

ESCAPE ROOM (SAMPLE RESPONSES)

Question 1

$$\begin{aligned}\sin^2 x &= \frac{1 - \cos(2x)}{2} \\ \frac{1 - \cos(2x)}{2} &= \frac{1 - (1 - 2\sin^2 x)}{2} \\ &= \frac{2\sin^2 x}{2} \\ &= \sin^2 x \Rightarrow \text{yes}\end{aligned}$$

Question 3

$$\begin{aligned}\frac{\cot^2 x - 1}{1 + \cot^2 x} &= 1 - 2\sin^2 x \\ \frac{\cot^2 x - 1}{1 + \cot^2 x} &= \frac{(\csc^2 x - 1) - 1}{1 + (\csc^2 x - 1)} \\ &= \frac{\csc^2 x - 2}{\csc^2 x} \\ &= 1 - \frac{2}{\csc^2 x} \\ &= 1 - 2\sin^2 x \Rightarrow \text{yes}\end{aligned}$$

Question 5

$$\begin{aligned}\tan(105^\circ) &= \tan(45^\circ + 60^\circ) \\ &= \frac{\tan(45^\circ) + \tan(60^\circ)}{1 - \tan(45^\circ)\tan(60^\circ)} \\ &= \frac{(1) + (\sqrt{3})}{1 - (1)(\sqrt{3})} \\ &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} = \frac{1 + 2\sqrt{3} + 3}{1 - 3} \\ &= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}\end{aligned}$$

Question 2

$$\begin{aligned}\cos\left(\frac{\pi}{2} - x\right) &= -\sin x \\ \cos\left(\frac{\pi}{2} - x\right) &= \cos\left(\frac{\pi}{2}\right)\cos(x) + \sin\left(\frac{\pi}{2}\right)\sin(x) \\ &= (0)\cos x + (1)\sin x \\ &= \sin x \\ &\neq -\sin x \Rightarrow \text{no}\end{aligned}$$

Question 4

$$\begin{aligned}\cos(15^\circ) &= \cos(45^\circ - 30^\circ) \\ &= \cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ) \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

Question 6

$$\begin{aligned}\sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) \\ &= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

Question 7: Below are four different approaches. There are more.

- $\cos(90^\circ) = \cos(2 \cdot 45^\circ) = \cos^2(45^\circ) - \sin^2(45^\circ) = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} - \frac{1}{2} = 0$
- $\cos(90^\circ) = \cos(2 \cdot 45^\circ) = 2\cos^2(45^\circ) - 1 = 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1 = 2 \cdot \frac{1}{2} - 1 = 0$
- $\cos(90^\circ) = \cos(2 \cdot 45^\circ) = 1 - 2\sin^2(45^\circ) = 1 - 2\left(\frac{\sqrt{2}}{2}\right)^2 = 1 - 2 \cdot \frac{1}{2} = 0$
- $\cos(90^\circ) = \cos(30^\circ + 60^\circ) = \cos(30^\circ)\cos(60^\circ) - \sin(30^\circ)\sin(60^\circ) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 0$

Question 8

$$\sin^2 \theta + \cos^2 \theta = -2 \sin \theta \cos \theta$$

$$1 = -\sin(2\theta)$$

$$-1 = \sin(2\theta)$$

$$\frac{3\pi}{2}, \frac{7\pi}{2} = 2\theta$$

$$\frac{3\pi}{4}, \frac{7\pi}{4} = \theta \Rightarrow G$$

Question 9

$$\sin\left(\frac{\pi}{6} + \theta\right) + \sin\left(\frac{\pi}{6} - \theta\right) = \frac{1}{2}$$

$$\left[\sin\left(\frac{\pi}{6}\right)\cos(\theta) + \sin(\theta)\cos\left(\frac{\pi}{6}\right)\right] + \left[\sin\left(\frac{\pi}{6}\right)\cos(\theta) - \sin(\theta)\cos\left(\frac{\pi}{6}\right)\right] = \frac{1}{2}$$

$$\left(\frac{1}{2}\right)\cos(\theta) + \left(\frac{1}{2}\right)\cos(\theta) = \frac{1}{2}$$

$$\cos(\theta) = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow D$$

Question 10

$$\cos(2\theta) - 2 = 5 \cos \theta$$

$$(2 \cos^2 \theta - 1) - 2 = 5 \cos \theta$$

$$2 \cos^2 \theta - 5 \cos \theta - 3 = 0$$

$$(\cos \theta - 3)(2 \cos \theta + 1) = 0$$

$$\cos \theta - 3 = 0 \text{ and } 2 \cos \theta + 1 = 0$$

$$\cos \theta = 3 \text{ and } \cos \theta = -\frac{1}{2}$$

$$\text{no sol. and } \theta = \frac{2\pi}{3}, \frac{4\pi}{3} \Rightarrow H$$

Question 11

- If $\tan \theta = \frac{15}{8}$, does $\sec \theta = \frac{17}{8}$?

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \left(\frac{15}{8}\right)^2 = \left(\frac{17}{8}\right)^2 ?$$

$$\frac{64}{64} + \frac{225}{64} = \frac{289}{64} ? \text{ yes } \Rightarrow \text{truth}$$

- $\cos(3\theta) = 4 \cos^3 \theta - 3 \cos \theta$

$$\cos(3\theta) = \cos(\theta + 2\theta)$$

$$= \cos(\theta) \cos(2\theta) - \sin(\theta) \sin(2\theta)$$

$$= \cos \theta [2 \cos^2 \theta - 1] - \sin \theta [2 \sin \theta \cos \theta]$$

$$= \cos \theta \cdot (2 \cos^2 \theta - 1 - 2 \sin^2 \theta)$$

$$= \cos \theta \cdot (2 \cos^2 \theta - 1 - 2[1 - \cos^2 \theta])$$

$$= \cos \theta \cdot (2 \cos^2 \theta - 1 - 2 + 2 \cos^2 \theta)$$

$$= \cos \theta \cdot (4 \cos^2 \theta - 3)$$

$$= 4 \cos^3 \theta - 3 \cos \theta \Rightarrow \text{truth}$$

- $\sin(2\theta) = \cos \theta$

$$2 \sin \theta \cos \theta = \cos \theta$$

$$2 \sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta (2 \sin \theta - 1) = 0$$

$$\cos \theta = 0 \text{ and } \sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ and } \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

> 3 solutions \Rightarrow lie