

The Sound of Polynomials

Adding and Subtracting Polynomials

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Grade Level	8th – 11th Grade	**Time Frame**	135–150 minutes
Subject	Mathematics	**Duration**	3–4 class periods
Course	Algebra 1, Algebra 2		

Essential Question

How can you use addition and subtraction to simplify polynomial expressions?

Summary

In this lesson, students will build on their knowledge of combining like terms and investigate adding and subtracting polynomial expressions, representing them visually using math manipulatives and algebraically using both horizontal and vertical methods. Students will watch a video of an education specialist from Ableton, who was a former tour musician, sharing the connection between polynomials, specifically Chebyshev polynomials, and sound. Students will then practice adding and subtracting polynomials and use a web tool to hear how their results sound.

Snapshot

Engage

Students make the connection between grouping like items and simplifying polynomial expressions.

Explore

Students use digital math manipulatives to explore modeling, adding, and subtracting polynomial expressions.

Explain

Students complete a Scavenger Hunt Notes activity to take notes on using addition and subtraction to simplify polynomial expressions.

Extend

Students watch a career-focused video to learn how polynomials relate to sound and then apply their knowledge to simplify polynomial expressions and listen to their results using a web tool.

Evaluate

Students demonstrate what they have learned about adding and subtracting polynomials in an Exit Ticket.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

A505: Add, subtract, and multiply polynomials

Oklahoma Academic Standards Mathematics (Algebra 1)

A1.A.3.2: Simplify polynomial expressions by adding, subtracting, or multiplying.

Oklahoma Academic Standards Mathematics (Algebra 1)

A2.A.2.2: Add, subtract, multiply, divide, and simplify polynomial expressions.

Attachments

- Clues—The Sound of Polynomials - Spanish.docx
- Clues—The Sound of Polynomials - Spanish.pdf
- Clues—The Sound of Polynomials.docx
- Clues—The Sound of Polynomials.pdf
- Exit Ticket—The Sound of Polynomials - Spanish.docx
- Exit Ticket—The Sound of Polynomials - Spanish.pdf
- Exit Ticket—The Sound of Polynomials.docx
- Exit Ticket—The Sound of Polynomials.pdf
- Learning Synths Playground—The Sound of Polynomials - Spanish.docx
- Learning Synths Playground—The Sound of Polynomials - Spanish.pdf
- Learning Synths Playground—The Sound of Polynomials.docx
- Learning Synths Playground—The Sound of Polynomials.pdf
- Lesson Slides—The Sound of Polynomials.pptx
- Note Catcher (Teacher Guide)—The Sound of Polynomials.docx
- Note Catcher (Teacher Guide)—The Sound of Polynomials.pdf
- Note Catcher—The Sound of Polynomials - Spanish.docx
- Note Catcher—The Sound of Polynomials - Spanish.pdf
- Note Catcher—The Sound of Polynomials.docx
- Note Catcher—The Sound of Polynomials.pdf
- Polynomials and Algebra Tiles—The Sound of Polynomials - Spanish.docx
- Polynomials and Algebra Tiles—The Sound of Polynomials - Spanish.pdf
- Polynomials and Algebra Tiles—The Sound of Polynomials.docx
- Polynomials and Algebra Tiles—The Sound of Polynomials.pdf
- Polynomials and Sound—The Sound of Polynomials - Spanish.docx
- Polynomials and Sound—The Sound of Polynomials - Spanish.pdf
- Polynomials and Sound—The Sound of Polynomials.docx
- Polynomials and Sound—The Sound of Polynomials.pdf

Materials

- Lesson Slides (attached)
- Polynomials and Algebra Tiles handout (attached; one per student; print two-sided)
- Clues handout (attached; one per group; print one-sided)
- Note Catcher handout (attached; one per student; print one-sided)
- Note Catcher (Teacher's Guide) document (attached)
- Polynomials and Sound handout (attached; one per student; print two-sided)
- Learning Synths Playground handout (optional; attached; one per student; print one-sided)
- Exit Ticket handout (attached; one half page per student; print one-sided)
- Pencils
- Notebook paper

- Algebra tiles (optional)
- Green cups (1 per pair)
- Yellow cups (1 per pair)
- Red cups (1 per pair)
- Earbuds (optional; one per student)
- Student devices with internet access
- Colored pencils
- Markers
- Clipboards (one per student)

10 minutes

Preparation

During the Explain phase of the lesson, which would begin on Day 2 if you have traditional 45-minute class periods, students are given clues to take notes. As students follow each clue, they are expected to scan the QR code or navigate to the URL to watch videos. Consider earbuds for students who are working independently to help with the background noise.

