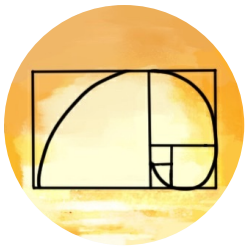

Are We Golden?

Investigating the Golden Ratio

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Grade Level	6th – 7th Grade	**Time Frame**	3 class periods
Subject	Mathematics	**Duration**	105–115 minutes
Course	Middle School Mathematics		

Essential Question

Are there common mathematical patterns around us?

Summary

In this lesson, students investigate the golden ratio by exploring the appeal of and interest in the golden rectangle. Students take a variety of measurements and use calculations to identify examples of the golden ratio and golden rectangle in many contexts throughout the lesson. Students identify the golden ratio in art then measure common objects and their own skeletal structure to determine where else the golden ratio is present.

Snapshot

Engage 1

Students examine similarities between paintings.

Explore 1

Students contemplate if there exists a rectangle that is most visually appealing and vote on a series of rectangles.

Explain 1

Students deepen their understanding of the golden ratio and golden rectangles.

Engage 2

Students discuss general size relationships in humans and which relationships may represent the golden ratio.

Explore 2

Students measure their bone lengths and record these measurements.

Explain 2

Students share approaches for estimating quotients, then use an approach to calculate ratios of their bone-length data.

Extend

Students identify common classroom objects that exhibit the golden ratio by taking objects' measurements and calculating proportions.

Evaluate

Students summarize their learning by writing their own explanation of the golden ratio.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

N 201: Perform one-operation computation with whole numbers and decimals

Attachments

- Class Average—Are We Golden.xlsx
- Data Collection—Are We Golden - Spanish.docx
- Data Collection—Are We Golden - Spanish.pdf
- Data Collection—Are We Golden.docx
- Data Collection—Are We Golden.pdf
- Journal Article—Are We Golden.pdf
- Lesson Slides—Are We Golden.pptx
- Ratio Calculations—Are We Golden - Spanish.docx
- Ratio Calculations—Are We Golden - Spanish.pdf
- Ratio Calculations—Are We Golden.docx
- Ratio Calculations—Are We Golden.pdf
- Strategy Harvest—Are We Golden - Spanish.docx
- Strategy Harvest—Are We Golden - Spanish.pdf
- Strategy Harvest—Are We Golden.docx
- Strategy Harvest—Are We Golden.pdf

Materials

- Lesson Slides (attached)
- Journal Article document (attached)
- Data Collection handout (attached; one per student)
- Strategy Harvest handout (attached; one per student)
- Ratio Calculations handout (attached; one per student)
- Class Average spreadsheet (attached)
- Pencil
- Paper
- Variety of measurement tools (e.g., rulers, meter sticks, tape measures, etc.)
- Rectangular objects to be measured (e.g., playing cards, cereal boxes, plastic ID cards, business cards, shoe boxes, etc.)

5 minutes

Engage 1

Teacher's Note: Concept Overview

Before starting the lesson, read the attached **Journal Article**. The article, “Are We Golden?” serves as notes to accompany the attached **Lesson Slides**.

The golden ratio is an irrational number that is often estimated as a proportion of components of a structure, organism, artwork, etc., that is often sought out in nature and art. Some find this ratio to be visually pleasing, even if they can’t explain why. This ratio can be expressed as the fraction (1+ √5) / (2) or approximated as a decimal as 1.618.

In this lesson, students look for the golden ratio in their own bodies and common objects.

Introduce the lesson using the attached **Lesson Slides**. Transition through **slide 3–4** to share the lesson’s essential question and learning objectives. Review these to the extent you deem necessary.

Move to **slide 5**, which shows DaVinci’s *Mona Lisa* and *Lady with an Ermine*. Ask students to share what they believe the images have in common. Use these paintings to capture curiosity and engage students. These paintings will be investigated further later in the lesson.

5 minutes

Explore 1

Show **slide 6** and conduct the "Rectangle Pageant." Present students with the four rectangles on the slide (A, B, C, and D) and ask students to vote on the rectangle that they find most visually pleasing. Consider tallying student votes on the slide, either below or beside the corresponding rectangle.

Ask students if they think there is a universally correct answer for which rectangle is most visually pleasing. If necessary, rephrase the question to ask students if another class of students was shown the same slide and asked the same question, would they crown the same rectangle as the winner of the rectangle pageant? Lead a brief class discussion on the topic.

Share with students that psychologists often wonder if different people find the same things visually appealing, and mathematicians wonder if there is a mathematical reason behind things being visually appealing.

Tell students that rectangle B is an example of a golden rectangle, which is the rectangle that psychologists and mathematicians both reference in search of that connection. Let students know that they are going to learn more about the golden rectangle during the next phase of the lesson.

