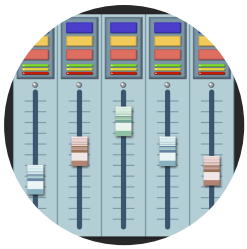

The Sound of Polynomials

Adding and Subtracting Polynomials

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Grade Level	9th – 11th Grade	**Time Frame**	120–140 minutes
Subject	Mathematics	**Duration**	3 class periods
Course	Algebra 1, Algebra 2		

Essential Question

How can you use addition and subtraction to simplify polynomial expressions?

Summary

In this lesson, students practice adding and subtracting polynomial expressions. They watch a video about a complicated food order and discuss the idea of combining like items to simplify the order. Students use math manipulatives to explore how to use addition and subtraction to simplify polynomial expressions. By participating in a Scavenger Hunt Notes activity, students create their own notes about adding and subtracting polynomials. Students watch an ICAP video of an education specialist from Ableton, who was a former tour musician. He shares his knowledge of the music industry, using technology to engineer and produce music, and he explains how Chebyshev polynomials of the first kind can be used to manipulate sound and music. After watching the video, students apply adding and subtracting polynomials to hear the sound of the simplified polynomial. Finally, students demonstrate what they have learned about adding and subtracting polynomials in an Exit Ticket.

Snapshot

Engage

Students make the connection of grouping like items and figure out how that concept applies to simplifying polynomial expressions.

Explore

Students use a digital math manipulative to explore using addition and subtraction to simplify polynomial expressions.

Explain

Students participate in a Scavenger Hunt Notes activity to complete notes on using addition and subtraction to simplify polynomial expressions.

Extend

Students watch an ICAP video to see how polynomial expressions affect or change sound, then work in groups to apply their knowledge to listen to their simplified polynomials using a web tool.

Evaluate

Students demonstrate what they have learned about adding and subtracting polynomials in an Exit Ticket.

Standards

Oklahoma Academic Standards Mathematics (Algebra 1)

A1.A.3.2: Simplify polynomial expressions by adding, subtracting, or multiplying.

Oklahoma Academic Standards Mathematics (Algebra 1)

A2.A.2.2: Add, subtract, multiply, divide, and simplify polynomial expressions.

Attachments

- Alternative Frayer Model—The Sound of Polynomials.docx
- Alternative Frayer Model—The Sound of Polynomials.pdf
- Alternative Scavenger Hunt QR Codes—The Sound of Polynomials.docx
- Alternative Scavenger Hunt QR Codes—The Sound of Polynomials.pdf
- Exit Ticket—The Sound of Polynomials - Spanish.docx
- Exit Ticket—The Sound of Polynomials - Spanish.pdf
- Exit Ticket—The Sound of Polynomials.docx
- Exit Ticket—The Sound of Polynomials.pdf
- Frayer Model (Teacher Guide)—The Sound of Polynomials.docx
- Frayer Model (Teacher Guide)—The Sound of Polynomials.pdf
- Frayer Model—The Sound of Polynomials - Spanish.docx
- Frayer Model—The Sound of Polynomials - Spanish.pdf
- Frayer Model—The Sound of Polynomials.docx
- Frayer Model—The Sound of Polynomials.pdf
- Learning Synths Playground—The Sound of Polynomials - Spanish.docx
- Learning Synths Playground—The Sound of Polynomials - Spanish.pdf
- Learning Synths Playground—The Sound of Polynomials.docx
- Learning Synths Playground—The Sound of Polynomials.pdf
- Lesson Slides—The Sound of Polynomials.pptx
- Polynomials and Algebra Tiles—The Sound of Polynomials - Spanish.docx
- Polynomials and Algebra Tiles—The Sound of Polynomials - Spanish.pdf
- Polynomials and Algebra Tiles—The Sound of Polynomials.docx
- Polynomials and Algebra Tiles—The Sound of Polynomials.pdf
- Polynomials and Sound—The Sound of Polynomials - Spanish.docx
- Polynomials and Sound—The Sound of Polynomials - Spanish.pdf
- Polynomials and Sound—The Sound of Polynomials.docx
- Polynomials and Sound—The Sound of Polynomials.pdf
- Scavenger Hunt QR Codes—The Sound of Polynomials.docx
- Scavenger Hunt QR Codes—The Sound of Polynomials.pdf

