EXIT TICKET SOLUTION KEY

1. Using the idea that the world around us is made of spacetime, explain why even light traveling at 3×10^8m/s cannot escape a black hole.

If there is enough mass at one location of spacetime, it will warp spacetime so much that even something traveling as fast as the speed of light will not be able to overcome the depression in space time. If there is a steep enough depression, then even light will have a decaying orbit and spiral into the center. The event horizon is the name given to the rim around a black hole where once something is inside it not even light is fast enough to get out. The equation below shows that if you know the speed of light and the mass of the black hole you can calculate where this event horizon would be located.

$$a_{Centripetal} = \frac{GM_{Sun}}{r^2} = \frac{v^2}{r}$$

Student responses do not need to mention the event horizon. Response just needs to explain that if there is enough mass to bend spacetime enough, then even light would not be moving fast enough to escape the orbit of the black hole where it could come out and be observed by someone.

2. Show the work for the derivation again and calculate the gravitational force of the sun on Jupiter, the average speed of Jupiter in the orbit around the sun, and the time in seconds that it takes for Jupiter to orbit the sun. Answers will vary from reality since the true orbit is not a perfect circle. M_S is the mass of the sun; M_J is the mass of Jupiter; and r is the distance from the center of mass of the sun to the center of mass of Jupiter.

($M_S = 1.99 \times 10^{30}$kg, $M_J = 1.9 \times 10^{27}$kg, r = 7.8×10^{11}m)

$$F_{g\ Jupiter} = \frac{GM_{Sun}M_{Jupiter}}{r^2} = \frac{(6.67\ x\ 10^{-11} Nm^2/kg^2)(1.99\ x\ 10^{30} kg)(1.9\ x\ 10^{27} kg)}{(7.8\ x\ 10^{11} m)^2} = 4.14\ x\ 10^{23} N$$

$$a_{Jupiter} = a_{centripetal} = \frac{F_{g\ on\ Jupiter}}{M_{Jupiter}} = \frac{4.14\ x\ 10^{23}\ N}{1.9\ x\ 10^{27} kg} = .00022\ m/s^2 = \frac{v^2}{r}$$

$$v = \sqrt{r \cdot a_c} = \sqrt{(7.8\ x\ 10^{11}\ m)(.00022\ m/s^2)} = 13{,}045 m/s = orbital\ speed$$

$$|v| = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{|v|} = \frac{2\pi \cdot (\ 7.8\ x\ 10^{11}\ m)}{|13{,}045\ m/s|} = 375{,}692{,}320 s = T$$