5 minutes

Explain 1

Display **slide 7** and share with students the definition of the golden ratio. Transition through **slides 8–10** and elaborate on the golden ratio and the golden rectangle. Offer additional explanations as needed to help students understand the concept.

Show **slide 11** and draw students' attention to the paintings on the slide. These paintings are the same ones presented earlier in this lesson, but now with overlays of the golden rectangle to help students understand where people see the golden ratio in the images. Facilitate a brief discussion about the proportions of the paintings. Share with students that many people will often see what they are looking for, so if someone looks for the golden ratio or golden rectangle within a painting, they can choose different parts to compare in order to find that ratio. However, even if a person finds the golden ratio or golden rectangle, it does not mean that the original artist used the golden ratio to create the work.

Transition through **slides 12–13** and let students know that the painting on the slides, Salvador Dali's *The Sacrament of the Last Supper*, was purposefully designed using the golden ratio. We know this because he left behind written records that describe his intentions for the piece.

5 minutes

Engage 2

Display **slide 14** and lead a brief, whole-class discussion. Ask students, “Are you aware that the length of your foot is relatively the same length as your forearm?” You can demonstrate this by removing your shoe and holding it up against your forearm (the length of your arm between your elbow joint and wrist).

Follow that question by asking students, “Do you know of any other comparisons like this?” Invite them to share out any responses.

Sample Student Responses

- The length of our arm span is the same as our height.
- The distance around your neck is half the distance around your waist. (**Although this is really only true for younger people.*)
- The length of your pinkie finger is the same length as the bridge of your nose.

As students share, have them estimate their claims to check which claims are reasonable. For example, one can quickly confirm that the length of one’s index finger is not reasonably the same length as one’s ear.

Tell the class that the next activity involves testing some of these claims by measuring different body dimensions and finding different proportions.

Teacher's Note: Body Measurements

Ensure that students understand that this activity involves measuring bone lengths and not other body parts, such as the hips or the waist. This will help reassure students who may have body image concerns.

20 minutes

Explore 2

Have students find a partner who they will feel comfortable measuring and who will feel comfortable measuring them. Display **slide 15** and give each pair one copy of the attached **Data Collection** handout. Tell students that they are expected to measure in centimeters. Have pairs select the measuring tools they think they need to accomplish the required measurements.

Show **slide 16** and have pairs work together to measure the indicated body dimensions. Have them record those values in the table on their handouts.

Teacher's Note: Facilitating the Activity

If students are unsure of how to use a ruler or other measuring tools, demonstrate how to use the tools to individual groups, or the whole class if needed.

During this activity, move around the room and answer questions about specific measurements. Students are likely to have difficulty measuring their height. They may not get a proper linear measurement of their height because they do not stand against a wall or lie on the floor to complete this measure. Consider pausing measurement activities and leading a whole class discussion about the procedure for measuring height. Consider also discussing how students should take the hip height measurement.

40 minutes

Explain 2

Display **slide 17** and explain to students that calculators only do exactly what humans ask them to do. Share with students that if humans type the problem incorrectly, they will get a correct answer for the wrong problem. Facilitate a brief discussion with students about their experience with calculators. Ask students if they have ever accidentally typed the wrong thing into a calculator. Ask how they can know if the answer on the calculator screen is correct. Invite several students to share out. As they share, emphasize the importance of relying on number sense to determine if the answer is reasonable.

Move to **slide 18** and give each student one copy of the attached **Strategy Harvest** handout. Read the following scenario on the slide, "A newborn baby is 20 inches long, and her legs are each 6 inches long. How could you estimate the ratio of the baby's body length to leg length?" Ask students to consider how they would find that estimate then write their answer in the "My Strategy" portion of their handout. Give students approximately three minutes to complete this task.

Introduce the Strategy Harvest instructional strategy and display **slide 19**. Have students find a partner and share the strategies they invented for estimating the ratio. Tell students to record their partner's strategy and name in the "Partner 1's Strategy" portion of the handout. Have students actively listen and encourage them to participate in a discussion by asking questions and offering feedback about their partner's strategy. Allow approximately three minutes to complete this activity.

Have students repeat this process two more times. Each time they should find a new partner and record their partner's strategy in the next space on their handout. After students complete their handouts, bring the whole class together and invite volunteers to share out one of their partner's strategies.

Give each pair one copy of the attached **Ratio Calculations** handout and a calculator. Display **slide 20** and have students use the measurements on their Data Collection handouts to find the ratios on their Ratio Calculations handouts. Have students record their values as fractions (for easy reference of which two numbers they used) and as a decimal approximation. Have students round their decimals to three decimal places.

Teacher's Note: Guiding the Activity

As students work, monitor their progress. If necessary, bring the class together and present another example. Ask students to consider if the newborn baby was 20.4 inches long with legs that were 5.8 inches long. Ask students if they would change their approach to estimating the ratio or body length to leg length. Remind them that estimating quotients before using a calculator will help them check if their ratio is reasonable.

