NAVIGATING THE GREAT DIVIDE—TEACHER RESOURCE

Example 1:

Teacher	Students
Step 1: Divide the first term of the dividend and divisor. $2x^3/2x$ *Guiding question: what number will I multiply by 2x to get* $2x^3$? The quotient is written at the top of the problem. *Be sure to line up the degrees to keep the problem neat.*	x^2 $2x+5\overline{)2x^3+19x^2+x-85}$
Step 2: Distribute x^2 into the divisor. Write the product below the dividend. *Again, notice how the degrees are lined up.*	x^2 $2x+5\overline{)2x^3+19x^2+x-85}$ $2x^3+5x^2$
Step 3: Subtract the product from the dividend. I put parentheses around the binomial to remind myself that I must subtract both terms, which may require me to change the signs.	x^2 $2x+5\overline{)2x^3+19x^2+x-85}$ $\underline{-(2x^3+5x^2)}$ $14x^2$
Step 4: We will bring down the next term and repeat the same three steps with the binomial $14x^2+x$.	x^2 $2x+5\overline{)2x^3+19x^2+x-85}$ $\underline{-(2x^3+5x^2)}$ ↓ $14x^2+x$
Step 5: Divide the first term of the binomial and the divisor. $14x^2/2x$ The quotient will be written on the top of the problem. *Guiding question: what number will I multiply by 2x to get* $14x^2$?	x^2+7x $2x+5\overline{)2x^3+19x^2+x-85}$ $\underline{-(2x^3+5x^2)}$ ↓ $14x^2+x$

Teacher	Students
Step 6: Distribute $7x$ into the divisor. Write the product below the dividend.	x^2+7x $2x+5\overline{)2x^3+19x^2+x-85}$ $-(2x^3+5x^2)$ ↓ $14x^2+x$ $\underline{14x^2+35x}$
Step 7: Subtract the product from the dividend. Again, I put parentheses around the binomial to remind myself that I must subtract both terms, which may require me to change the signs.	x^2+7x $2x+5\overline{)2x^3+19x^2+x-85}$ $-(2x^3+5x^2)$ ↓ $14x^2+x$ $-(\underline{14x^2+35x})$ $-34x$
Step 8: We will bring down the next term and repeat the same three steps with the binomial $-34x-85$.	x^2+7x $2x+5\overline{)2x^3+19x^2+x-85}$ $-(2x^3+5x^2)$ ↓ $14x^2+x$ $-(\underline{14x^2+35x})$ ↓ $-34x-85$
Step 9: Divide the first term of the binomial and the divisor. $-34x/2x$ The quotient will be written on the top of the problem. *Guiding question: what number will I multiply by 2x to get* $-34x$?	$x^2+7x-17$ $2x+5\overline{)2x^3+19x^2+x-85}$ $-(2x^3+5x^2)$ ↓ $14x^2+x$ $-(\underline{14x^2+35x})$ ↓ $-34x-85$
Step 10: Distribute -17 into the divisor. Write the product below the dividend.	$x^2+7x-17$ $2x+5\overline{)2x^3+19x^2+x-85}$ $-(2x^3+5x^2)$ ↓ $14x^2+x$ $-(\underline{14x^2+35x})$ ↓ $-34x-85$ $\underline{-34x-85}$

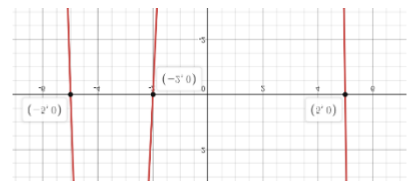
Teacher	Students
Step 11: Subtract the product from the dividend. Again, I put parentheses around the binomial to remind myself that I must subtract both terms, which may require me to change the signs.	$2x+5\overline{)2x^3+19x^2+x-85}$ quotient $x^2+7x-17$ $-(2x^3+5x^2)$ ↓ $14x^2+x$ $-(14x^2+35x)$ ↓ $-34x-85$ $-(34x-85)$ 0
Answer: At this point in the problem, I notice that I do not have any more terms to bring down AND I got zero after subtracting. Because there are no more terms to bring down, I know I have my answer. Because the difference is zero, I know that the answer has no remainder.	$x^2+7x-17$

