

Countdown to Congruence

Investigating Triangle Congruence Theorems

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Grade Level	9th – 10th Grade	**Time Frame**	3 class periods
Subject	Mathematics	**Duration**	120–135 minutes
Course	Geometry		

Essential Question

How can we justify that two triangles are congruent?

Summary

Students use manipulatives to investigate triangle congruence and complete proofs using triangle congruence theorems in this game-show themed lesson.

Snapshot

Engage

Students consider the minimum amount of information needed to prove that two triangles are congruent.

Explore 1

Students explore possible congruence theorems: SSS, SSA, and SAS.

Explain 1

Students write conjectures based on their observations then write the congruence theorems in their own words.

Explore 2

Students explore possible congruence theorems: AAA, AAS, and ASA.

Explain 2

Students write conjectures based on their observations then write the congruence theorems in their own words.

Extend

Students apply their knowledge of triangle congruence to complete proofs.

Evaluate

Students demonstrate their understanding by identifying the minimum information needed to determine triangle congruence and by applying the appropriate congruence theorem.

Standards

ACT College and Career Readiness Standards - Mathematics (6-12)

G603: Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles

Oklahoma Academic Standards Mathematics (Geometry)

G.2D.1.9: Construct logical arguments to prove triangle congruence (SSS, SAS, ASA, AAS and HL).

Attachments

- Lesson Slides—Countdown to Congruence.pptx
- Let's Make a Proof (Sample Responses)—Countdown to Congruence.docx
- Let's Make a Proof (Sample Responses)—Countdown to Congruence.pdf
- Let's Make a Proof—Countdown to Congruence - Spanish.docx
- Let's Make a Proof—Countdown to Congruence - Spanish.pdf
- Let's Make a Proof—Countdown to Congruence.docx
- Let's Make a Proof—Countdown to Congruence.pdf
- Phone a Friend—Countdown to Congruence - Spanish.docx
- Phone a Friend—Countdown to Congruence - Spanish.pdf
- Phone a Friend—Countdown to Congruence.docx
- Phone a Friend—Countdown to Congruence.pdf
- Triangle Congruence Theorems (Sample Response)—Countdown to Congruence.pdf
- Triangle Congruence Theorems—Countdown to Congruence - Spanish.pdf
- Triangle Congruence Theorems—Countdown to Congruence.pdf

Materials

- Lesson Slides (attached)
- Triangle Congruence Theorems handout (attached; one per student; printed front only)
- Triangle Congruence Theorems (Sample Response) document (attached; for teacher use)
- Let’s Make a Proof handout (attached; one per pair; printed front/back)
- Let’s Make a Proof (Sample Responses) document (attached; for teacher use)
- Phone a Friend handout (attached; one-half per student; printed front only)
- Pencils
- Paper
- AngLegs (one set per two pairs)
- Patty Paper or straws (optional)
- Scissors (optional; one per pair)
- Protractor (optional; one per pair)
- Tape (optional)

5 minutes

Engage

Teacher's Note: Lesson Order

The order of this lesson is as follows: Engage, Explore 1, Explain 1, Explore 2, Explain 2, Extend, Evaluate.

The lesson is structured this way for students to explore three possible triangle congruences at a time. It is chunked for cognitive load and for time. For a traditional 45-minute class period, this lesson has natural breaks after Explore 1 and again after Explore 2.

This lesson is themed around game shows, so try to play the role of the game-show host. Consider playing game-show music from your favorite show in the background or reading through the activity directions in an overly dramatic way.

Introduce the lesson using the attached **Lesson Slides**. Use the Bell Ringers strategy to begin class.

Display **slide 3** and help students get into the game-show zone by reading the following prompt to students like a game-show host (i.e., in an overly dramatic and loud way): *Who wants to be a MATH MASTER?* Then read the question and answer choices from the slide: *What can we conclude if two triangles are congruent? Is it: (A) All corresponding angles are equal in measure; (B) All corresponding sides are equal in measure; (C) Areas are equal; or (D) All of the above?* Have students independently answer the question on a piece of notebook paper or elsewhere if you have a classroom norm for bellwork. After a minute, ask the class what they thought the correct answer was and help them understand that option (D) *All of the above* was the correct choice.

