Are We Golden?

What do our skeletons have in common mathematically with nature, Greek statues, the Parthenon, and Leonardo da Vinci's artwork? *Fascinating Fibonaccis: Mystery and Magic in Numbers* (Garland 1987) sheds light on the answer to this question and provides the opportunity for readers to discover other wonderful connections among mathematics, art, and nature. Making connections within mathematics and between mathematics and other content areas is one of the NCTM's goals for students. In my opinion, it is perhaps one of the most important goals. When students begin to make connections in mathematics and also between mathematics and other aspects of their world, they begin to see its beauty and its mystery. It becomes intriguing and meaningful to them, and their learning begins to have personal relevance.

The activity in this article begins with the question posed in the first sentence. It gives students the opportunity to make meaningful connections among mathematics, nature, and art and to develop proportional reasoning and measurement skills. Although I have not engaged my students in an in-depth exploration of da Vinci's artwork or Greek statues, we have explored more common objects and measurements that exhibit the golden ratio to help answer this question. The golden ratio is a special number that is derived mathematically from

$$\frac{1+\sqrt{5}}{2},$$

which produces 1.618033989 . . . , commonly used as the approximation 1.618. The golden ratio, also known as the divine proportion, golden number, or

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Investigations with the Golden Ratio

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golden section, has been of interest to many people through the centuries, as it can readily be found in architecture, art, and everyday items. Two quantities are said to be golden if their sum is to the larger quantity as the larger is to the smaller quantity. Mathematically, that relationship can be represented with the following equation,

$$\frac{a+b}{a}=\frac{a}{b};$$

where $a > b$. For a rectangle, if l is the length of the longer sides and w is the length of the shorter sides, the relationship is expressed as

$$\frac{l+w}{l}=\frac{l}{w}.$$

As a middle school mathematics teacher for several years, I used an exploration of the golden ratio as an activity with students in both algebra and geometry. I have also used it with prospective teachers in an intermediate mathematics methods course. This article shares examples from a group of prospective teachers. With both age groups, this activity and the embedded discussion have typically been met with great enthusiasm and surprise by the mathematical results of the investigations. As students participated in the following activity and the discussion that ensued, they made mathematical connections; discovered relationships among mathematics, nature, and art; conducted linear measurements; and applied proportional thinking through the investigation of the golden ratio.

SETTING THE STAGE

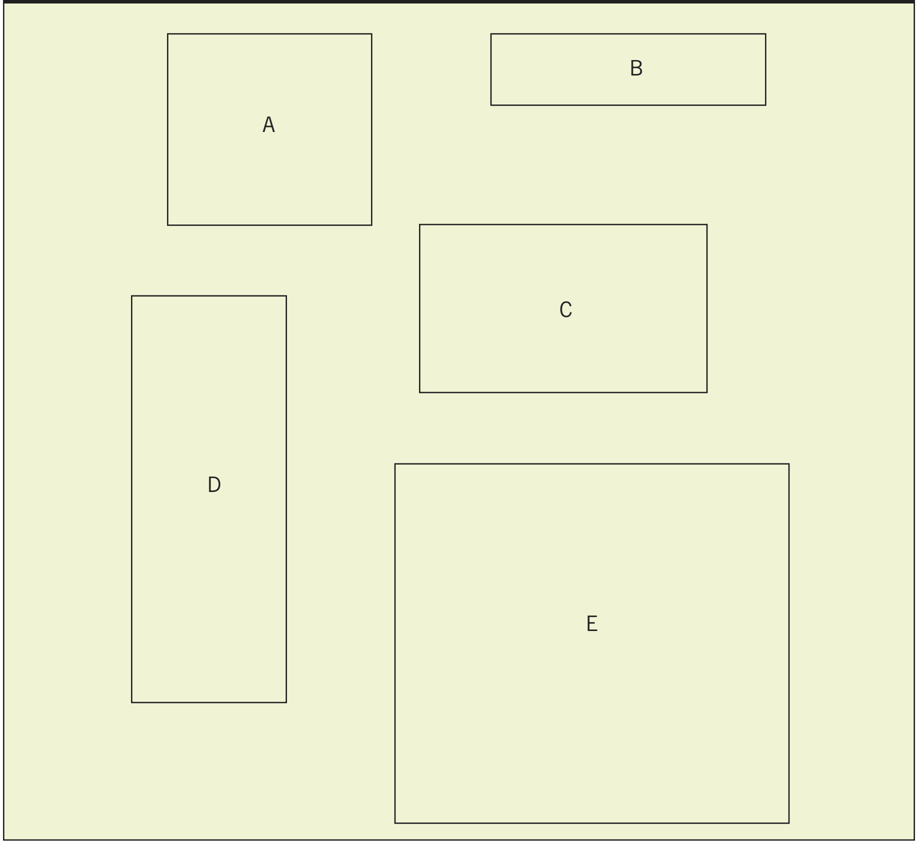
Typically, the students did not have many responses to the question posed at the beginning of this article. After initial discussion, I announced, "We are going to examine a famous picture of a man." I placed a transparency of Leonardo da Vinci's *Vitruvian Man* on the overhead projector, and asked, "Does anyone know the name of this famous drawing?" (See mathforum.org/alejandre/frisbie/math/leonardo.html for the image.) No student knew the name. "Has anyone seen this drawing before? If so, where?" Most students had seen the drawing in a doctor's office, in a university computer or technology department, or in a business office. I then asked, "What do you notice mathematically about the Vitruvian man?"