Materials

- Lesson Slides (attached)
- Polynomials and Algebra Tiles handout (attached, 1 per student, 2-sided)
- Scavenger Hunt QR Codes handout (attached, print 1-sided, print on colored paper)
- Frayer Model handout (attached, 1 per student)
- Frayer Model (Teacher Guide) document (optional, attached)
- Alternative Scavenger Hunt QR Codes handout (optional, attached, print 1-sided, print on colored paper)
- Alternative Frayer Model handout (optional, attached, 1 per student)
- Polynomials and Sound handout (attached, 1 per student)
- Learning Synths Playground handout (optional, attached)
- Exit Ticket handout (cut in half, one half page per student)

- Pencils
- Clipboards (one per student)
- Devices with Internet access
- Algebra tiles (optional)
- Green cups (1 per group)
- Yellow cups (1 per group)
- Red cups (1 per group)
- Blank sheet of paper (1 per student)
- Earbuds (1 per student, optional)
- Colored pencils
- Markers

10 minutes

Engage

Present the lesson using the attached **Lesson Slides**.

Display **slide 3** and ask students to watch a video about ordering at a fast-food restaurant. Show students the **MADtv Fast Food Ordering** video.

Embedded video
https://youtube.com/watch?v=M2T7Z8PwESY

Teacher's Note: Video

It is recommended that you stop the video at 4:01. After this point, the customer insults the waitress.

Once the students have watched the video, display **slide 4** and ask students to figure out an easier way the customer could have communicated his order.

If students don't mention grouping things that have been ordered, guide them to the possible answer of grouping like items.

Possible Student Responses

He could have grouped liked items together.

He could have written his order down.

Share **slides 5**–**6** to introduce the essential question and the learning objective to connect the idea of grouping like items to simplifying polynomials.

30 minutes

Explore

Display **slide 7** and inform students they will be using an online math manipulative to answer questions about the given polynomials. Have students navigate to k20.ou.edu/algebratiles to access the Mathsbot dienes blocks, also known as base-ten blocks. Follow the steps itemized below:

1. Instruct students to use these resources to represent terms of the polynomial expressions.
2. Inform students that to create a negative, they will need to click the " x-1" button at the top of the tool bar.
3. Advise them to combine like terms by dropping a negative (or red block) on top of the positive block.
4. Let them know that this action will cause the blocks to disappear or cancel each other out.
5. Instruct them to get rid of a block by clicking on the block and selecting "delete."
6. Instruct them to hit "refresh" in the web browser to clear their screen.
7. Advise them that the blocks on the screen are very large and can be made smaller by using the "negative magnifying glass" in the top right corner of their screen.

Consider modeling these steps for the class and have the students mimic at the same time. Note that this modeling will give students the opportunity to practice using the tool they will use for the activity.

Alternative Activity Options

You can have the students use algebra tiles instead of online manipulatives. If you choose algebra tiles and the students are unfamiliar with algebra tiles, it is highly recommended that you use this Introducing Algebra Tiles to Students activity to help them explore and understand how algebra tiles are used.

An alternative technology option is GeoGebra. You can provide students with the link to this activity called 3D Algebra Tiles Illustrator.

Display **slide 8** and introduce the Try It, Talk It, Color It, Check It instructional strategy. Let students know that as they work on the problem, they will first try it on their own, discuss it with a partner, and then choose a colored cup to represent their confidence in efficiently summarizing how many blocks of each kind they have all together. They will have three colored cups. Tell the students that the colors represent the following confidence levels:

- Green: They can summarize their thought processes.
- Yellow: They are a bit uncertain about how to summarize their thought processes.
- Red: They need help summarizing their thought processes.

Pass out the **Polynomials and Algebra Tiles** handout, the green, yellow, and red cups, and a device to each student. Inform students to complete only the *addition* side of the handout. Give students time to work through the addition side of their handout before moving on. Clear up any misconceptions as they work.

After 10 minutes, you can unhide **slide 9** for students to check their answer and share their process.

