

Trig Identities, Part 3

Sum, Difference, and Double-Angle Identities

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Grade Level	11th – 12th Grade	**Time Frame**	75-90 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Precalculus		

Essential Question

How do we use trigonometric identities?

Summary

Students discover the sum and difference identities for sine, cosine, and tangent. They discover the double-angle identities for sine and cosine. They will then use those identities to practice simplifying and verifying trigonometric identities during this archeologist-themed lesson. The prerequisite knowledge for this lesson is knowing the reciprocal, quotient, and Pythagorean identities, which are included in the Trig Identities, Part 1 lesson, and knowing how to determine whether a function is even or odd. This lesson is the third lesson of four in a "Trig Identities" lesson series.

Snapshot

Engage

Students algebraically prove whether functions are even or odd or neither.

Explore

Students discover the sum identity for sine through a scaffolded proof.

Explain

Students prove the sum identity for cosine and deepen their understanding using the Inverted Pyramid strategy.

Extend

Students apply what they have learned to discover the remaining sum, difference, and double-angle identities.

Evaluate

Students demonstrate their understanding by independently verifying a trigonometric identity.

Standards

Oklahoma Academic Standards Mathematics (Precalculus)

PC.T.3.1: Algebraically manipulate the structure of a trigonometric expression to identify ways to rewrite it.

PC.T.3.2: Choose and produce an equivalent form of an expression to explain the properties of the quantity represented by the expression.

Attachments

- Artifacts (Sample Responses)—Trig Identities, Part 3.docx
- Artifacts (Sample Responses)—Trig Identities, Part 3.pdf
- Artifacts Check—Trig Identities, Part 3 - Spanish.docx
- Artifacts Check—Trig Identities, Part 3 - Spanish.pdf
- Artifacts Check—Trig Identities, Part 3.docx
- Artifacts Check—Trig Identities, Part 3.pdf
- Artifacts—Trig Identities, Part 3 - Spanish.docx
- Artifacts—Trig Identities, Part 3 - Spanish.pdf
- Artifacts—Trig Identities, Part 3.docx
- Artifacts—Trig Identities, Part 3.pdf
- Even, Odd, or Neither—Trig Identities, Part 3 - Spanish.docx
- Even, Odd, or Neither—Trig Identities, Part 3 - Spanish.pdf
- Even, Odd, or Neither—Trig Identities, Part 3.docx
- Even, Odd, or Neither—Trig Identities, Part 3.pdf
- Into the Unknown (Sample Responses)—Trig Identities, Part 3.docx
- Into the Unknown (Sample Responses)—Trig Identities, Part 3.pdf
- Into the Unknown—Trig Identities, Part 3 - Spanish.docx
- Into the Unknown—Trig Identities, Part 3 - Spanish.pdf
- Into the Unknown—Trig Identities, Part 3.docx
- Into the Unknown—Trig Identities, Part 3.pdf
- Lesson Slides—Trig Identities, Part 3.pptx
- The Scrolls (Sample Responses)—Trig Identities, Part 3.docx
- The Scrolls (Sample Responses)—Trig Identities, Part 3.pdf
- The Scrolls—Trig Identities, Part 3 - Spanish.docx
- The Scrolls—Trig Identities, Part 3 - Spanish.pdf
- The Scrolls—Trig Identities, Part 3.docx
- The Scrolls—Trig Identities, Part 3.pdf

Materials

- Lesson Slides (attached)
- Even, Odd, or Neither handout (attached; one per student; printed front/back)
- The Scrolls handout (attached; one per pair; printed front only)
- The Scrolls (Sample Responses) document (attached; for teacher use)
- Into the Unknown handout (attached; one per pair; printed front/back)
- Into the Unknown (Sample Responses) document (attached; for teacher use)
- Artifacts handout (attached; one per group of 4; printed front only)
- Artifacts (Sample Responses) document (attached; for teacher use)
- Artifacts Check document (attached; one set; printed front only)
- Pencil
- Paper
- Manila Envelopes (3, letter size: 9” x 12”)