Students recognized the circle and square, of course, so I followed with questions about whether the circle is inscribed in the square or vice versa. This invoked discussion among the students about what it means for a figure to be inscribed or circumscribed. As the discussion unfolded, I asked students to draw examples on the board to explain their thoughts and ideas. I then continued with the following questions:

- "What are the characteristics of a square?"
- "What are the characteristics of a circle?"
- "What is da Vinci implying about the human body using a square and a circle in this drawing?"
- "If da Vinci's *Vitruvian Man* is as tall as his arm span, is he implying a one-to-one correspondence between a person's height and the length of his or her arm span?"

Although a few students responded that they had heard about the height-to-arm-span relationship, many had not, so we measured a few people and found that this correspondence seemed reasonable. Discussion ensued about other proportional relationships existing between measurements of different body parts. For example, Shelly said, "You know, your foot is supposed to be the same length as your forearm, wrist to elbow." Many students pulled off their shoes and checked. Todd added, "I have also heard that your nose is the same length as your pinky finger." Everyone in class tested this statement, as well. Another student heard that you could shop for jeans without trying them on: If one-half the waist fits around your neck, the jeans will fit around your waist. When this idea was introduced, we put our hands around our necks, then around our waists, then to our necks again, and laughed in disbelief.

Fig. 1 A sample set of rectangles used for the "rectangle pageant"



INTRODUCING THE GOLDEN RATIO

Following our discussion, I conducted what I call a "rectangle pageant." The students were presented with two sets of rectangles. The rectangles in each set were labeled as containing one golden rectangle. Students were asked to look at each set and choose the one rectangle that they believed to be the most beautiful and pleasing (Garland 1987, 1997; Serra 1989). The first set of rectangles (see **fig. 1**) was displayed on the overhead projector. A vote was taken by a show of hands, and a frequency chart was constructed to record the votes. The process was repeated with the second set of rectangles. Although my students have typically chosen the rectangles that exhibit the golden ratio, this particular class chose "taller and skinnier" rectangles. (Someone suggested that the tall and skinny choices resulted from students watching the Ms. America pageant the night before.) Once we completed the "rectangle pageant," we spent several minutes discussing the rectangles around us that we encounter regularly: windows, floor and ceiling tiles, student identification cards, playing cards, columns throughout the classroom, and doors. Based on the rectangles that exhibited the golden ratio from the pageant, we considered which rectangles in our daily life might be golden, as well. We discussed the definition of, and what it means for an object to exhibit, the golden ratio. I explained to the students that psychologists have used similar techniques to determine if people are more inclined to favor rectangles or objects that exhibit the golden ratio. Finally we discussed the belief by many that the golden ratio can be found in many other pieces of artwork beyond the *Vitruvian Man*, such as Greek statuary, the Parthenon, and a variety of common objects. The class looked at pictures of the Parthenon and Greek statues and at playing cards as we talked.

ARE *WE* GOLDEN?—THE ACTIVITY

Are *we* golden? We investigated that question to complete this activity. Students were required to work with a partner to obtain a variety of measurements of various parts of their bodies. Students were given an **activity sheet** (see "Investigate the Golden Ratio") that directed what measurements they should find. This activity sheet was a modified version of information about the golden ratio found in *Discovering Geometry* (Serra 1989). Students were given yarn, adding-machine tape, yardsticks, rulers, and flexible measuring tapes. All students did not have the same measuring device; since they were expected to negotiate how to measure their body parts within their groups, they were not directed how best to attain any given measurement.

Questions arose immediately as students began to measure their overall height. Amy asked, "I must be doing something wrong, because I am not this tall. What am I doing wrong? Do I need to be measuring with something else?" Jon added, "I have measured twice, and I got two different numbers. How can I get this to work?" At this point, the room became quiet as several students tried to hear what others were saying. Although measuring one's height is not difficult, this setting provided a challenge for most of the students because they tried to measure their own height standing in the middle of the room. If they used a yardstick, they did not hold it vertically (parallel to their body) the entire time. If they used a flexible measuring device, they tended to measure the curves of their body, which created inconsistencies in the results. We discussed the curve issue and ways in which students might measure the height more accurately.