Display **slide 10** and inform students they will continue working on simplifying polynomials using algebra tiles and the Try It, Talk It, Color It, Check It instructional strategy. But this time, they will focus on *subtraction* on the back of the handout. Remind students that, as they work on the problem, they should first try it on their own, discuss it with a partner, and then choose a color to represent their confidence in efficiently summarizing their thought process.

Have students use the dienes blocks on the Mathsbot website they used for addition. Give students time to work through the subtraction side of their handouts before moving on. Clear up any misconceptions as they work.

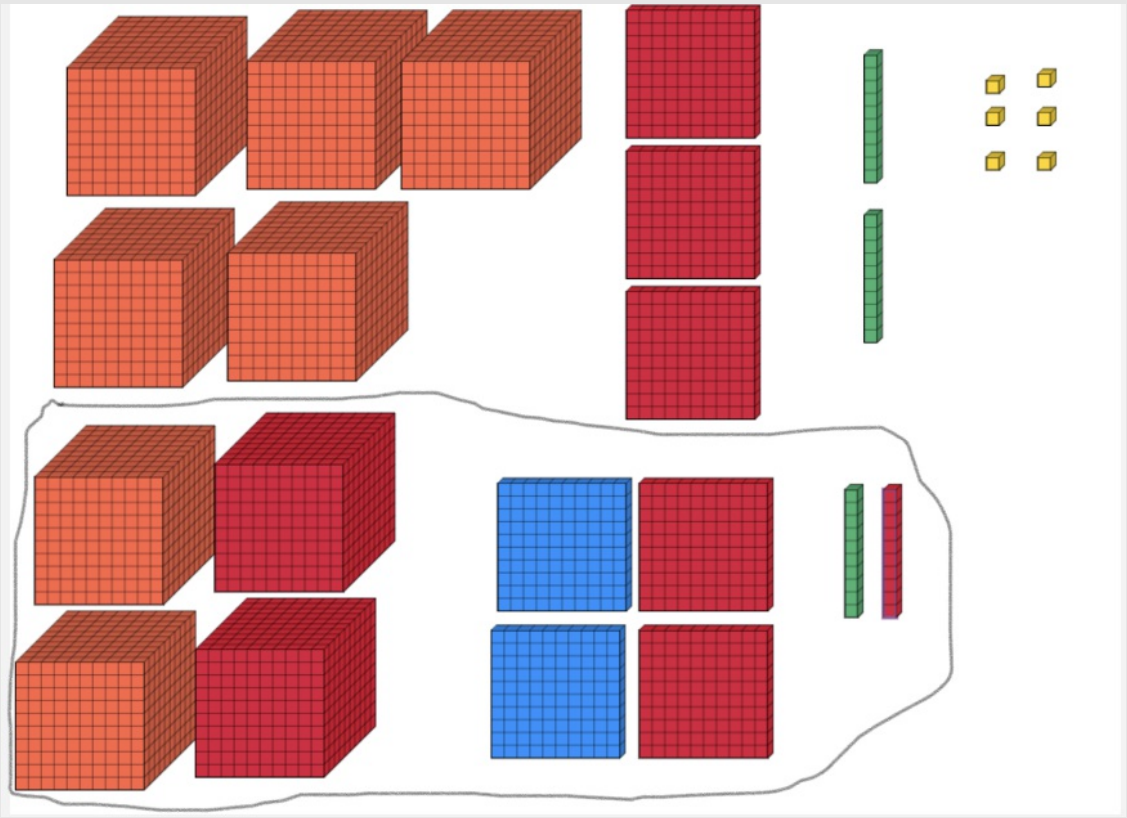
Teacher's Note: Subtraction

There are two options to modeling subtraction of polynomials.

Option 1

Representing "adding the opposite" with algebra tiles. The problem can ask to subtract an amount that is not there to subtract.