15 minutes

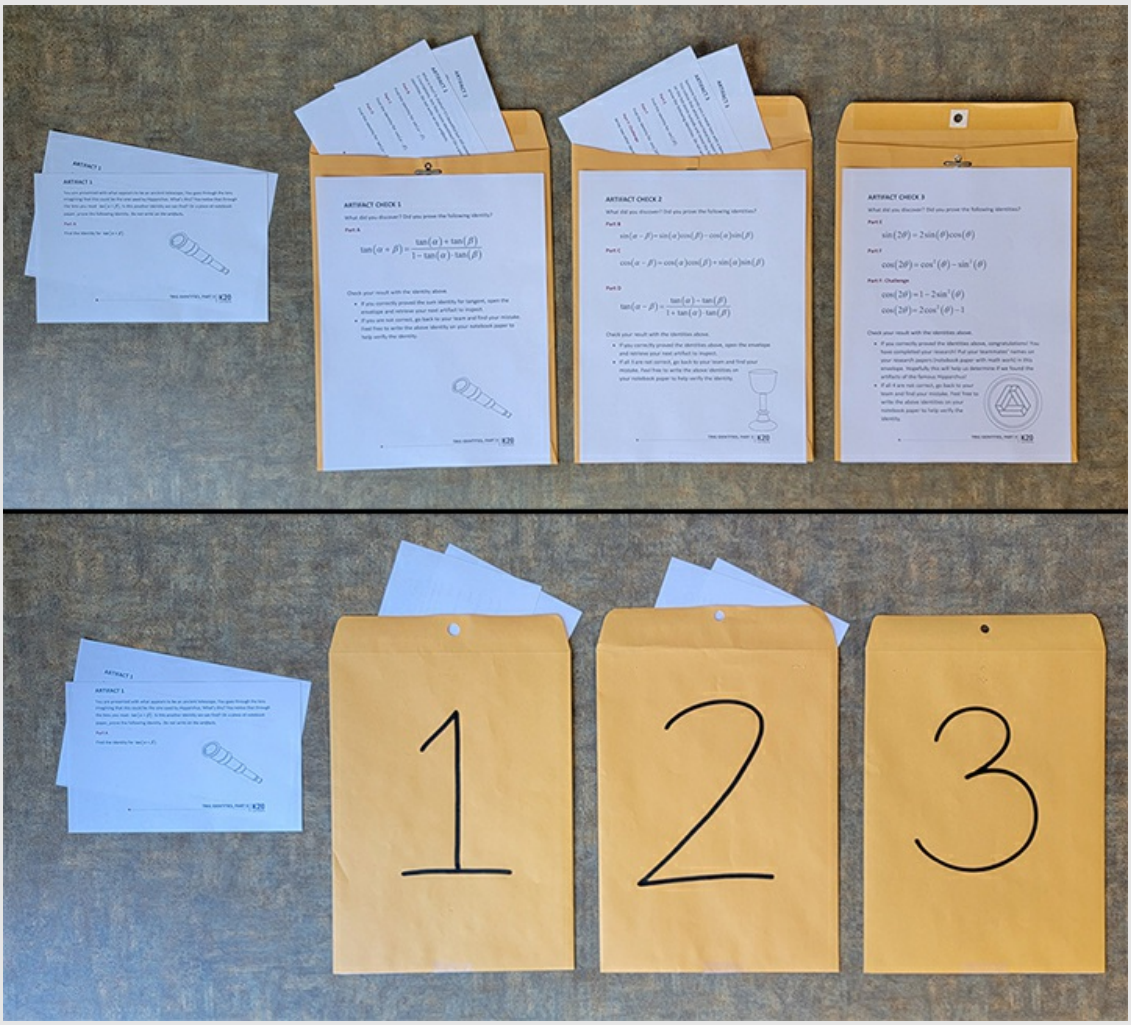
Engage

Teacher's Note: Lesson Preparation

Before the lesson, numerically label the three manila envelopes by writing one number on the front of each envelope: 1, 2, and 3. Print the attached **Artifacts Check** document. Attach page 1 to the back of envelope #1; attach page 2 to the back of back of envelope #2; and attach page 3 to the back of back of envelope #3.

Print the attached **Artifacts** handout; make one copy per every four students in your class. For example, if you have a class of 30 students, print 8 copies of this handout. Cut page 1 in half and put both halves into a stack to distribute later. Repeat this for all of the copies of page 1. Cut page 2 and all of its copies in half and put all halves into envelope #1. Repeat this for page 3 and place all halves into envelope #2. Envelope #3 will be empty. Students will be asked to put their work in that envelope at the end of the Extend activity.

Consider printing on cardstock paper or even laminating the Artifacts handouts, especially if you plan to reuse these handouts. Each page of the Artifacts handout could be printed in a different color to help monitor progress. These steps might also help to ensure that students do not write on these handouts. See the Extend portion of the lesson for more details.



Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Transition through **slides 5-6** and help set the scene for this "archeology discovery" lesson by reading the scenario on the slides:

- *Wake Up! The taxi to take you to the airport is on its way.*
- *Pack your bag and put on your archeologist's hat.*
- *The meter is running!*
- *Your ticket says you're going to Rhodes, Greece.*
- *You have it on good authority that some ancient trigonometry artifacts have been found there.*
- *You have 19 hours and 42 minutes before arriving. It's time to catch up on some basics.*

But first some history:

- Hipparchus (huh·paar·kuhs), a Greek astronomer, was fascinated with both astronomy and mathematics. He is thought to have written many works using trigonometry and is known as the Founder of Trigonometry.
- He recorded observations during the second century BCE and was exceptionally precise. One of his accomplishments was estimating the length of a year within seven minutes of accuracy.

Teacher's Note: Historical Accuracy

The information in this lesson about Hipparchus is accurate based on what historians know about this ancient mathematician. There are hardly any of his works remaining today, which is what inspired the archaeologist-theme of this lesson.

Move to **slide 7** and let students know that they need to brush up on some basics before they "arrive at the dig site." Remind students of the symmetric properties (graphical) of even and odd functions, then give each student a copy of the attached **Even, Odd, or Neither** handout.

Instruct students to follow the directions on their handout. Here students are to use the graph to identify the type of symmetry, if there is any, and record that information into the center column. Then they are to use that information to predict whether the function will be even, odd, or neither (neither even nor odd). Lastly, they are to find $f(-x)$ and prove that the function is even, odd, or neither in the last column on the right.

As students complete the first row, transition to **slide 8** so students can check their work. Circulate the room to monitor student progress and transition through **slides 9-11** as students finish each question so they can continue to check their work.

Use what you hear as you circulate the room and use students' questions to determine whether students need a quick refresher on how to algebraically prove if a function is even or odd.

Teacher's Note: Guiding the Activity

Depending on your pacing, students may or may not have proven if the sine or cosine functions are even or odd. If this is new information to your students, consider working through those questions as a class or in small groups instead of individually. Regardless of their background, try to ask guiding questions to help students understand as opposed to just directly giving students the answer.

10 minutes

Explore

Transition through **slides 12-13** and read the scenario on the slides:

- *Through your research, you have learned that, unfortunately, all but one of Hipparchus' works have been lost and very little is known of his life.*
- *When you get to the dig site, you are momentarily dazzled by the beauty of the island.*
- *But the team has unearthed a scroll that needs your attention.*
- *Being buried in rubble for centuries has taken its toll.*
- *So, you unroll the scroll very carefully, inch by inch.*
- *Could this be one of his lost writings?*
- *We're not sure how long this scroll is going to hold up in the wind from the Aegean Sea.*

Move to **slide 14** and dramatically read the scenario:

- *Oh no! It has been torn into two pieces!*

Ask students to find a partner or assign partners. Have students determine who will be "Student A" and who will be "Student B."

Give each pair a copy of the attached **The Scrolls** handout. This is a two-page handout, one page labeled "Student A" and the other "Student B." Direct students to independently follow the directions on their handout.

Teacher's Note: Guiding the Activity

The work for both students is of similar difficulty. If you have assigned partners, consider giving the stronger of the two math students the "Student B" page of the handout.

As students complete their handout, move to **slide 15** and give each pair of students a copy of the attached **Into the Unknown** handout. Direct their attention to the front of their handout: Step 3, and instruct them to work together, using the information on their The Scrolls handouts, to prove the sum identity for sine.

As students work, circulate the room and check progress. Allow students to have a healthy struggle with this activity but use guiding questions to keep students encouraged. If students are stuck, have them talk through how they did their work on The Scrolls handout. Use the attached **The Scrolls (Sample Responses)** document to see what student work should look like. If your students are missing any of that information, that is likely why they are stuck on the proof for the sum identity for sine. Ask prompting questions to ensure that students have all of the pieces of information they need for this proof.

Teacher's Note: Guiding the Activity

On the The Scrolls handout, Student A should have equations for the sin(x), cos(x), sin(y), cos(y) at a minimum. So, for example, if they are missing an equation for the sin(y), ask prompting questions, which might look like the following:

- Did you find the sine and cosine of both angles: x and y?
- What is the sin(y)?

Similarly, if Student B only found the sin(y) and is missing the cos(y) or does not have any equations including angle y, try to ask more questions than you give answers. Prompting questions could include:

- What is the cos(y)?
- Could you write your equations using angles x or y?

