

Slope Fields

Meteorology and Wind Maps

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Grade Level	12th Grade	**Time Frame**	60-75 minutes
Subject	Mathematics	**Duration**	1-2 class period(s)
Course	AP Calculus		

Essential Question

How do we create and use slope fields?

Summary

In this lesson, students will explore slope fields by visualizing a toy boat being moved by the current of a stream and a paper airplane moving through the air. Students will watch an interview of a meteorology researcher, then learn how to create and interpret slope fields given a differential equation. Then they will apply their understanding to match differential equations to their slope fields and practice an Advanced Placement type free response problem.

Snapshot

Engage

Students make observations while visualizing the movement of a toy boat in a stream and a paper airplane moving through the air.

Explore

Students watch an interview of a professional and explore how meteorologists use mathematics.

Explain

Students complete guided notes with the class and formalize their understanding of how to create slope fields and sketch solution curves.

Extend

Students apply what they have learned to match slope fields with their differential equations.

Evaluate

Students demonstrate their understanding of creating and interpreting slope fields by answering a free response question.

Standards

AP Calculus AB and BC Course and Exam Description (AP Calculus AB & BC (2020))

FUN-7.C: Estimate solutions to differential equations.
FUN-7.C.1: A slope field is a graphical representation of a differential equation on a finite set of points in the plane.
FUN-7.C.2: Slope fields provide information about the behavior of solutions to first-order differential equations.
FUN-7.C.3: Solutions to differential equations are functions or families of functions.

Attachments

- Free Response (Sample Responses)—Slope Fields.docx
- Free Response (Sample Responses)—Slope Fields.pdf
- Free Response—Slope Fields - Spanish.docx
- Free Response—Slope Fields - Spanish.pdf
- Free Response—Slope Fields.docx
- Free Response—Slope Fields.pdf
- Guided Notes (Sample Responses)—Slope Fields.docx
- Guided Notes (Sample Responses)—Slope Fields.pdf
- Guided Notes—Slope Fields - Spanish.docx
- Guided Notes—Slope Fields - Spanish.pdf
- Guided Notes—Slope Fields.docx
- Guided Notes—Slope Fields.pdf
- Slope Calculations—Slope Fields - Spanish.docx
- Slope Calculations—Slope Fields - Spanish.pdf
- Slope Calculations—Slope Fields.docx
- Slope Calculations—Slope Fields.pdf

Materials

- Guided Notes handout (attached; one per student; printed front/back)
- Guided Notes (Sample Responses) document (attached; for teacher use)
- Slope Calculations handout (attached; one per student; printed front only)
- Free Response handout (attached; one per student; printed front only)
- Free Response (Sample Responses) document (attached; for teacher use)
- Pencils
- Paper
- Student devices with internet access

15 minutes

Engage

Teacher's Note: Desmos Classroom Activity Preparation

To use this Desmos Classroom activity, select the following link: "Slope Fields." Create an account or sign in under the "Activity Sessions" heading. After you log in, the "Assign" dropdown button will be active. Click the arrow next to the word "Assign," then select "Single Session Code." After making some setting selections, select "Create Invitation Code" and give the session code to students. For more information about previewing and assigning a Desmos Classroom activity, go to https://k20center.ou.edu/externalapps/using-activities/.

For more detailed information about Desmos features and how-to tips, go to https://k20center.ou.edu/externalapps/desmos-home-page/.

To set up the activity's pacing for students, select "View Dashboard" (next to the session code). In the upper-left corner of your screen, select the icon above the word "Pacing." Desmos Classroom should then prompt you to select the first and last screens that you want students to see. When prompted to set a range, select screens 1 and 4. Select "Restrict to Screens 1–4" to confirm your selection. This allows students to access only screens 1–4 at this time. For more information about teacher pacing, go to https://k20center.ou.edu/externalapps/pacing-activities/.

Provide students with your session code. Then, have students go to student.ampify.com/join and enter the session code.

Teacher's Note: Sign-in Options

If students sign in with their Google or Desmos accounts, then their progress is saved, and they can resume the activity or view their work later. If students continue without signing in, they can complete the activity, but they must do so in one sitting. It is strongly recommended that students sign in; otherwise, they risk losing their work.