1. At this point, students need to add in the amount of zero pairs that is being subtracted, which does not change the value of the problem.
2. Then they can take away the amount they need. Consider the problem, "Build **$(5x^3 - 3x^2 + 2x + 6)$** and take away **$(-2x^3 + 2x^2 - x + 2)$.**"
3. We do not have 2 negative x-cubes (x^3 's) to subtract from 5 positive x-cubes (x^3's).
4. Therefore, we need to add in two zero pairs so that we can take away 2 negative x^3's from 5 positive x^3's.
5. We are now left with 7 positive x-cubes (x^3's). This will be done for any term that cannot be subtracted.
6. See the video "Using algebra tiles adding and subtracting directed numbers using zero pairs" for reference.



Option 2

Another option is to first change the subtraction to addition by adding the opposite or distributing the negative one to each term in the second polynomial and then model the addition problem.

After 10 minutes, you can unhide **slide 11** for students to check their answer and share their process.

Teacher's Note: Extra Problems to use for Explore

The examples below were chosen because they will easily work with the dienes blocks on the Mathsbot website to further explore.

$(3x^3 + 2x^2 - 5x + 7) + (2x^3 - x^2 + 4x - 3) = 5x^3 + x^2 – x + 4$

$(2x^2 - 6x + 5) + (7x^2 - x - 4) = 9x^2 – 7x + 1$

$(2t^3 + 8t^2 - t - 4) - (4t^3 - t^2 + 2) = -2t^3 + 9t^2 – t – 6$

$(-4x^3 + 2x^2 - 2x + 8) - (x^3 - 2x^2 + 4x - 1) = -5x^3 + 4x^2 – 6x + 9$

40 minutes

Explain

Teacher's Note: Setting up the Activity

Prior to starting this activity, hang up the **Scavenger Hunt QR Codes** handout pages around the room. Decide whether you want the students to work through the scavenger hunt independently, with a partner, or in a group of 3.

If a student is unsure of an answer, encourage them to skip a prompt and move to a different QR code.

Consider earbuds for students who are working independently to help with the background noise.

Start day 2 with a review of the essential question and learning objective (**slides 5** and **6**). Then, go through the following steps:

1. Inform students that they will be taking notes on what they explored the day before.
2. Display **slide 12** and introduce the students to the Scavenger Hunt Notes instructional strategy.
3. Explain to the students that there are QR codes taped around the room to help them take notes.
4. Have the students start anywhere they would like. Encourage them to spread out.
5. Pass out a **Frayer Model** handout and clipboard to each student.
6. Make sure each student or group of students has a device to scan the QR codes.
7. Using the QR codes on the wall, have students walk around the room filling in the boxes of their Frayer Model with the information from the corresponding videos.
8. During the scavenger hunt, in the **top-left corner** of the Frayer Model handout, ask students to write a definition of *polynomial expression.* This concept is addressed in the video at station 1.
9. In the **top-right corner** of the Frayer Model handout, have students write the steps for simplifying polynomials by adding or subtracting.
10. Ask them to identify what is the most important thing to remember when subtracting polynomials. This concept is addressed at stations 2 and 3.
11. In the **bottom left and right corners** of the Frayer Model handout, ask students to simplify the polynomial expressions on the Scavenger Hunt QR Codes handout using either the vertical or horizontal methods.
12. Inform students that both station 4, *adding*, and station 5, *subtracting*, have two QR codes.
13. Instruct them to watch both videos before deciding whether they want to solve the problem horizontally or vertically on their Frayer Model.
14. Provide students with 20–30 minutes to complete the activity.

Once all students have completed the Scavenger Hunt, display **slide 13** and facilitate a class discussion to review the Frayer Model.

Ask students to share what they put on their Frayer Model to help them understand what a polynomial expression is. Use **slide 14** to reveal the answers, then have them check their work.

Then, display **slides 15** and **16** to go over the steps of simplifying polynomials. Remind students of the most important thing to remember when subtracting polynomials. Click slide 16 a second time to share the answer with the students.

Use **slides 17–20** to show the answers of the problems in the addition and subtraction sections of the Frayer Model.

Teacher's Note: Guided Discussion

This part of the lesson is the big idea of this concept. Ask guiding questions to have students explain key understandings when it comes to adding and subtracting polynomials. Having students share these key understandings will show if there are any misconceptions that need to be addressed. If they cannot explain why to any of these steps, this is the time to explicitly go over how to simplify polynomials.