Move to **slide 4** and ask the class: *What is the minimum number of parts (sides and angles) you would need to measure to prove congruence?*

After a minute of giving students time to think and respond, tell them that the answer to that question is actually what they are going to learn during this lesson. Share the lesson's essential question on **slide 5** and the learning objectives on **slide 6**. Review each of these with your class to the extent you feel necessary.

Transition to **slide 7** and facilitate a brief discussion about why we need to measure more than one or two parts of each triangle to determine congruence. Let students know that they are going to need to further investigate to determine if measuring three parts is the minimum.

Sample Student Responses:

You should expect students at this point to say that, in order to be certain of congruence, one would need to measure and compare all six parts of the triangles: each of the three sides and each of the three angles. Consider guiding students who seem to be certain of needing to measure "everything" that "there might be an easier way." Mathematicians are known for finding a pattern to save themselves time.

35 minutes

Explore 1

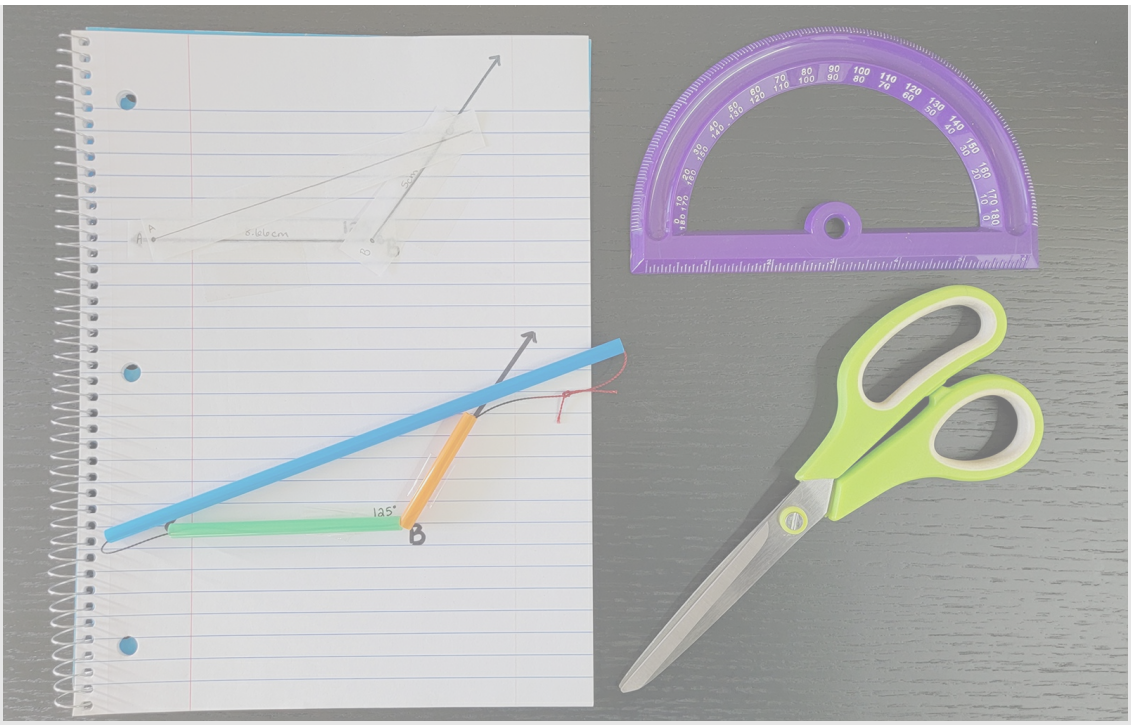
Teacher's Note: Activity Preparation

During the Explore portions of this lesson, students will be exploring the congruence of triangles using AngLegs. A typical AngLegs set has 74 pieces: 2 protractors and 12 of each color/length legs with 6 different colors/lengths. (Some AngLegs sets also contain activity cards, and so include more pieces.) Each pair will need at least three AngLegs of each color/length and a protractor. This means that one set of AngLegs could be divided into 2–4 mini-sets; 2 additional protractors would be needed if the purchased set is divided into 4 mini-sets, since a purchased set typically only contains 2 protractors. Printing this protractor from inchcalculator.com could also be a solution if there are not enough protractors. Consider creating AngLegs mini-sets (as described above) for pairs to use during this activity. This can be done by putting the needed materials into zip-top bags for easy distribution. An example of a mini-set is in the image below.



