GRAVITY HOLDS SOLAR SYSTEM TOGETHER TEACHER'S GUIDE

Reimagining the fabric around the PVC as the fabric of spacetime

1. What stretches the fabric?

    Mass placed on the fabric depresses the fabric.

One mass alone does not create a gravitational force. One mass creates a gravitational field (depression in spacetime), which you can use to predict how big the force will be when you put a certain mass at a certain location in the gravitational field ($F_g = \frac{GM_1M_2}{r^2}$).

2. If we imagine the mass at the center of the table is the sun (M_{Sun}), then write an equation for the force that the object (M_{Object}) experiences at any location away from the center of the sun (r) because of the depression in spacetime.

$$F_{g\ on\ Object} = \frac{GM_{Sun}M_{Object}}{r^2}$$

Treating r as one value for the whole sun works because the sun has a spherical mass distribution. If the mass was spread out differently the equation would need to take the different distances for different pieces of mass into account.

3. If the object is released from rest, explain the motion of the object, and write the equation for the object's acceleration?

The object accelerates straight towards the sun following the depression in spacetime. The object accelerates more as spacetime is depressed more as the object approaches the sun. So, the speed gets faster and faster at a faster and faster rate.

$$F_{g\ on\ Object} = \frac{GM_{Sun}M_{Object}}{r^2} = M_{Object}\ a_{Object}$$

$$\frac{F_{g\ on\ Object}}{M_{Object}} = \frac{GM_{Sun}}{r^2} = a_{Object}$$

4. If we imagine that the marble is thrown tangentially to the earth, explain the motion of the earth, and write the equation for the earth's acceleration?

The object wants to continue in a straight path, but the force of gravity (bent spacetime) becomes a centripetal force that changes the direction of the earth. The Earth goes into orbit around the sun.

$$F_{g\ on\ Earth} = \frac{GM_{Sun}M_{Earth}}{r^2} = M_{Earth}\ a_{Earth} = M_{Earth}\ a_{Centripetal}$$

$$\frac{F_{g\ on\ Earth}}{M_{Earth}} = \frac{GM_{Sun}}{r^2} = a_{Centripetal} = a_{Earth}$$

5. Since the acceleration of the earth is centripetal, show an equation to find the speed of the earth as during the orbit assuming the orbit is circular.

$$a_{Centripetal} = \frac{GM_{Sun}}{r^2} = \frac{v^2}{r}$$

$$speed = |v| = \sqrt{\frac{GM_{Sun}}{r}}$$

6. Use this speed to find the time that it takes the earth to orbit the sun.

$$speed = |v| = \sqrt{\frac{GM_{Sun}}{r}} = \frac{2\pi r}{T}$$

Square both sides from above: $\frac{GM_{Sun}}{r} = \frac{4\pi^2 r^2}{T^2}$

Isolate for period: $T = 2\pi\sqrt{\frac{r^3}{GM_{Sun}}}$

7. Show the work for the derivation again and calculate the gravitational force of the sun on the earth, the average speed of the earth in the orbit around the sun, and the time in days that it takes for the earth to orbit the sun. Answers will vary from reality since the true orbit is not a perfect circle.

   ($Mass_{Sun} = 1.99 \times 10^{30}$ kg, $Mass_{Earth} = 6 \times 10^{24}$ kg, $r_{Earth\ to\ Sun} = 1.5 \times 10^{11}$ m)

$$F_{g\ Earth} = \frac{GM_{Sun}M_{Earth}}{r^2} = \frac{(6.67\ x\ 10^{-11}Nm^2/kg^2)(1.99\ x\ 10^{30}kg)(6\ x\ 10^{24}kg)}{(1.5\ x\ 10^{11}m)^2} = 3.539\ x\ 10^{22}N$$

$$a_{Earth} = a_{centripetal} = \frac{F_{g\ on\ Earth}}{M_{Earth}} = \frac{3.539\ x\ 10^{22}\ N}{6\ x\ 10^{24}kg} = .0059\ m/s^2 = \frac{v^2}{r}$$

$$v = \sqrt{r \cdot a_c} = \sqrt{(1.5\ x\ 10^{11}\ m)(.0059\ m/s^2)} = 29{,}748.95 m/s = orbital\ speed$$

$$|v| = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{|v|} = \frac{2\pi \cdot (\ 1.5\ x\ 10^{11}\ m)}{|29{,}748.95\ m/s|} = 31{,}681{,}044s = T$$

$$T = 21{,}681{,}044s\ x\ \frac{1\ hr}{3{,}600s}\ x\ \frac{1\ day}{24\ hr} = 366.7\ days = T$$

