Are We Golden?

Fibonacci Sequence and Patterns in Nature

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Published by *K20 Center*

Grade Level	6th – 7th Grade	**Time Frame**	2-3 class period(s)
Subject	Mathematics	**Duration**	140 minutes

Essential Question

Are there common mathematical patterns expressed in nature?

Summary

This lesson teaches students about the golden ratio and Fibonacci numbers. The lesson focuses on finding the golden ratio in art, nature, and common objects, as well as in their own skeletal structure. Students take measurements and use calculations to identify examples of the golden ratio both inside and outside the classroom.

Snapshot

Engage

Students examine various images that seemingly have nothing in common, but upon further investigation, patterns they all share are revealed.

Explore

Students discuss general size relationships in humans, then they measure each other looking for these relationships in their own bodies.

Explain

Students analyze the measurement data collected in the Explore phase, looking for patterns and relationships.

Extend

Students search for other common objects in the classroom that exhibit the golden ratio.

Evaluate

Students find three objects in their homes that exhibit the golden ratio. They take pictures or draw sketches of these objects.

Standards

Oklahoma Academic Standards for Mathematics (Grade 6)

6.N.3.3: Apply the relationship between ratios, equivalent fractions and percents to solve problems in various contexts, including those involving mixture and concentrations.

Oklahoma Academic Standards for Mathematics (Grade 6)

7.A.1: Understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.

Attachments

- Are We Golden Teacher Slides—Are We Golden - Spanish.pptx
- Are We Golden Teacher Slides—Are We Golden.pptx
- Article—Are We Golden.pdf
- Student Data Collection Excel File—Are We Golden - Spanish.xlsx
- Student Data Collection Excel File—Are We Golden.xlsx
- Student Data Collection Handout—Are We Golden.docx
- Student Data Collection Handout—Are We Golden.pdf
- Teacher Data Collection Excel File—Are We Golden - Spanish.xlsx
- Teacher Data Collection Excel File—Are We Golden.xlsx

Materials

- "Are We Golden?" teacher slides (attached)
- Student Data Collection Excel spreadsheet (attached)
- Student Data Collection PDF handout (attached)
- Teacher data collection spreadsheet (attached)
- Variety of measurement tools: rulers, meter sticks, tape measures, etc.
- Rectangular objects to be measured, such as playing cards, cereal boxes, plasticID cards (the same shape and size as a credit card), or business cards. Make sure to also include objects that do not exhibit the golden ratio.

Engage

Teacher's Note: Concept Overview

Before starting this lesson, it is recommended that you read the attached article "Are We Golden?" The article serves as notes to go along with the attached teacher slideshow. The golden ratio is a proportion between components of a structure, organism, artwork, etc., that is often found in nature and art. Typically, people find this ratio to be visually pleasing, even if it is hard to explain why. It can be expressed as a fraction by (1+ v5) / (2) or as a decimal with 1.618. In this lesson, students will look for the golden ratio in their own bodies and common objects.

Introduce the lesson and present the attached teacher slideshow. Show students a variety of images, beginning with DaVinci's "The Mona Lisa" and "Vitruvian Man" on slide four. Ask them to discuss what they believe the images have in common. Be sure to note students' responses as you progress through each set of images on **slides 5-6**.

Next, transition to **slide 7** and conduct the "Rectangle Pageant." There are four rectangles shown (A, B, C, and D). Ask students to vote on the rectangle they believe is the most pleasing to their eye.

Teacher's Note: Tallying Votes

Slide 8 contains a template for tallying student votes for the most pleasing rectangle. The rectangle exhibiting the golden ratio is rectangle B. Once the voting is complete, students typically will have voted rectangle B as the most visually pleasing. If rectangle B doesn't win, you can talk about how psychologists have conducted this type of test as part of a larger study and that most groups of people typically select the rectangle that exhibits the golden ratio. The rectangle can be measured to demonstrate and show the golden ratio as a fraction and then as a decimal value. To show the golden ratio based on measurements, measure both sides of the rectangle and plug the measurements into this formula: (l / w). The result should give you about 1.6, the golden ratio.