Print the attached **Clues** handout. The file has two pages, where the first page contains the clues, and the second page contains the citations for videos. If trying to save paper, consider only printing the first page, as the students will not need the last page.

10 minutes

Engage

Present the lesson using the attached **Lesson Slides**.

Display **slide 3** and play the *MADtv Fast Food Ordering* video, which is a video about ordering at a fast-food restaurant.

Embedded video
https://youtube.com/watch?v=M2T7Z8PwESY

Teacher's Note: Video

It is recommended that you stop the video at 4:01. After this point, the customer insults the waitress.

Once the students have watched the video, display **slide 4** and ask students to figure out an easier way the customer could have communicated his order.

Sample Student Responses

- He could have grouped liked items together.
- He could have written his order down.

If students do not mention grouping things that have been ordered, guide them to the possible answer of grouping like items.

Transition through **slides 5–6** to introduce the essential question and the learning objective to connect the idea of grouping like items to simplifying polynomials.

30 minutes

Explore

Display **slide 7** and inform students they will be using online math manipulatives to answer questions about the given polynomials. Have students navigate to k20.ou.edu/AlgebraTiles to access the Mathsbot dienes blocks, also known as base-ten blocks. Tell students that they will be using this resource to represent the terms of their polynomial expressions.

If this is your students' first time using this tool, show **slide 8** and ask them to model: $x^2 - 3x + x$. Use the information below to help your students become familiar with how they are expected to use the tool:

- To create a negative block, select the "Negate" button at the top of the toolbar (after selecting the block they want to make negative). The resulting block will turn red.
- Students can combine like terms by dragging and dropping a negative (red) block on top of the positive block. Let them know that this action will cause the blocks to disappear or "cancel each other out."
- To get rid of an unwanted block, have them select that block and then select the "Delete" button at the top of the toolbar.
- To clear their screen, they can select the refresh icon in their web browser or select the "Tidy" button at the top of the toolbar.
- Advise them that the blocks on the screen are very large and can be made smaller by using the "Zoom out" button (which looks like a magnifying glass with a negative symbol) in the top-right corner of the toolbar.

Use the "Exchange" and "Group" buttons when using these blocks as base-ten blocks. Similarly, use the "Rotate" button to model multiplication as area.

Consider modeling these steps for the class and have the students imitate your actions at the same time. Note that this modeling will give students the opportunity to practice using the tool they will use for the activity.

Alternative Activity Options

You can have the students use algebra tiles instead of online manipulatives. If you choose algebra tiles and the students are unfamiliar with algebra tiles, it is highly recommended that you use this Introducing Algebra Tiles to Students activity to help them explore and understand how algebra tiles are used.

An alternative technology option is GeoGebra. You can provide students with the link to the "3D Algebra Tiles Illustrator" activity: geogebra.org/m/usVjhq2T.

Display **slide 9** and introduce the Try It, Talk It, Color It, Check It strategy. Let students know that as they work on the problem, they will first try it on their own, then discuss it with a partner. Then they choose a colored cup to represent their confidence in efficiently summarizing how many blocks of each kind they have all together. Explain to students that they will have three colored cups that represent the following confidence levels:

- Green: They can summarize their thought processes.
- Yellow: They are a bit uncertain about how to summarize their thought processes.
- Red: They need help summarizing their thought processes.

Pass out the **Polynomials and Algebra Tiles** handout to each student. Direct students to complete only the "Adding Polynomials" section of their handout. Once students begin, give pairs the green, yellow, and red cups. Clear up any misconceptions as they work.

After 10 minutes, you can unhide and show **slide 10** for students to check their answer and share their process or model the example and work through it together as a class. Use the colors of the cups as a gauge to make this decision.