I asked the following questions: "What is linear measurement?" "How can we measure a linear distance or height of something that is curved?" The students answered: "We can stand against a wall, mark our height, and then measure the wall" and "We could also lie down on the floor and measure next to our bodies" (i.e., use the measuring tape stretched taut next to their bodies to find the linear distance from the top of their heads to the bottom of their feet). Following this discussion, the students measured their height and collected the following list of data:

- Overall height (labeled B)
- Navel height, navel to floor (labeled N)
- Length of the index finger (labeled F)
- Length of the distance from the fingertip to the big knuckle of the index finger (labeled K)
- Length of the leg (labeled L)
- Length of the distance from the hip to the kneecap (labeled H)
- Length of the arm (labeled A)
- Length of the distance from the fingertip to the elbow (labeled E)
- Length of the distance from the top of the head to the level of the bottom of the chin (labeled X)
- Length of the distance from the bottom of the ear to the level of the bottom of the chin (labeled Y)

Fig. 2 Measurements and ratios computed by students

Name	Express Each Ratio in Both Its Fraction and Decimal Form				
	B/N	F/K	L/H	A/E	X/Y
1. Pauline	65/39 $1.\overline{66}$	2.5/1.5 $1.\overline{66}$	34/17.25 1.97	27/10 2.7	8.25/4.5 $1.8\overline{3}$
2. Kelly	72/44 $1.\overline{63}$	2.75/1.5 $1.8\overline{3}$	36/19 1.89	27/12 2.25	8/4 2.0
3. Rachel	65/38 1.71	2.5/1.5 $1.\overline{66}$	32.5/16 2.03	25.5/11.25 $2.2\overline{6}$	8.5/4.25 1.88
4. Dawn	63.5/38 1.67	2.25/1.5 1.5	32/17 1.88	24/11 $2.\overline{18}$	8/4.5 $1.7\overline{7}$
5. Lindsay	64.75/38 1.7	2.75/1.75 1.57	33/15 2.2	25.5/10.5 2.43	8.5/4.25 1.88

Students worked with their partner to obtain the measurements, recorded the information and added it to their data chart, and resolved any other difficulties and challenges with measurement through discussion.

Once all data were collected and recorded, students were asked to represent the ratios as decimals. Students used calculators to complete the computations. Each group prepared a poster or overhead-projector transparency of their data, ratios, and final decimal representations of the ratios. Each group presented its data to the class and shared some aspect of how the measurements were attained or how difficulties were overcome. (See **fig. 2**.)

As each group presented its data, the numbers 1.61, 1.62, 1.57, and 1.54 were repeatedly heard. Students were shocked, surprised, and dismayed that the results were so similar. Often, the ratio related to the measurement of the profile and the jawline was the number most removed from the golden ratio (1.8 to 2.2). This discrepancy in the data and findings provided a wonderful opportunity to discuss proportions and the effects of same-sized-measurement errors on different ratios. For example, this measuring error affects the ratio of the profile to

jawline quite differently than it would the ratio of the height to the length of the leg. If a student measured the profile as 7.5 inches when it was actually 9 inches and the jawline was 3.5 inches, the ratio would be 2.14, whereas it should have been 9/3.5 ≈ 2.57. This produces a difference of 2.57 – 2.14 = 0.43 because of the initial error of 1.5 inches in measurement. That same 1.5-inch error when measuring height only has a slight effect. If the height is 63 inches, but it was incorrectly measured as 61.5 inches and the length of the leg was 38 inches, an incorrect ratio of height to leg would result in 61.5/38 = 1.62 instead of an expected ratio of 63/38 = 1.66. The same measurement error would result in a difference in the ratio of only 0.04.

The students examined the issue of the effect of same-sized measurement error and the use of appropriate units of measurement in the following discussion and questions. "Why do we think this ratio is so much different than the others we found?" "Are our skulls not golden?" Lisa shared, "It was really challenging to measure the jawline; we were not sure whether to measure the angle of the jawline or measure straight down." Kristina said, "Also, a little mistake on such a small measure really messes things up as compared to making a mistake when measuring the length of our legs. Our group measured in inches and rounded; maybe centimeters would be better, more accurate." Through the experience of hands-on measurement and calculating ratios in this activity, the students came to understand the effects of same-sized-measurement errors on different ratios.