Example 2:

Teacher	Students
Step 1: Imagine for a moment that you are completing this division problem $32\overline{)603}$ Is the zero in the dividend important to the problem? How so? If you were to remove the zero, would the answer change? Notice in the original problem, it says $6x^2-3$; the first term has a power of 2 and the second terms has a power of zero. There is no term with a power of 1. Just like in $32\overline{)603}$, the term with no value is still important to the problem, so we are going to rewrite the problem and include the power of 1. This will be accomplished by adding 0x.	$3x+2\overline{)6x^2+0x-3}$

Teacher	Students
Step 2: Divide the first term of the dividend and divisor. $6x^2/3x$ *Guiding question: what number will I multiply by 3x to get* $6x^2$? The quotient is written at the top of the problem. *Be sure to line up the degrees to keep the problem neat.*	$2x$ $3x+2\overline{)6x^2+0x-3}$ $-(6x^2+4x)$
Step 3: Distribute $2x$ into the divisor. Write the product below the dividend. Subtract the product from the dividend. I put parentheses around the binomial to remind myself that I will need to subtract both terms, which may require me to change the signs.	$2x$ $3x+2\overline{)6x^2+0x-3}$ $-(6x^2+4x)$ $-4x$
Step 4: We will bring down the next term and repeat the same three steps with the binomial $-4x-3$.	$2x$ $3x+2\overline{)6x^2+0x-3}$ $-(6x^2+4x)\downarrow$ $-4x-3$
Answer: Divide the first term of the dividend and divisor. $-4x/3x$ *Guiding question: what number will I multiply by 3x to get* $-4x$? (There is no number that I can multiply by 3x to ger -4x) At this point in the problem, I cannot go any further, so what does that mean for the binomial that is left over? The left-over value is called the remainder and will be added to the quotient to complete the answer.	$2x+\frac{-4x-3}{3x+2}$

Example 3:

Teacher	Students
If one factor is x+5, Identify all other factors of this function through Graphing, Factoring, and Division.	$x^3+2x^2-25x-50$
Let's start by graphing this function in the calculator and using our table to find the zeros of the graph. *Teacher's note: The terms zero, root, solution, and x-intercepts are used interchangeably throughout the lesson to help students understand they have the same meaning.*	
Zeros are the x-intercepts of our graph.	$x=-5 \quad x=-2 \quad x=5$
How can we write these zeros as a factor? To change the zero into a factor, we would need to move the -5 to the other side of the equation and set it equal to zero, making the first factor $x+5=0$ or just $x+5$. (This was given to us in the question.)	$x=-5 \quad x=-2 \quad x=5$ $x=-5$ $+5 \quad +5$ $x+5=0$
Do the same thing to the other zeros. Move the number to the other side of the equation so that the equation is equal to zero to write the factors. *Guiding question: what pattern do you notice from turning the zeros into factors?*	$x+5 \quad x+2 \quad x-5$
We want to factor this function.	$x^3+2x^2-25x-50$
I see the first two terms have an x^2 in common, and the second two terms share a common factor. This tells me that we would be able to factor by grouping.	$(x^3+2x^2)+(-25x-50)$