Display **slide 11** and direct students' attention to the "Subtracting Polynomials" section of their handout. Tell students that they are going to continue working on simplifying polynomials using algebra tiles and the Try It, Talk It, Color It, Check It strategy. Again, clear up any misconceptions as they work.

Teacher's Note: Subtracting Polynomials

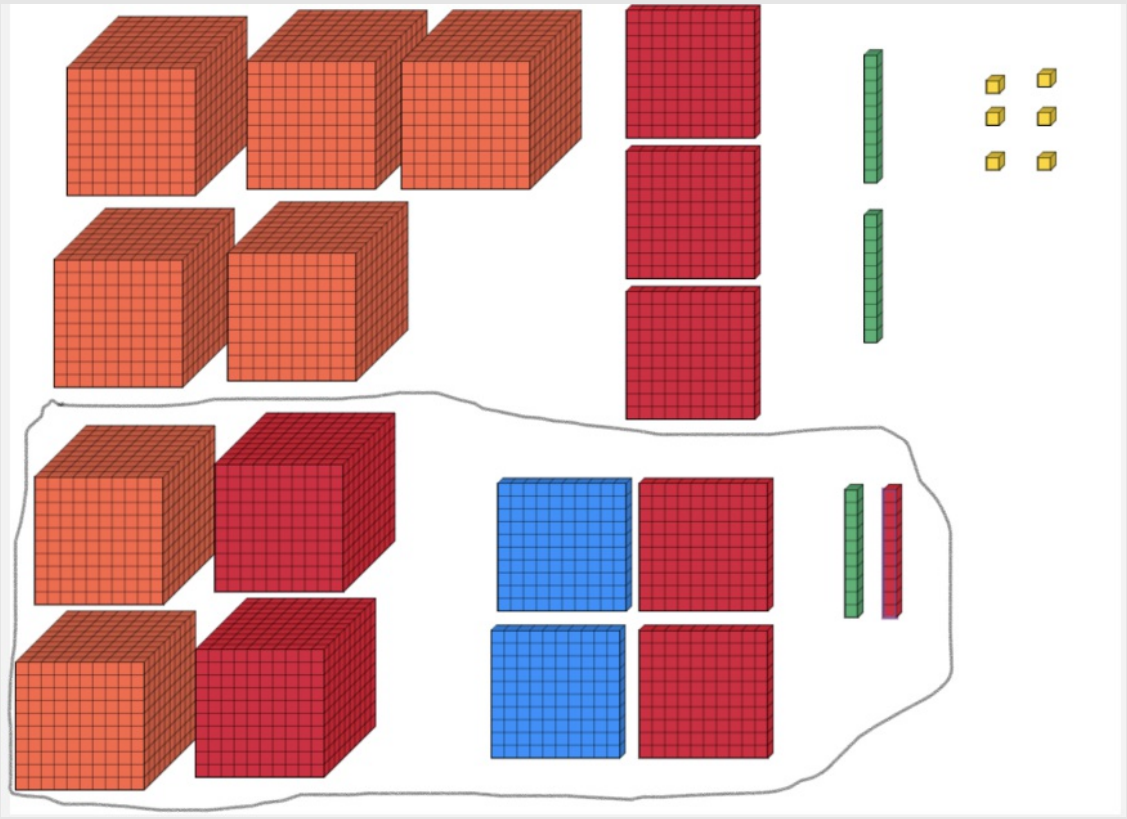
Students are asked to build $(5x^3 - 3x^2 + 2x + 6)$ and take away $(-2x^3 + 2x^2 - x + 2)$. There are two traditional options to modeling subtraction of polynomials.

Option 1: Representing "Taking Away" Something Unseen

Focusing on the first terms of each expression, the problem is asking students to "take away" two negative *x*-cubes $(-2x^3)$ from five positive five cubes $(5x^3)$, and there are not any negative *x*-cubes to "take away." This can be modeled by adding *zero pairs*:

1. Beginning with $(5x^3) - (-2x^3)$, add zero pairs $(+2x^3)$ and $(-2x^3)$ to the first expression: $(5x^3 + 2x^3 - 2x^3) - (-2x^3)$. Now there are two negative *x*-cubes to "take away."
2. Take away the two negative *x*-cubes: $(5x^3 + 2x^3)$.
3. Continue to simplify the expression by combining like terms: $7x^3$.

