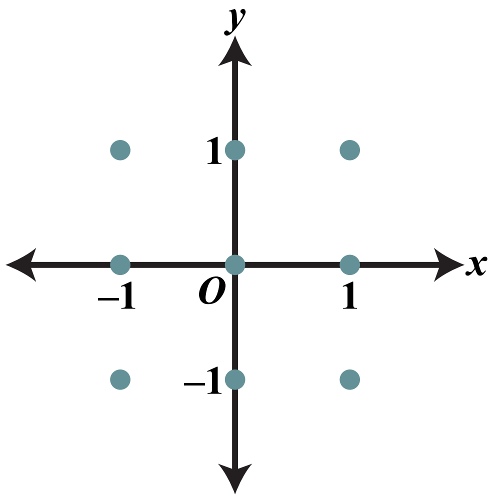
## FREE RESPONSE (SAMPLE RESPONSES)

*This problem is intended to be solved **without** the use of a calculator.*

Consider the curve defined by the equation $\frac{dy}{dx} = (y+1)^2 \sin\left(\frac{\pi}{2}x\right)$.

**(a)** On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.




**(b)** There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of $c$.

**(c)** Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

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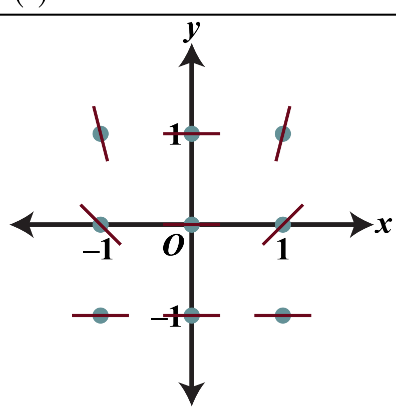
**(a)**




$(-1, 1) \Rightarrow -4$ $\quad (0, 1) \Rightarrow 0$ $\quad (1, 1) \Rightarrow 4$

$(-1, 0) \Rightarrow -1$ $\quad (0, 0) \Rightarrow 0$ $\quad (1, 0) \Rightarrow 1$

$(-1, -1) \Rightarrow 0$ $\quad (0, -1) \Rightarrow 0$ $\quad (1, -1) \Rightarrow 0$

**+2:** { **+1:** zero slopes; **+1:** all other slopes }

**(b)** $c$ quals $-1$ because the slope when $y=-1$ is constantly zero and the slope of $y=c$ would also be constantly zero.

**+1:** $c=-1$

**(c)** $\frac{dy}{dx}=(y+1)^2\sin\left(\frac{\pi}{2}x\right)$

$$\int (y+1)^{-2}\,dy=\int \sin\left(\frac{\pi}{2}x\right)dx$$

$$-(y+1)^{-1}=-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)+c$$

$$\frac{-1}{y+1}=-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)+c$$

$$y+1=\frac{-1}{-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)+c}$$

$$y+1=\frac{\pi}{2\cos\left(\frac{\pi}{2}x\right)+c}$$

$$(0)+1=\frac{\pi}{2\cos\left(\frac{\pi}{2}(1)\right)+c}$$

$$1=\frac{\pi}{0+c}$$

$$c=\pi$$

$$y=\frac{\pi}{2\cos\left(\frac{\pi}{2}x\right)+\pi}-1$$

**+6:**
- **+1:** separates variables
- **+2:** antiderivatives
- **+1:** constant of integration
- **+1:** uses initial condition
- **+1:** answer

**Note:** If missing constant of integration, maximum of 3/6 points: 1-2-0-0-0.

**Note:** If no separation of variables, 0/6 points.