Following the group presentations, the class discussed what they found interesting or surprising. Reflecting on the activity as a whole, Michelle talked about her surprise: "I couldn't believe it because I always thought that I was built strangely, that my arms are too long, but I ended up with really close to the same proportions as everyone in my group, everyone in class for that matter." Sherri said, "When we started this activity, I thought this is really cool because this is all about me, and even though we were working in a group, I was using my measurements. But by the time we finished the activity, I realized that the activity was even more cool because it wasn't about me, it was about all of us."

CONCLUSION

This activity gave students the opportunity to engage in both whole-group discussion and small-group collaboration. While students determined a variety of linear measures related to their bodies and calculated ratios, they were also delightfully surprised that mathematics could be so interesting. Throughout the activity, the students commented that they did not realize that mathematics could be found in nature, art, and even biology. Students grappled with the linear measurement of their bodies and made decisions about what standard measure would provide the most accurate measure. Likewise, they used proportional reasoning to make sense of the varying amount of impact caused by small errors in measurement.

This activity can easily lead to a variety of extensions, including investigations of the Fibonacci sequence and its relevance in nature (see Garland 1987 and 1997 for activities). Once students have explored the Fibonacci sequence in nature, they can examine the relationship between the sequence and the golden ratio. This extension helps students understand the connections between mathematics and other content areas. Technology can also be integrated through the use of spreadsheets (Baugh and Raymond 2003). Students can create a spreadsheet that will calculate their ratios as they enter their measurement data, and all student data can be represented on one sheet.

Throughout my years as a middle school mathematics teacher and currently as I work with teacher candidates in mathematics methods courses, this activity has had an impact on students and their appreciation for mathematics. It also provides an opportunity for them to discover that mathematics is connected to them personally and to their world. As students begin to see mathematics as something more than a set of disconnected algorithms and concepts but rather as something embedded in their lives and their surroundings, it becomes more intriguing, interesting, and personally relevant.

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activity sheet

Name ______________________________

Investigate the Golden Ratio

Are we golden? Is the golden ratio somewhere in each of us? Form groups of four or five and use the table, directions, and skeletal diagrams to determine if you are golden.

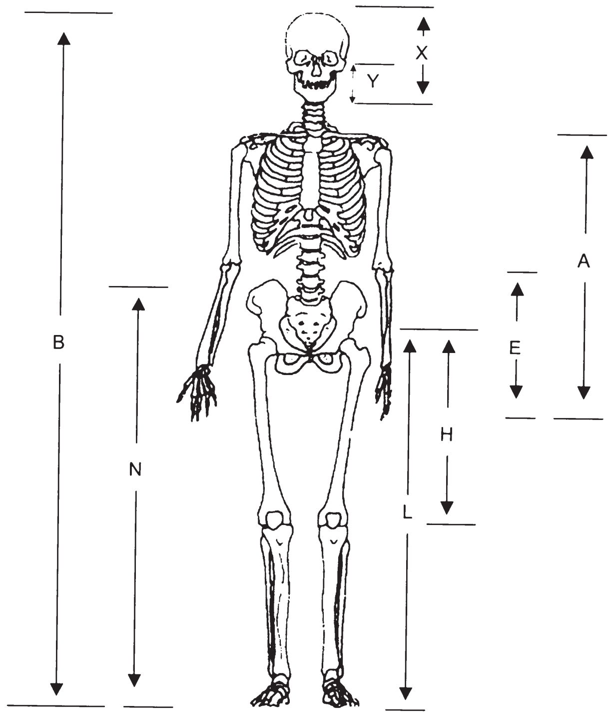
Step 1: Measure the height (B) and the navel height (N) of each member of your group. Calculate the ratio B/N. Record them in your table.

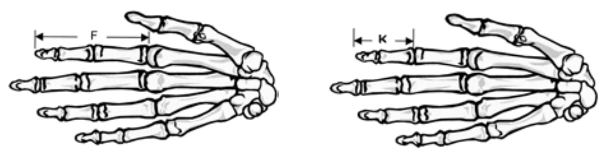
Step 2: Measure the length (F) of an index finger and the distance (K) from the fingertip to the big knuckle of each member of your group. Calculate the ratio F/K. Record them in your table.

Step 3: Measure the length (L) of a leg and the distance (H) from the hip to the kneecap of everyone in your group. Calculate and record the ratio L/H.

Step 4: Measure the length (A) of an arm and the distance (E) from the fingertips to the elbow of everyone in your group. Calculate and record the ratio A/E.

Step 5: Measure the length (X) of a profile (the top of the head to the level of the bottom of the chin) and the length (Y) (the bottom of the ear to the level of the bottom of the chin). Calculate and record the ratio X/Y.





Name	Express Each Ratio in Both Its Fraction and Decimal Form				
	B/N	F/K	L/H	A/E	X/Y
1.					
2.					
3.					
4.					
5.					

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