Applying this idea to the remainder of the problem would make the first expression look like the following: $5x^3 + (+2x^3 - 2x^3) - 3x^2 + (+2x^2 - 2x^2) + 2x + (+x - x) + 6$.



For students needing more support, consider having them watch this *Using Algebra Tiles Adding and Subtracting Directed Numbers Using Zero Pairs* video.

Option 2: Representing "Adding the Opposite"

Another option is to algebraically rewrite the expression:

- from: $(5x^3 - 3x^2 + 2x + 6) - (-2x^3 + 2x^2 - x + 2)$
- to: $(5x^3 - 3x^2 + 2x + 6) + -(-2x^3 + 2x^2 - x + 2)$.

Now one can think about modeling the expressions by "adding the opposite" of each term in the second polynomial or distributing the negative one to each term in the second polynomial and then model the addition problem.

After 10 minutes, you can unhide and show **slide 12** (option 1) or **slide 13** (option 2) for students to check their answer and share their process or model the example and work through it together as a class. Use the colors of the cups as a gauge to make this decision.

This is a natural place in the lesson to pause for the day. If that is the case, consider reminding students to bring their earbuds/headphones to class tomorrow, as they will be watching videos.

Teacher's Note: Extra Problems to Use for Explore

If you notice that your students need additional practice, consider using the following examples. The examples below were chosen because they will easily work with the dienes blocks on the Mathsbot website to further explore.

- $(3x^3 + 2x^2 - 5x + 7) + (2x^3 - x^2 + 4x - 3) = 5x^3 + x^2 - x + 4$
- $(2x^2 - 6x + 5) + (7x^2 - x - 4) = 9x^2 - 7x + 1$
- $(2t^3 + 8t^2 - t - 4) - (4t^3 - t^2 + 2) = -2t^3 + 9t^2 - t - 6$
- $(-4x^3 + 2x^2 - 2x + 8) - (x^3 - 2x^2 + 4x - 1) = -5x^3 + 4x^2 - 6x + 9$

40 minutes

Explain

Start Day 2 by reviewing the essential question and learning objective (**slides 5–6**). Inform students that they will be taking notes on what they explored the day before.

Display **slide 14** and introduce the Scavenger Hunt Notes strategy. Have students get into groups of 2–3 or assign groups. Give each student a copy of the attached **Note Catcher** handout and give each group a copy of the attached **Clues** handout. Explain to the students that on their Clues handout, each clue has directions about watching and both a QR code and a URL to access a video. Tell students they are to use what they learn from those videos to take notes in the corresponding space of their handout. Direct students' attention to Clues #2 and #4; point out that these clues have directions for them to watch two videos, and that they should watch both for each clue (not just pick one to watch). Direct students to use their clues to complete their notes.

During the scavenger hunt, students complete their Note Catcher handout given the following clues:

- **Clue #1:** Students take notes on the definition of a *polynomial expression*.
- **Clue #2:** Students take notes on the steps for adding and simplifying polynomial expressions. Students take notes on subtracting polynomials and identify the most important thing to remember when subtracting polynomials.
- **Clue #3:** Students take notes on adding polynomial expressions using the vertical and horizontal methods. On their Note Catcher, students are asked to practice simplifying an expression using their preferred method.
- **Clue #4:** Students take notes on subtracting polynomial expressions using the vertical and horizontal methods. On their Note Catcher, students are asked to practice simplifying an expression using their preferred method.

Provide students with 20–30 minutes to complete the activity.

Once students are done taking notes, ask students to share what they wrote on their handout for defining a *polynomial expression* (Clue #1). Then show **slide 15** to reveal the definition. Give students time to adjust their notes as needed. Move to **slide 16** to review *terms*, *coefficients*, *exponents*, and *constant* using the example on the slide: $2x^3 + 5x - 7$.

Similarly, ask for volunteers to share the steps for adding and simplifying algebraic expressions (Clue #2). Then transition through **slides 17–18** to reveal the steps. Give students time to adjust their notes as needed but also use the prompts on the slides to ask questions and check for understanding.