15 minutes

Explain

Display **slide 16** and make sure that the whole class discovered the correct sum identity for sine. Give students time to check their work and ask questions.

Teacher's Note: Guiding the Activity

This is the time in the lesson to clear up any misconceptions and ensure that students understand how to prove the sum identities for sine and cosine.

Use the attached **Into the Unknown (Sample Responses)** document for possible student responses for these two proofs.

Show **slide 17** and direct students' attention to the back of their handout: Step 4. Prompt students to work with their partner to find the sum identity for the cosine function. Encourage them to use all of their handouts to make notes and record their proof on the Into the Unknown handout.

As students complete their proofs, move to **slide 18** and introduce the Inverted Pyramid strategy. Have each pair of students find another pair of students (creating groups of four) to compare their results and reasoning. Direct students to use this time to make any needed corrections.

After a few minutes, transition to **slide 19** and bring the class together for a whole-group discussion. Work through the proof together and have a volunteer from each group share the next step. For example, one student from the first group shares how they started their proof, then a volunteer from another group shares how they arrived at step 2, then a volunteer from another group shares what step 3 should look like, and so on.

30 minutes

Extend

Teacher's Note: Preparation

Place the prepared manila envelopes in dedicated places around the room. Depending on space and quantity of students consider some of the following options:

- To ensure a large quantity of students is not all in one place, position the three envelopes in three different corners of the room.
- Consider creating two sets of prepared manila envelopes for larger quantities of students.

See the teacher's note at the beginning of Engage for more information on how to prepare the manila envelopes.

Display **slide 20** and read the scenario on the slide:

- *The scroll has started to crumble.*
- *Luckily, you have collected the information you needed.*
- *There are some additional artifacts that have been brought to your attention. What else can we prove?*

Show **slide 21** and preview the activity with the class by explaining that they will continue to work in pairs and are about to receive their first artifact. This artifact will require that they complete a proof for the sum identity for tangent. Once they think they have completed the proof, one person from each team will take their answer to envelope #1, flip over the envelope, and follow the directions, which is to check their answer, and if correct, take the next artifact (piece of paper) out of envelope #1 back to their partner. If they are incorrect, they are to write the identity, go back to their partner and fix any mistakes to verify the identity. This will repeat for envelopes #2 and #3, except for the third envelope, there is nothing to remove from the envelope. Instead they are directed to put their research papers (notebook paper with math work) inside the empty envelope.

Direct students to get out notebook paper to show their work. Distribute a copy of the Artifact 1 half-page of the attached **Artifacts** handout to each pair of students. Remind students to **not** write on the artifact (handout), but to show all of their work (and final answer) on their notebook paper (research paper). Tell students that they will need their previous answers to answer later questions, which is why they are recording all of their work on their notebook paper.

To add some excitement to this activity, play the "Indiana Jones" soundtrack on the slide as students work. There is more music than you need for the activity in this video. However, feel free to find alternative music if you prefer.

Embedded video

https://youtube.com/watch?v=vzulc7RlNpA

As students work, circulate the room. Again this is the time for students to have a healthy struggle with this activity but use guiding questions to keep students encouraged. Let time determine how much assistance you give a particular pair. In other words, if students have been working on Artifact 1 for 10 minutes give much more assistance than to the students who have only been working on it for 3-5 minutes.

Once students complete the activity, have them sort the Artifacts handouts into three stacks to reuse for a later class or next school year.

Use the **Artifacts (Sample Responses)** document to evaluate students' work. Use their work to identify misunderstandings.

Teacher's Note: Give Praise

Take a moment to praise your students for their hard work. They have just independently proven 10 trigonometric identities! Some students will feel satisfied or a sense of accomplishment while others may not. For those who do, let them know that if they found this fun that proofs are a large part of a mathematics degree and maybe that is something they should consider for a field of study.

5 minutes

Evaluate

Display **slide 22** and use the Exit Ticket strategy to individually assess what students have learned from the lesson. Have students verify the identity on a piece of notebook paper.

Collect student responses and use them to see which misconceptions persist before moving on to the next lesson: "Trig Identities, Part 4."

Alternative Pacing

After collecting the Exit Ticket handout, and if there is enough time, consider unhiding and reviewing the solutions on **slides 23-24**. You may choose to assign the Exit Ticket as homework and review the solutions on the slides as bellwork during the next class period.

Resources

- Ambient Worlds. (2021, September 26). *Indiana Jones* [Video]. YouTube. https://youtu.be/vzulc7RlNpA
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Inverted pyramid. Strategies. https://learn.k20center.ou.edu/strategy/173