Alternatively, pairs could instead use:

- a pencil, a compass, and protractor on paper
- a pencil, protractor, scissors, tape, and Patty Paper
- a pencil, protractor, scissors, tape or string, and straws



Any of the options will yield the same results. However, using the alternative supplies will likely increase the time of the exploration activities.

If this is your first time using AngLegs, check out this "hand2mind - AngLegs" video.

Have students find a partner or assign partners, then move to **slide 8.** Ask pairs to provide all combinations of three adjacent parts they can think of (you may suggest S for side and A for angle). Allow them to work in pairs to create a list. Choose two pairs to write their lists on the board, then other groups can add to what they have. Discuss the possibilities together.

Teacher's Note: Guiding the Activity

The list that students might create is: SSS, AAA, SAA, SSA, SAS, ASA, AAS, ASS. Point out to students that AAS is equivalent to SAA (two angles and a non-included side), and that ASS is equivalent to SSA (two sides and a non-included angle).

Come to a consensus: *Are these the only six ways that the same three parts of two triangles may be compared?* (Yes, as any additional ones would be equivalent.) Then show **slide 9**. Tell students that they will investigate all six possibilities to see which ones prove congruence.

Display **slide 10** and tell students that they are going to only focus on SSS, SSA, and SAS during this first Explore portion of the lesson. Then move to **slide 11** and again, try to get into character as a game-show host. Tell students that they are going to play, "Can You Match My Triangle?" where you, the host, will give the class three properties about your triangle, and they are to use their AngLegs or alternative materials to try to create a triangle that matches yours.

Provide each pair with one AngLegs mini-set containing three legs of each color/length and a protractor. Have students get out a pencil and piece of scratch paper. During this activity, students will be using the Inverted Pyramid strategy.

Transition to **slide 12**. Begin round 1 by reading the first triangle's three properties from the slide: side *AB* is 8.66 cm in length; side *BC* is 10 cm in length; and side *AC* is 7.07 cm in length. Let students know that the lengths of the AngLegs are printed on the legs. Start the 3-minute timer on the slide. As students work, circulate the room to make sure students are understanding the task. This is the time for you to ask more guiding questions than give answers. So if a student asks something like, "Is this right?" consider replying with something like, "We'll have to wait and see" or "Do you think there is more than one right answer?"

Once the timer expires, or students are done, move to **slide 13** and direct students to find another pair and compare their triangles.

After giving students a minute or two to determine if their pair of triangles are congruent or not, transition to **slide 14**. Have students hold up their triangles and see if they are all the same (congruent) or if at least one is different (a counterexample). Then ask the class which congruence possibility they just tested.

Display **slide 15** to share your triangle. Did everyone's triangle match? Students should conclude that all of the triangles are congruent and that they tested the Side–Side–Side (SSS) possibility.

Repeat these steps using **slides 16–20** for students to test the Side–Side–Angle (SSA) possibility for round 2. Students should conclude that SSA does not determine congruence, as someone should have a counterexample in the class.

Teacher's Note: Guiding the Activity

In the unlikely event that everyone's triangles are congruent when testing Side–Side–Angle (SSA), use **slide 19** or unhide **slide 20** to show the counterexample implying that no one was able to match your triangle. If you notice that there is at least one counterexample in the class, there is no need to unhide any slides.

You also might consider physically making the triangles, using the AngLegs or alternative materials, to hold up and show the class.

For round 3, again repeat these steps using **slides 21–24** for students to test the Side–Angle–Side (SAS) possibility. Students should conclude that all of the triangles are congruent.

10 minutes

Explain 1

Display **slide 25** and let students know that based on their observations they are going to be making conjectures—statements that seem true but have not been formally proven.

Give each student a copy of the attached **Triangle Congruence Theorems** handout and show **slide 26**. Facilitate a discussion about how to complete the given conjectures. Have students jot down notes about the conjectures on the backs of their handouts. If needed, use this time to clarify the difference between *included* and *non-included* angles.

Teacher's Note: Providing Extra Support

If students are struggling with the difference between the included and non-included angles, encourage students to sketch the triangles being described on the slide. Remind students that they read a triangle either clockwise or counterclockwise. With SAS, the A (angle) is **in** the middle of the two Ss (sides), so it is referring to the **in**cluded angle, while with SSA, the A is **not in** the middle, so it is referring to the **non-in**cluded angle.