Ask the class what the most important difference is between **subtracting** and **adding** polynomials. Then move to **slide 19** to give the reminder of the most important thing to remember when subtracting polynomials: distributing that negative sign when removing the parentheses. Use this time to emphasize that "distributing that negative symbol" is multiplying by negative one.

Transition through **slides 20–23** to share what the work would look like for adding and subtracting their expressions using the horizontal and vertical methods (Clue #3 and Clue #4). Give students time to adjust their notes as needed.

Teacher's Note: Guided Discussion

Ask guiding questions to have students explain key understandings when it comes to adding and subtracting polynomials. Use student responses as a formative assessment to determine if students understand how to simplify polynomial expressions or have misconceptions that need to be addressed. If students cannot explain why to any of these processes, this is the time to explicitly go over how to simplify polynomials and resolve misunderstandings before proceeding with the lesson. For more assistance, use the **Note Catcher (Teacher's Guide)** document to help facilitate this discussion.

45 minutes

Extend

Teacher's Note: Activity Preparation

During this phase of the lesson, students watch a video as a whole class, then divide into groups of no more than four, to simplify polynomial expressions and enter their results into a web app to actually hear how each new polynomial impacts a pre-recorded sample. In preparation, consider how groups of four can best hear sound from their devices without distracting other groups.

Since students will play different sounds from the website, consider having students use earbuds so that they can hear the sound changes and not disturb others around them. It could get loud. Alternatively, students could be strategically placed around the room to avoid a clashing of sounds if students are having to share one device within their group.

Teacher's Note: Lesson Timing

This is a 3-day lesson on polynomials; if time is a concern, consider shortening the Extend phase of this lesson by having the class just watch the video of Dustin Ragland, who shares the real-world connection between polynomials and music.

Display **slide 24** and briefly introduce the students to the *Chebyshev polynomials*. *Chebyshev polynomials* are a special set of polynomials named after Pafnuty Chebyshev, a Russian mathematician. Share that these polynomials have an effect on music when they are added and subtracted.

Show **slide 25** and inform students they will watch a video of Dustin Ragland, who is an *Education Specialist for Ableton*. Then play the video on the slide. In this video, Dustin shares his knowledge of the music industry and using technology to engineer and produce music. He also explains how Chebyshev polynomials of the first kind can be used to manipulate sound and music. As students watch the video, consider pausing the video as students see the questions on the screen and facilitate brief discussion about what they think the answer to each question will be.

Embedded video
https://youtube.com/watch?v=H4vF2556Sig

Once the video ends, divide students into groups of no more than four. Display **slide 26** and give each student a copy of the attached **Polynomials and Sound** handout. Have groups simplify the four expressions on their handout. Each expression is a sum or difference of Chebyshev polynomials. Encourage students to show their work on a piece of notebook paper and then record their simplified expression on their handout.

Optional Checking Work

If you would like students to check their work before listening to the results, unhide **slides 27–30**. Then transition through these slides to show the steps and simplified expressions for each question.

Once students have simplified the four expressions on their handout, direct their attention to the "Web Tool" section of the handout where they have the steps for using the web tool to now hear their polynomials. Have students navigate to k20center.ou.edu/e-learning/polynomials-sound/ with their devices. Transition through **slides 31–33** to walk through the steps of entering the $T_8(x)$ Chebyshev polynomial into the web tool. Adjusting the slider to the far right turns on the *noeffect* and *effect* volumes, while adjusting the slider to the far left turns off the *noeffect* and *effect* volumes. Have students think about how to enter negative coefficients and constants, which are the guiding questions on slide 32. Try to ask prompting questions instead of directly telling them how to enter this information.