Once students have sufficient understanding of the task and have completed at least half of their tables, transition to **slide 21**. Notify students that they should refer to this slide when they are ready to find the average ratio in the final row of their tables.

Show **slide 22** and have one member from each team report the averages for each student. As they share out, collect this data for the whole class in the attached **Class Average** spreadsheet.

Teacher's Note: Working with Spreadsheets

If you have access to Excel or Google Sheets, consider recording the averages from each pair so a classroom average can be quickly and easily calculated. The top row of the attached Class Average spreadsheet will calculate the class average ratio as you enter values into each row. There is no need to edit the top row, unless you have 40 or more students in a class.

Initiate a whole-class discussion about students' findings. Ask the class to determine who the "Golden Genius" is, which would be the student whose ratio is closest to the golden ratio.

Teacher's Note: Questions to Drive Discussion

Guide the whole-class discussion using the following questions:

- Were you surprised by your findings?
- Does it seem plausible that the golden ratio appears in the measurements of people of different heights? For example, someone in our class that is 5 feet and 1 inch tall *and* someone who 5 feet and 8 inches tall?
    - Why do you think that is possible?
- Do you think the golden ratio would appear in measurements of people of different heights if we were to measure 10 four-year-old children?
- Do you think the golden ratio would appear in measurements of teenagers in a different country, like Japan, where the average height is a few inches below the average height of Americans?
- When we measured our own bodies at the beginning, we saw that they had similar proportions to the golden ratio. Do you think that influences what we find beautiful?

20 minutes

Extend

Show **slide 23** and have students rejoin their original partners. Have them work together to measure and identify at least three rectangular objects in the room that they think exhibit the golden ratio. Notify students that the rectangular objects do not have to be movable, and may include cabinets, tiles, windows, doors, or any other object they believe exhibits the golden ratio.

Have students record their chosen objects and their measurements on a piece of notebook paper, along with an explanation of whether or not the objects exhibited the golden ratio.

Once students are done measuring and calculating their proportions, bring the class together and have students share out which objects they classified as golden rectangles.

Teacher's Note: Finding Objects

Remind students to carefully select objects based on their knowledge of the golden ratio. Ideally, they will be able to "see" the golden ratio and notice it in objects before measuring them.

If you have the ability to let students into the hallway or outside, encourage them to find objects around the school building or outdoors that exhibit the golden ratio.

Optional Alternative or Additional Activity

Ask students to locate three objects outside of school that they believe exhibit the golden ratio. Have them photograph or sketch each object. Have them then measure each item and record the measurements to confirm if the objects are examples of the golden ratio. Encourage students to record their measurements by editing their photographs or labeling their sketches.

Have them submit their findings by email, through your school's Learning Management System (LMS), or by turning in their sketches.

Optional Technology Integration

If students do not have measuring tools at home, consider having them upload their image to a tech tool like GeoGebra and have them use the tool to take digital measurements.

5 minutes

Evaluate

Display **slide 24** and introduce the Two-Minute Paper instructional strategy. Ask students to respond to the question on the slide. Have them explain the golden ratio to someone not in their class and have them provide an example of the ratio. Start the 2-minute timer on the slide.

Teacher's Note: Completing the Assessment

You may need to give your students more than two minutes to finish their responses to the prompt, but ensure that you collect students' responses at the end of the class period.

Resources

- Dalí, S. (1955). *The sacrament of the Last Supper* [Painting]. National Gallery of Art, Washington D.C., United States. https://www.nga.gov/collection/art-object-page.46590.html
- Da Vinci, L. (1490). *Lady with an ermine* [Painting]. Czartoryski Museum, Kraków, Poland. https://commons.wikimedia.org/wiki/File:The_Lady_with_an_Ermine.jpg
- Da Vinci, L. (1506). *Mona Lisa* [Painting]. Musée du Louvre, Paris, France. https://commons.wikimedia.org/wiki/File:France-003324_-_Mona_Lisa_(16236519171).jpg
- K20 Center. (n.d.). GeoGebra. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2352
- K20 Center. (n.d.). Google sheets. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2855
- K20 Center. (n.d.). Strategy harvest. Strategies. https://learn.k20center.ou.edu/strategy/135
- K20 Center. (n.d.). Two-minute paper. Strategies. https://learn.k20center.ou.edu/strategy/152
- K20 Center. (2021, September 21). *K20 Center 2 minute timer* [Video]. YouTube. https://youtu.be/HcEEAnwOt2c?si=IjQ0Z14eT1CEtWwS
- Reeder, S. (2007, October). Are we golden?. *Mathematics Teaching in the Middle School, 3*(13), 150–155. http://www.shastacoe.org/uploaded/scmp2/are_we_golden_copy_x.pdf