It is not exactly a year because the orbit is not a perfect circle.

8. Show the work for the derivation again and calculate the gravitational force of the sun on Mercury, the average speed of Mercury in the orbit around the sun, and the time in seconds that it takes for Mercury to orbit the sun. Answers will vary from reality since the true orbit is not a perfect circle.

(Mass$_{Sun}$ = 1.99 x 10^{30} kg, Mass$_{Mercury}$ = 3.28 x 10^{23} kg, r$_{Mercury\ to\ Sun}$ = 5.8 x 10^{10} m)

$$F_{g\ Mercury} = \frac{GM_{Sun}M_{Mercury}}{r^2} = \frac{(6.67\ x\ 10^{-11}Nm^2/kg^2)(1.99\ x\ 10^{30}kg)(3.28\ x\ 10^{23}\ kg)}{(5.8\ x\ 10^{10}m)^2} = 1.29\ x\ 10^{22}N$$

$$a_{Mercury} = a_{centripetal} = \frac{F_{g\ on\ Mercury}}{M_{Mercury}} = \frac{1.29\ x\ 10^{22}\ N}{3.28\ x\ 10^{23}kg} = .039\ m/s^2 = \frac{v^2}{r}$$

$$v = \sqrt{r \cdot a_c} = \sqrt{(5.8\ x\ 10^{10}\ m)(.039\ m/s^2)} = 47{,}838.3m/s = orbital\ speed$$

$$|v| = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{|v|} = \frac{2\pi \cdot (\ 5.8\ x\ 10^{10}\ m)}{|47{,}838.3\ m/s|} = 7{,}617{,}849.8s = T$$

9. Explain why the orbit of Mercury is so much shorter than the orbit of the earth.

Mercury is closer to the sun, which means that spacetime is curved more, and that it has more centripetal acceleration. To maintain its orbit, it needs to have a faster tangential speed, and so the faster speed around the shorter loop means that its period is much shorter than the earth's period.

Claim	The period (time it takes the planet to go around the sun) of Mercury (7,617,849.8s) is much shorter than the period of the Earth (31,681,044s).
Evidence	Mercury (r = 5.8 x 10^{10}m) is closer to the sun than the Earth (1.5 x 10^{11}m). The centripetal acceleration of Mercury (.039 m/s^2) is bigger than the centripetal acceleration of the Earth (.0059 m/s^2). The orbital speed of Mercury (47,838.3m/s) is bigger than the orbital speed of the Earth (29,748.95m/s).
Reasoning	Mercury being closer to the sun and moving with a larger orbital speed than the Earth creates a larger centripetal acceleration for mercury. Spacetime is warped more when the planet is closer to the sun and so the centripetal acceleration is larger. Mercury does have less mass (3.28 x 10^{23} kg) than the Earth (6 x 10^{24} kg), but the mass of the planet cancels in the centripetal acceleration calculation. Distance from the sun is directly proportional to period, and the distance to Mercury is less than the distance to the Earth. Orbital speed is indirectly proportional to period, and the orbital speed of Mercury is greater than the orbital speed of the Earth ($T = \frac{2\pi r}{\lvert v \rvert}$). Both of these factors lead to the period of Mercury being less than the period of the Earth. Mercury's orbital speed must be bigger than the Earth to maintain a stable orbit because spacetime is more warped closer to the sun. So, having a smaller distance to travel in the orbit and a faster speed lead to the shorter time to complete the orbit.