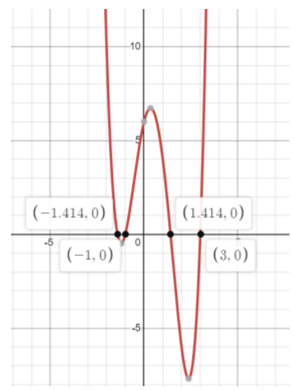
Teacher	Students
I can pull out the x^2 from the left set to factor common terms.	$(x^3+2x^2)+(-25x-50)$ $x^2(x+2)$
I can pull the -25 out of the right set to factor common terms.	$(x^3+2x^2)+(-25x-50)$ $x^2(x+2)-25(x+2)$
Rewrite and factor common terms again. $(x+2)$ cannot be factored. (x^2-25) can be factored.	$(x+2)(x^2-25)$
Factor (x^2-25). Factors of 25 are 5 and 5. The 25 is negative so that tells us we are multiplying one positive and one negative number.	$(x+2)(x^2-25)$ $\downarrow$ $(x+5)(x-5)$
What we are left with cannot be factored anymore. Notice that you got the same answer through graphing and through factoring!	$(x+2)(x+5)(x-5)$
	$x+5\overline{)x^3+2x^2-25x-50}$
Step 1: Divide the first term of the dividend and divisor. x^3/x *Guiding question: what will I multiply by x to get* x^3? The quotient is written at the top of the problem. *Be sure to line up the degrees to keep the problem neat.*	$\begin{array}{r} x^2 \\ x+5\overline{)x^3+2x^2-25x-50} \end{array}$

Teacher	Students
Step 2: Distribute x^2 into the divisor. Write the product below the dividend. *Again, notice how the degrees are lined up.*	$\begin{array}{r} x^2 \\ x+5\overline{)x^3+2x^2-25x-50} \\ -(x^3+5x^2) \end{array}$
Step 3: Subtract the product from the dividend. I put parentheses around the binomial to remind myself that I will need to subtract both terms, which may require me to change the signs.	$\begin{array}{r} x^2 \\ x+5\overline{)x^3+2x^2-25x-50} \\ -(x^3+5x^2) \\ -3x^2 \end{array}$
Step 4: We will bring down the next term and repeat the same three steps with the binomial $-3x^2-25x$.	$\begin{array}{r} x^2 \\ x+5\overline{)x^3+2x^2-25x-50} \\ -(x^3+5x^2) \downarrow \\ -3x^2-25x \end{array}$
Step 5: Divide the first term of the dividend and divisor. $-3x^2/x$ *Guiding question: what will I multiply by x to get* $-3x^2$? The quotient is written at the top of the problem. *Be sure to line up the degrees to keep the problem neat.* **Step 6**: Distribute $-3x$ into the divisor. Write the product below the dividend. *Again, notice how the degrees are lined up.*	$\begin{array}{r} x^2 -3x \\ x+5\overline{)x^3+2x^2-25x-50} \\ -(x^3+5x^2) \downarrow \\ -3x^2-25x \\ -(-3x^2-15x) \end{array}$

Teacher	Students
Step 7: Subtract the product from the dividend. I put parentheses around the binomial to remind myself that I will need to subtract both terms, which may require me to change the signs.	$x^2 \quad -3x$ $x+5\overline{)x^3+2x^2-25x-50}$ $-(x^3+5x^2) \quad \downarrow$ $-3x^2-25x$ $-(-3x^2-15x)$ $-10x$
Step 8: We will bring down the next term and repeat the same three steps with the binomial $-10x-50$.	$x^2 \quad -3x$ $x+5\overline{)x^3+2x^2-25x-50}$ $-(x^3+5x^2) \quad \downarrow$ $-3x^2-25x$ $-(-3x^2-15x) \quad \downarrow$ $-10x-50$
Step 9: Divide the first term of the dividend and divisor. $-10x/x$ *Guiding question: what will I multiply by x to get* $-10x$? The quotient is written at the top of the problem. *Be sure to line up the degrees to keep the problem neat.* **Step 10**: Distribute -10 into the divisor. Write the product below the dividend. *Again, notice how the degrees are lined up.*	$x^2 \quad -3x \quad -10$ $x+5\overline{)x^3+2x^2-25x-50}$ $-(x^3+5x^2) \quad \downarrow$ $-3x^2-25x$ $-(-3x^2-15x) \quad \downarrow$ $-10x-50$ $-(-10x-50)$