Show students the definition of the golden ratio on **slide 9**. Offer your own explanation, then show students **slides 10–12**, which show the same images they saw earlier but with diagrams and overlays illustrating the golden ratio. Discuss these items and their relationship to the golden ratio.

Next, discuss how this ratio not only can be found and has been purposely included in man-made objects but that it also shows up in nature. Transition to **slide 13** and show the images from nature with diagrams indicating the golden ratio.

Teacher's Note: Introducing Fibonacci Numbers And The Golden Spiral

In the teacher slides, you will notice there are several tiled squares/rectangles with a spiral drawn in them. Each rectangle is an example of the golden ratio. Within the largest rectangle, a series of smaller rectangles and squares can be seen. All the rectangles exhibit the golden ratio and the spiral is said to be a "golden spiral" because it gets wider by a factor of 1.6 (the golden number) for every quarter turn it makes. The spiral is based on the Fibonacci sequence, named for the Italian mathematician who introduced this concept to the Western world in the early 13th century. The Fibonacci sequence is the series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on. In the sequence, the next number is always found by adding up the two numbers before it. If you take any two successive (one after the other) Fibonacci Numbers, their ratio is very close to the golden ratio: e.g. 3/2=1.5 or 13/8=1.625. When inscribed on a sequence of golden rectangles, you can see the relationship between a golden spiral and a series of golden rectangles. You'll notice that the arc of the spiral passes through the corners of the squares.

Explore

Prior to allowing students to begin the Explore portion of the lesson, lead a whole-class discussion asking the following questions:

- *Are all of you aware that the length of your foot is relatively the same length as your forearm?* (You can demonstrate this by removing your shoe and holding it up against your arm between your elbow joint and your wrist.)
- *Do any of you know any other comparisons like these?*

Teacher's Note: Possible Responses

The following are some possible responses that demonstrate physical examples of the golden ratio: "The length of our arm span is the same as our height," "The distance around one's neck is half the measurement of his/her waist," (though this really is only true when we are younger), and "The length of your pinkie finger is the same length as the bridge of your nose."

Share responses and allow students to verify them if needed. Next, tell them that today's activity will involve examining measurements of their bodies that express another relationship.

Teacher's Note: Body Measurements

Make sure students understand that the activity will only include measuring bone lengths and not other body parts, such as the hips or the waist. This will help reassure students who have body image concerns.

Have students select a partner with whom they will feel comfortable measuring and being measured.

Students will work in groups of two and in teams of two groups (so a total of four students per team). Students have two options for recording measurements and ratios. If the technology is available, they can record them on the attached **Student Data Collection Excel File** (this file can also be opened and used in Google Sheets if preferred). If you prefer for students to record the data manually, you will also find PDF and Word versions attached. Each pair of students should select the measuring tools they think they will need to accomplish the required measurements.

Have students complete all required measurements with their partner and record them on the team's Student Data Collection Excel File, either in digital or paper form. Measurements should be recorded in centimeters and each ratio of measurements should be expressed as a decimal.

Teacher's Note: Facilitating The Activity

More detailed instructions on each measurement can be found for your reference on the last page of the attached article, "Are We Golden?" During the measurement activity, move around the room and answer questions about specific measurements. It is not uncommon for students to have difficulty measuring their height. They often do not get a linear measurement of their height because they do not stand up against a wall or lie on the floor to complete this measurement. You may want to pull the class together and stop all measurement activities to discuss how height is best measured. Also, discuss with the class as a whole where the "hip height" measurement should be taken.

Explain

Once all groups have completed their measurements, recorded all ratios, and expressed them as a decimal, have students use a calculator to find the average of each of the columns on their worksheet. Have one member from each team report the averages for each column while you collect the data for the class.

Teacher's Note: Working With Spreadsheets

If Excel or Google Sheets are available, it is a good idea to record the averages from each team so a classroom average can be quickly and easily calculated. A **Teacher Data Collection Excel** template is attached with formulas for averages in the correct cells. It was made for a classroom with six groups, but it can be altered to fit the needs for any class size. If Excel cannot be used, then assign one column to each team to find the average. The decimal expression of the ratio of measurements should be close to the golden ratio.