Introduce the lesson using **screens 1-2**, which display the lesson's essential question and learning objectives. Review each of these with your class to the extent you feel necessary.

Instruct students to find a partner or assign students partners.

Direct students' attention to **screen 3**. Here, students are given a slope field and asked to imagine that it represents wind blowing and they are throwing a paper airplane and considering the path it would follow. Do not yet give students the academic language of *slope field*. Allow students to engage with the interactive graph for a couple of minutes. After a few minutes, ask the class what they noticed.

Then direct their attention to **screen 4** and repeat. Here, students are given a different slope field and asked to image it representing the current of a stream and they are placing their toy boat in the water and watching it move. After allowing students time to interact with the graph, ask again for volunteers to share what they observed. Students are not yet expected to explain their observations using proper academic language; they will learn the correct terminology during the Explain portion of the lesson.

Sample Student Responses:

- The path was dependent on the point the plane started.
- The little lines seemed to direct the plane, telling it which way to go (up/down).
- The paths for the airplane were all very similar in shape.
- The paths for the boat were all really different.
- If the boat started below that $y = -x - 1$ line, then the boat went down; if it started above, it went up.
- The little lines with negative slopes seemed to make the boat move down, while the lines with positive slopes moved the boat up.

On the Dashboard, press the orange plus sign three times to allow students to progress to **screens 5-7**. Direct students' attention to screen 5 and share that the air flow and current were examples of *slope fields*. And that the paths or curves were examples of *solutions* to slope fields. Again, students will see formal definitions during the Explain portion of the lesson. At this point, they just need enough vocabulary knowledge to match some slope fields to their corresponding algebraic solutions.

Introduce students to the Card Matching strategy and direct their attention to screen 6. Here students are asked to imagine the shape of the path that their boat would create if they put it into a stream where they could see the current. The solution cards are all parent graphs that students should be familiar with. After a couple of minutes of students working, let them know that they can check their work on screen 7. Screen 7 will not have any results if there are zero correct card matches.

Teacher's Note: Guiding the Activity

Giving students the concrete idea of a toy boat moving down a stream can really help kids visualize the movement that is represented by these static slope fields.

15 minutes

Explore

Press the orange plus sign on the Dashboard to allow students to progress to **screen 8**. Have students watch the "Meteorology and Mathematics" video where Dr. Scott Salesky, a meteorology researcher and professor at the University of Oklahoma, shares how math is used in meteorology. Consider taking a moment to explain to students that students need to understand the "simple" things before they can understand the "complex" things of this world, which is why understanding slope fields is the "simple thing" and understanding wind maps is the "complex thing"—we do not start learning how to run before we learn how to walk.

Embedded video

https://youtube.com/watch?v=YTctNK8EcNk

On the Dashboard, press the orange plus sign once to allow students to progress to **screen 9**. Here students are given a brief introduction to vector fields and vorticity, which are used by meteorologists to make weather predictions. Students are asked to interpret graphs and make an educated guess of which graph would be described as having *positive vorticity*. Once students select a graph, justify their reasoning, and click the "Submit" button, they will see feedback about their selection. Regardless of their selection, encourage students to not stress to answer correctly; remind them that they are practicing their justification skills on this screen. In other words, students should not lose any points or credit for making an incorrect guess on this screen.

Teacher's Note: Purpose

Students are not expected to take notes or memorize information about vorticity or vector fields. The purpose here is for students to practice interpreting graphs, imagining that movement represented by a graph, and justifying their reasoning. Students are not expected to know with certainty which graph has positive vorticity; students need to be able to justify their thinking.

15 minutes

Explain

Teacher's Note: Pacing the Lesson

During a traditional 45-minute class period, this lesson will naturally need to pause and resume the next day. A smooth transition could be to pause and resume the lesson during the Explain portion. Finish the first day after completing the first example from the Guided Notes. Consider sending students home thinking about the second example. The next class period can begin with students sharing their ideas for example 2.

Depending on your classroom routines, you could finish the Explain portion during the first day and give the Extend and Evaluate activities as homework.