Teacher	Students
Step 11: Subtract the product from the dividend. I put parentheses around the binomial to remind myself that I will need to subtract both terms, which may require me to change the signs.	$x^2 \quad -3x \quad -10$ $x+5\overline{)x^3+2x^2-25x-50}$ $\underline{-(x^3+5x^2)} \downarrow$ $-3x^2-25x$ $\underline{-(-3x^2-15x)} \downarrow$ $-10x-50$ $\underline{-(-10x-50)}$ 0
Based on the division we did, these are the factors. If this correct? Can we factor anymore?	$(x+5)(x^2-3x-10)$
Let's decide what two numbers add to be -3 and multiply to be -10.	$(x+5)(x^2-3x-10)$ $\downarrow$ $(x+\_)(x-\_)$
	$(x+5)(x^2-3x-10)$ $\downarrow$ $(x+2)(x-5)$
What do you notice about the answer when I find the factors by graphing, factoring, and dividing? You should notice that you get the same answer every time. So, these three processes can be used interchangeably. We will practice this more in the next part of the lesson.	$(x+5)(x+2)(x-5)$

Example 4:

Teacher	Students
Find the roots of $y = x^4 - 2x^3 - 5x^2 + 4x + 6$ *At this point in the lesson, students might be wondering why they need to learn polynomial long division because they already know how to solve by factoring. Let's try this example to see how polynomial long division could be helpful.*	$x^4 - 2x^3 - 5x^2 + 4x + 6$
Step 1: Can we factor? No, there are too many terms	
Step 2: Let's try graphing! From the graph we can see that x=-1 and x=3 are roots of the graph, but we need to algebraically find the other two zeros.	
Step 3: Division Based on the graph, it looks like x=3 is a solution; therefore, x-3 is a factor. Let's see if that's true.	$x-3\overline{)x^4 - 2x^3 - 5x^2 + 4x + 6}$

Teacher	Students
At this point we are going to solve this polynomial long division problem the same way we did in the previous examples. *Refer back to previous examples to clarify the reasoning for each step.* **There is no remainder, which means that x-2 is a factor.**	$x^3 + x^2 - 2x - 2$ $x - 3 \overline{) x^4 - 2x^3 - 5x^2 + 4x + 6}$ $-(x^4 - 3x^3) \downarrow$ $x^3 - 5x^2$ $-(x^3 - 3x^2) \downarrow$ $-2x^2 + 4x$ $-(-2x^2 + 6x) \downarrow$ $-2x + 6$ $-(-2x + 6)$ 0
Because we divided, we were able to break up the original polynomial into two manageable pieces. *The first set parentheses is the divisor, and the second set of parentheses is the quotient.*	$(x-3)(x^3 + x^2 - 2x - 2)$
The second set of parentheses has 4 terms, so we could factor by grouping.	$(x-3)(x^3 + x^2 - 2x - 2)$
We can use parentheses to group the first two terms and the last two terms. For $x^3 + x^2$, you can pull out x^2 as a common term leaving you with $(x+1)$. For $-2x-2$, you can pull out -2 as a common term leaving you with $(x+1)$	$(x-3)[(x^3 + x^2)(-2x-2)]$ $(x-3)[x^2(x+1) - 2(x+1)]$

Teacher	Students
We keep $x-3$ as a factor, $x+1$ is now a factor, and we combine the common factors to make x^2-2.	$(x-3)[(x^3+x^2)(-2x-2)]$ $(x-3)[x^2(x+1)-2(x+1)]$ $(x-3)(x+1)(x^2-2)$
To find your zeros, set all factors equal to zero and solve.	$x-3=0$ $x+1=0$ $x^2-2=0$
We could see these two zeros graphically and were able to confirm them algebraically.	$x-3=0$ $x+1=0$ $x^2-2=0$ $x=3$ $x=-1$
This last factor is the ± 1.414 that we saw in the graph. But we needed to use long division to find the exact answer.	$x-3=0$ $x+1=0$ $x^2-2=0$ $x=3$ $x=-1$ $x^2=2$ $x=\pm\sqrt{2}$