*

Part B

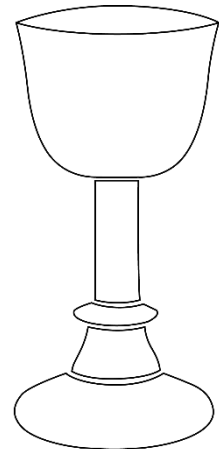
Find the identity for $\sin(\alpha - \beta)$.

Part C

Find the identity for $\cos(\alpha - \beta)$.

Part D

Find the identity for $\tan(\alpha - \beta)$.



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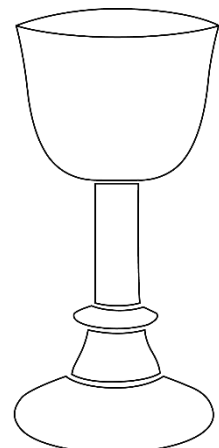
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Find the identity for $\tan(\alpha - \beta)$.



ARTIFACT 3

Someone hands you a metal item with a beautiful Penrose triangle design. What's happening? You notice that where your hand has touched the metal, it is changing color. You place it on the hot stone outside and reveal the following...identities? On a piece of notebook paper, prove the following identities. *Do not write on the artifacts.*

Part E

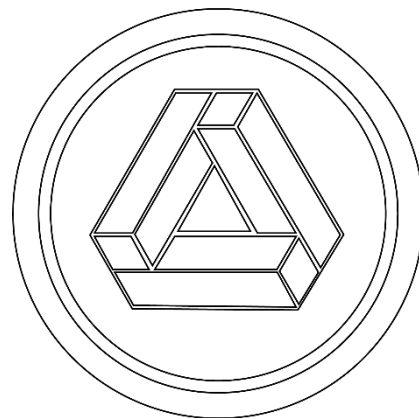
Find the identity for $\sin(\alpha - \beta)$.

Part F

Find the identity for $\cos(2\theta)$.

Part F: Challenge

Write two other identities for $\cos(2\theta)$ using what you know.



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Part F

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