Teacher's Note: Guiding the Activity

The web tool shows a polynomial with five terms, where only the last exponent can be changed to zero. Use the guidelines below to help students navigate this activity:

- **Slide 32:** $128x^8 + -256x^6 + 160x^4 + -32x^2 + 1x^0$
- **Slide 35:** $16x^5 + -20x^3 + 5x^1 + 0x^1 + 0x^1$
- **Handout Q1 (also Slide 28):** $16x^5 + -16x^3 + 2x^1 + 0x^1 + 0x^1$
- **Handout Q2 (also Slide 29):** $-32x^6 + 56x^4 + -26x^2 + 0x^1 + 2x^0$
- **Handout Q3 (also Slide 30):** $128x^8 + -288x^6 + 208x^4 + -50x^2 + 2x^0$
- **Handout Q4 (also Slide 31):** $256x^9 + -512x^7 + 320x^5 + -64x^3 + 2x^1$

Once the class has entered the polynomial and there is a unified sound in the room, display **slide 34** and have students adjust the sliders such that one is to the far right (on) while the other is to the far left (off) and listen carefully. Ask for volunteers to describe how the sound changes. Remind students that the "before" is when the *noeffect volume* slider is to the far right (on), and the "change" is when the *effect volume* slider is to the far right (on), while the other is off.

Teacher's Note: Web Tool Tips

As students use this web tool, encourage them to turn both the *effect* and *noeffect volume* sliders to the far left (turning them both off) as they switch between which is to the far right. Turning on both *effect* and *noeffect* volumes will make unpleasant sounds.

Alternatively, consider having two students collaborate where one has the *noeffect* volume on and the other student has the *effect* volume on, so they can compare the two sounds at the same time.

If students need additional practice using the web tool, move to **slide 35** and have students try to enter the $T_5(x)$ Chebyshev polynomial into the web tool. Otherwise, skip this slide. Students can use the back of their handout for quick reference regarding how to use the web tool.

Again, once you are hearing a unified sound in the room for the $T_5(x)$ Chebyshev polynomial, indicating that students are understanding how to use the web tool, show **slide 36**. Now is an excellent time to use earbuds if students have not been already. Direct students to now individually enter the results from their handout and listen for the changes in sound.

Display **slide 37**. Hold a quick class discussion and have a couple of students share their answers about how they would describe the change of their sound. Use the following questions to help with sharing out for one subtraction and one addition problem:

- How are the sounds different?
- What descriptive words would you use for each sound? For example, would *nasal*, *cold*, *scary*, *happy*, etc., describe the change in sound?

Optional Differentiation

If time allows, consider challenging students by asking them to predict what they think the graphs of their results might look like. Have students check their predictions using a graphing utility.

Then, as an introduction or review of analyzing graphs of polynomials, consider having students identify critical features of the resulting graphs such as domain, range, intervals of increase or decrease, etc.

Optional Extension

There are a variety of other math equations in music besides Chebyshev polynomials. If you have students who are interested in music and want to explore more about math and music, unhide **slide 38** and play the *Sound Designer and Film Composer* video of Nicholas Poss, film composer and a professor at the Academy of Contemporary Music (a.k.a. ACM@UCO). Here he describes how math is linked to sound effects and how instruments are made and the importance of pursuing math with music.

Embedded video
https://youtube.com/watch?v=pwC2gWQEnFw

After watching the video, pass out a copy of the attached **Learning Synths Playground** handout to each student and direct students to LearningSynths.Ableton.com/en/Playground, which is a website made by Ableton. This website has a simple synthesizer instrument, where each of these controls represent physical modules and is based on a math equation. (For reference, those equations are shown on the handout.) Have students play with the different sliders and listen for how the sound changes. Ask them if some modules work together better than others and if so, why they think that is the case.

10 minutes

Evaluate

Display **slide 39** and use the Exit Ticket strategy to individually assess what students have learned from the lesson. Give each student a half-sheet of the attached **Exit Ticket** handout or have students use a piece of notebook paper to answer the questions on the slide.

Teacher's Note: Reflection Source

One of the reflection questions asks students to pinpoint what they do not understand and identify the source of their confusion. This question is adapted from Heick, T. (2022, June 25). *50 learning reflection questions for students*. TeachThought. https://www.teachthought.com/learning/reflection-questions-for-students/.

Collect student responses and use them to determine whether they need additional practice or are ready for the next lesson. Use the hidden **slide 40** that has the simplified expressions to check students' work or as needed.

Resources

- Abelton. (n.d.). *Learning Synths.* [Web app]. https://learningsynths.ableton.com/en/playground
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- Teachings in Education. (2019, October 26). *Subtracting polynomials: Algebra animations.* [Video]. YouTube. https://www.youtube.com/watch?v=7GtBobbFrB4