Give each student a copy of the **Guided Notes** handout, then press the orange plus sign on the Dashboard three times to allow students to progress to **screens 10-12**. Direct students to use Desmos to complete the handout.

On screen 10, students are given definitions for *differential equations*, *lineal elements*, and *slope fields* (*direction fields*). As much as it is important to have formal definitions, in summary, students need to know that differential equations are basically equations with differentials such as *dy*, *dx*, etc. They do not need to memorize the name for the line segments that make up a slope field, *lineal elements*, but they need to know that slope fields are the graphical representation of differential equations.

Direct students' attention to screen 11. Let students know that they are going to look at a differential equation that they know the solution to first, so they can just focus on the new concept of slope fields. Students will likely wonder if they can algebraically find the solution to a differential equation, then why would one need slope fields? The reality is that most differential equations can not be solved algebraically, so that is why slope fields are so important. Remind students to make notes on their handout for the first example.

As students finish screen 12, reading the explanation of how to interpret a slope field and what a family of solutions could look like, have the class come back together for questions and discussion. Ask for volunteers to explain in their own words what those lineal elements (line segments) represent. Ensure students understand that the differential equation represents the slope at each point and those segments are like slopes of miniature tangent lines of the solution/function. Use student responses to help clarify any misunderstandings.

On the Dashboard, click the orange "Stop" button; now students can complete the Desmos activity at their own pace. On **screens 13-18**, students are to use the table to input slope values for the indicated points. What they enter will generate the lineal elements on the graph. Each screen has students complete one column of work at a time and provides feedback as they go. As students work, circulate the room to answer any questions.

As students complete screen 18, bring the class together and ask for a couple of volunteers to summarize what they did for the second example. How did they generate the slope field? How did they sketch the solution curve? It can often be helpful to show students that sketching the solution curve from left to right is often challenging and that it is okay to start at the initial condition and sketch away from that point, whether it be left or right.

Have students add their completed Guided Notes to their math notebook if that is a classroom norm.

10 minutes

Extend

Give each student a copy of the attached **Slope Calculations** handout. On **screens 19-22**, students are expected to complete two Card Matching activities. Have students work with their partner to match the differential equation cards with their corresponding slope field cards. The second set of cards is more challenging than the first set. Encourage students to use the handout to show work as needed. Each handout has space to show work for six differential equations, which is the quantity of differential equations in each set of cards. If you prefer, you can give students two copies of the handout, one for each set of cards.

The Desmos activity has a built-in self-check on **screens 20 and 22**, so students can check their card matches. These screens will not have any results if there are zero correct card matches.

20 minutes

Evaluate

Teacher's Note: Prerequisite Knowledge

The free response provided asks students to (a) sketch a slope field, (b) find a general solution based on the slope field, and (c) solve the differential equation using separation of variables. If students have not yet learned how to integrate using the method of separation of variables, then have students only complete parts a and b of the free response question, skipping part c.

Once students have completed screen 22, direct their attention to **screen 23**. Give each student a copy of the attached **Free Response** handout and direct students to work independently so that you can individually assess what students have learned from this lesson. This question is intended to be solved without the use of a calculator.

Give students 10–15 minutes to answer the free response question. Then use the attached **Free Response (Sample Responses)** document to review the sample response and scoring guidelines with the class. Be sure to help students understand how to earn points on exam questions. Consider displaying this document in the same way you would display a slide deck instead of printing one for each student.

Point out to students that there is not a point awarded to the value of *c*. This is likely because the value of *c* will vary based on how the problem is approached. For example, if *c* was beside the fraction instead of within the fraction, then the value of *c* would not be the same value as what is shown in the sample response. Instead, the final answer is what the rubric is grading (as well as using the initial condition to get a *c*-value), because "solving for *c*," is just a step to get to that final result. As long as a *c* value is found, then that part of the problem has been addressed.

Teacher's Note: Source

This free response question is adapted from the 2006 AP Calculus AB (Form B) Question 5. The scoring rubric has been modified accordingly.

Resources

- K20 Center. (n.d.). Card matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- K20 Center. (n.d.). Desmos classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081
- YouTube. (2024). *K20 ICAP-meteorology and mathematics.* [Video]. YouTube. https://youtu.be/YTctNK8EcNk