Move to **slide 27** and share with students that due to time they are not going to formally prove the congruence theorems, but instead appreciate that mathematicians already have proven SSS and SAS as triangle congruence theorems. Have students label one space on the front of their handout SSS and mark the triangles accordingly, then write the theorem in their own words. Repeat this for SAS; use the images on the slide to help direct students where to write. Depending on time and the needs of your students, you might consider collaborating as a class and agreeing upon the language used to write the theorems. Use the attached **Triangle Congruence Theorems (Sample Response)** document as needed.

Sample Student Responses:

- If two triangles have 3 pairs of congruent sides, then the triangles are congruent.
- If all of the sides of one triangle are congruent to all of the sides of another triangle, then the triangles are congruent.
- If 2 sides and the included angle of a triangle are congruent to the corresponding 2 sides and included angle of another triangle, then the 2 triangles are congruent.
- If 2 triangles have 2 pairs of congruent sides and the angles between them are congruent, then the triangles are congruent.

35 minutes

Explore 2

Display **slide 28** and tell students that they are going to only focus on AAA, AAS, and ASA during this second Explore portion of the lesson. Then move to **slide 29** and again, try to get into character as a game-show host. Tell students that they are going to play the *challenge* rounds of "Can You Match My Triangle?" These investigations will likely take longer than the previous investigations, which is why students will be given more time.

As before, provide each pair with one mini-set of AngLegs containing three legs of each color/length and a protractor. Then have pairs form groups of four, with two mini-sets of AngLegs. Students need to have access to two protractors, which is why students are now in groups of four. If you have enough protractors for each pair to have two, consider having students work in pairs for more engagement. Have students get out a pencil and piece of scratch paper; they can use the same scratch paper from the earlier exploration.

Transition to **slide 30**. Begin challenge round 1 by reading the first triangle's three properties from the slide: the measure of angle *A* is 30°, the measure of angle *B* is 60°, and the measure of angle *C* is 90°. Start the 5-minute timer on the slide. As students work, circulate the room to make sure students are understanding the task. This is the time for you to ask more guiding questions than give answers.

Teacher's Note: Guiding the Activity

Students may already know the third angle conjecture and argue that AAA is true because of that. Clarify that, from the conjecture, all we know for certain is that the *angles* are congruent and ask, "Does that guarantee the *triangles* are congruent, too?"

However, if the third angle conjecture has been investigated rigorously in your class, there may be no need to investigate AAA. If that is the case, skip the first challenge round.

Once the timer expires, or students are done, move to **slide 31** and have students hold up their triangles and see if they are all the same (congruent) or if at least one is different (a counterexample). Then ask the class which congruence possibility they just tested.

Use **slide 32** or unhide **slide 33** to share your triangle. If everyone's triangles happen to be congruent, show the counterexample implying that no one was able to match your triangle. If you notice that there is at least one counterexample in the class, there is no need to unhide any slides. Students should conclude that AAA does not determine congruence.

Repeat these steps using **slides 34–36** for students to test the Angle–Angle–Side (AAS) possibility for challenge round 2. Students should conclude that all of the triangles are congruent.

For challenge round 3, again repeat these steps using **slides 37–39** for students to test the Angle–Side–Angle (ASA) possibility. Students should conclude that all of the triangles are congruent.

10 minutes

Explain 2

Display **slide 40**. Facilitate a discussion about how to complete the given conjectures. Have students jot down notes about the conjectures on the backs of their Triangle Congruence Theorems handouts. Be sure that students know what the difference between *included* and *non-included* sides is.

Move to **slide 41** and share with students that, due to time, they are not going to formally prove the congruence theorems but instead appreciate that mathematicians already have proven ASA and AAS as triangle congruence theorems. Have students complete their handout by writing the theorem names, labeling the triangles, and writing the theorems in their own words. Depending on the needs of your students and time, you might consider collaborating as a class and agreeing upon the language used to write the theorems.

Sample Student Responses:

- If 2 angles and the non-included side of a triangle are congruent to the corresponding angles and side of another triangle, then the triangles are congruent.
- If 2 triangles have 2 pairs of congruent angles and the sides not between them are congruent, then the triangles are congruent.
- If 2 triangles have 2 pairs of congruent angles and the sides between them are also congruent, then the triangles are congruent.
- If 2 angles and the included side of a triangle are congruent to those of another triangle, then the 2 triangles are congruent.