During this time, monitor and manage the reporting of the averages either on the board or in a spreadsheet. Once reporting is complete, initiate and engage a whole-class discussion regarding the findings.

Teacher's Note: Questions To Drive Discussion

Ask students questions such as the following to drive the whole-class discussion: "Were you surprised by your findings?", "Does it seem plausible that the golden ratio shows up in the measurements of people of different heights, for example, someone in our class that is 5-feet-1 *and* someone whois 5-feet-8?", "Why do you think that is possible?", "Do you think the same would be true if we were to measure 10 four-year-olds?", "What if we were to measure teenagers in another country, say Japan, where theaverage height is a few inches below the average height of Americans?"

This discussion should revolve around the idea that bones demonstrate linear growth and that the skeletal system overall grows at the same proportional rate. For example, our right arm does not decide to grow for a few years while everything else remains the same. Our skeleton grows at a continual rate. While the rate is not constant throughout our life (e.g., we have growth spurts and periods of slower growth), our body's growth is proportional. Finally, ask students to reflect on the Engage phase of this lesson: Ask students, "Do you think the fact that our own bodies reflect the golden ratio influences what we determine is beautiful or pleasing?" Refer to the rectangle from the rectangle pageant. Be explicit in pointing out that many of the relationships in their bodies have the same ratio as the rectangle they all thought was visually appealing at the beginning of this lesson. It is also the same ratio as many of the pictures of art and architecture they looked at in the slides.

Extend

Students should rejoin their original partners and work together to measure and identify at least three rectangular objects in the room that exhibit the golden ratio. Encourage students to identify objects that have not already been demonstrated, such as smartphones. The rectangular objects do not have to be movable. They could be cabinets, tiles, a window, a door, or whatever they see in the room they think exhibits the golden ratio.

In their notebooks, have students record the objects they identified and their measurements, along with an explanation of whether or not they exhibited the golden ratio.

Teacher's Note: Finding Objects In The Classroom

The students should measure a variety of items to find three that exhibit the golden ratio. You can facilitate this activity by reminding students to try to carefully select the objects they measure, based on their sense of the golden ratio at this point. Hopefully, they can begin to "see" the golden ratio and notice it in objects even before measuring them.

Optional Activity: Other Measurements

If you have the ability to let students into the hallway or outside, see if students can find objects around the school building or in nature that exhibit the golden ratio.

Evaluate

Ask students to locate three objects at home that they believe exhibit the golden ratio and photograph them with a digital camera or smartphone. If access to a digital camera is not available, encourage students to sketch and describe what they found. They should measure each item and record the measurements in order to confirm that the object is an example of the golden ratio. Have students email you their photos or turn in their sketches. Be sure they include the measurements in the body of the email or written on the sketches.

Optional Activity: GeoGebra

If you have access to the math app, GeoGebra, encourage students to upload their photos and use the tools to make their measurements digitally.

Finally, have students use a Two-Minute Paper strategy using the following prompt: Explain to a friend who was not in our class today what the golden ratio and share a few examples of where it can be found in nature and in man-made objects.

Teacher's Note: Completing The Assessment

The teacher may give students more than two minutes to finish their responses to the prompt, but the responses should be collected at the end of the class period.

Resources

- Free math apps—used by over 100 Million Students & Teachers Worldwide. (n.d.). GeoGebra. https://www.geogebra.org/?lang=en
- K20 Center. (n.d.). Two-minute paper. Strategies. https://learn.k20center.ou.edu/strategy/d9908066f654727934df7bf4f506cf73
- Reeder, S. (2007, October). Are we golden? Mathematics teaching in the middle school. 3(13), 150-155. http://www.shastacoe.org/uploaded/scmp2/are_we_golden_copy_x.pdf
- TheBITK. (2013, April 05). Nature by Numbers - By Cristobal Vila - A display of the Fibonacci Sequence/Spiral. https://www.youtube.com/watch?v=BaXjWXXwQTk
- K20 Center. (n.d.). GeoGebra. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2352