For more assistance, use the **Frayer Model (Teacher Guide)** document to help facilitate this activity.

More Challenging Frayer Model and Scavenger Hunt

If the students need less scaffolding, use the **Alternative Frayer Model** handout and the **Alternative Scavenger Hunt QR Codes** handout found in the facilitator resources. This option will enable students to create their own definition and steps. Have students watch all the videos and take notes on their own notebook paper. Then, in groups of 2–3, have the students fill out the Frayer Model using their notes from the Scavenger Hunt and what they did in the Explore. Choose the option that best fits your students' needs.

30 minutes

Extend

Teacher's Note: Activity Preparation

During this phase of the lesson, students watch an ICAP video as a whole class, then divide into groups of no more than four, to simplify polynomial expressions and enter their results into a web app to actually hear how each new polynomial impacts a pre-recorded sample. In preparation, consider the following grouping suggestion and how groups of four can best hear sound from their device without distracting other groups.

***Grouping*:** If you need a quick method for sorting students into groups of four, try using a variation of the strategy Pick a Card, Any Card. Remove the face cards and any additional unnecessary cards from the deck. Shuffle the deck, then give each student a card. Then have students find other students with cards of the same numerical value. Since there are only four cards with the same number, each group will have no more than four students. For example, if you have 30 students in class, remove the face cards and the nines and tens cards. You will be left with 32 cards: four sets with values 1–8.

***Sound/Volume*:** Since students will play different sounds from the website, consider having students use headphones so that they can hear the sound changes and not disturb others around them. It could get loud. Alternatively, students could be strategically placed around the room to avoid a clashing of sounds if students are having to share one device within their group.

Display **slide 21** and introduce the students to the Chebyshev polynomials and the effects these polynomials have on music when they are added and subtracted.

Display **slide 22** and inform students they will watch an ICAP video of Dustin Ragland, an "Education Specialist for Ableton," where he shares his knowledge of the music industry, using technology to engineer and produce music. He explains how Chebyshev polynomials of the first kind can be used to manipulate sound and music. As students watch the video, consider pausing the video as students see the questions on the screen and facilitate brief discussion about what they think the answer to each question will be.

Embedded video

https://youtube.com/watch?v=H4vF2556Sig

After watching the ICAP video, pass out the **Learning Synths Playground** handout and direct students to the website "Learning Synths Playground" made by Ableton. This website has a simple synthesizer instrument, but each of these modules is based on a math equation. (For reference, those equations are shown on the front of this page.) Have students play with the different sliders and see how the sound changes. Ask them: Do some modules work together better than others? Why do you think that is the case?

Optional Differentiation

If time allows, consider challenging students by asking them to predict what they think the graphs of their results might look like. Have students check their predictions using a graphing utility.

Then, as an introduction or review of analyzing graphs of polynomials, consider having students identify critical features of the resulting graphs such as domain, range, intervals of increase or decrease, etc.

10 minutes

Evaluate

Display **slide 34** and use the Exit Ticket strategy to individually assess what students have learned from the lesson. Give each student a half-sheet of the attached **Exit Ticket** handout or give students a sticky note, an index card, etc., for them to write their response.

Collect student responses and use them to determine whether they need additional practice or are ready for the next lesson.

Resources

- Abelton. (n.d.). *Learning Synths* [Web app]. https://learningsynths.ableton.com/en/playground
- Brzezinski, T. (2017). *3D algebra tiles illustrator* [Web app]. GeoGebra. https://www.geogebra.org/m/usVjhq2T
- Carter, S. (2022). *Introducing algebra tiles to students* [Blog]. https://mathequalslove.net/introducing-algebra-tiles/
- CK-12 Foundation. (2017). *Adding and subtracting polynomials vertically* [Video]. YouTube. https://youtu.be/FYk_PxwANSE
- Khan Academy. (2021). *The parts of polynomial expressions: Polynomial and rational functions: Algebra II: Khan Academy* [Video]. YouTube. https://youtu.be/REiDXCN0lGU
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- Teachings in Education. (2019). *Adding polynomials: Algebra animations* [Video]. YouTube. https://youtu.be/9y4nsLOB