Transition through **slides 42–43** to share with students the Hypotenuse–Leg (HL) congruence theorem in as much detail as you feel necessary. There is empty space at the bottom of the front page of their Triangle Congruence Theorems handout for notes on this theorem.

Teacher's Note: Purpose

Since the HL congruence theorem is essentially a shortcut to the SSS congruence theorem, students do not need to memorize it. However, if a student is trying to earn a higher score on a timed test, like the ACT, then encourage those students to also commit this one to memory.

Before continuing the lesson, make sure students understand that the minimum number of parts they would need to measure to determine triangle congruence is three.

20 minutes

Extend

Have each student find a new partner—someone they have not yet worked with during this lesson—or assign students new partners. Working with different peers encourages the development of academic vocabulary and encourages students to consider different approaches to a problem. Display **slide 44** and give each pair a copy of the attached **Let's Make a Proof** handout. Again, this is a good time to continue your role as the overly excited game-show host.

Direct pairs to collaborate to complete the given two-column proofs. Here students will work through four proofs that will increase in difficulty as they progress. The first proof has quite a few pieces of the proof completed for the students, while the last proof only gives the hint of how many steps (rows) there are in the proof. Use the attached **Let's Make a Proof (Sample Responses)** document for reference as needed. Remember, there is some flexibility in the order students write a proof.

Differentiation

If students need additional support, consider printing the Let's Make a Proof (Sample Responses) document and cutting the table into strips of paper for students to then sequence instead of writing in the proof.

Give them time to piece together the strips in a logical order, encouraging them to think about how they constructed their triangles in their previous investigations. After they have completed the proofs, have them write a justification of their sequence. If time allows, pairs could compare their work and come to consensus.

5 minutes

Evaluate

Use the Exit Tickets strategy to individually assess what students have learned from the lesson. Show **slide 45** and give each student a copy of the attached **Phone a Friend** half-page handout. Read the prompt from the slide (again, in your best game-show host voice): *Your friend is on Who Wants to be a Math Master? and is unsure about one of the questions. Luckily for them, they have the option to call a friend: you!* Students are asked to determine the unnecessary information and which congruence theorem proves congruence.

Sample Student Responses:

This question has multiple correct responses; when reviewing students' work, check that their reasoning is accurate.

- (1) We do not need the information about angles *C* and *F*. (2) I would recommend using the SAS congruence theorem. (3) Side *AB*, angle *A*, and side *AC* being congruent to side *DE*, angle *D*, and side *DF* proves that triangle *ABC* is congruent to triangle *DEF* by the SAS congruence theorem.
- (1) We do not need the information about side *AB* and side *DE*. (2) I would recommend using the ASA congruence theorem. (3) Angle *A*, side *AC*, and angle *C*, being congruent angle *D*, side *DF*, and angle *F* proves that triangle *ABC* is congruent to triangle *DEF* by the ASA congruence theorem.
- (1) We do not need the information about side *AC* and side *DF*. (2) I would recommend using the AAS congruence theorem. (3) Angle *C*, angle *A*, and side *AB* being congruent angle *F*, angle *D*, and side *DE* proves that triangle *ABC* is congruent to triangle *DEF* by the AAS congruence theorem.

Optional Addition

If you would like to give students additional practice in preparation for the ACT, consider unhiding **slide 46** and sharing it with the class. It is appropriate to give students just a minute or two to complete this ACT-style question, as that is about how long they have when taking the ACT. Then use the hidden **slide 47** as you see fit: to share the sample response with the class or use it to check students' work.

Resources

- hand2mind. (2022, January 11). *AngLegs* [Video]. YouTube. https://youtu.be/PNW6iNixdmw?si=J356ZNBJO15y8xq-
- Inchcalculator. (n.d.). *Standard 6" protractor US letter*. https://www.inchcalculator.com/wp-content/uploads/2016/04/protractor.pdf
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Inverted pyramid. Strategies. https://learn.k20center.ou.edu/strategy/173
- K20 Center. (2021, September 21). K20 Center 3 minute timer [Video]. YouTube. https://youtu.be/ilSP02KPau0?si=oew-VIocHqA5Tn8z
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- Serra, M. (2003). *Discovering geometry: An investigative approach* (pp. 219–299). Key Curriculum Press.